ON THE ADDED MASS AND DAMPING OF POROUS CYLINDERS

NON HARMONIC MOTION

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At the last workshop (Molin, 1989) a method was presented to obtain the added mass and damping coefficients of porous or slotted cylinders. Subsequently to the workshop experiments carried out at ENSM wave tank confirmed the validity of the proposed theory (Molin and Legras, 1990). Nevertheless the obtained results suffer from a major deficiency : they are restricted to the case of harmonic motion. As a quadratic discharge law is applied on the cylinder wall they cannot be extrapolated to non harmonic motions. It is this problem that we consider here.

2 D POROUS CYLINDER

As in the previous paper we start with the simple case of a two-dimensional porous cylinder, undergoing forced motion with velocity $U(t)$ along the Ox axis in a quiescent perfect fluid. In the fluid domains interior and exterior to the cylinder the velocity potential can be written as :

$$\Phi^i(R,\theta,t) = \sum_{m=0}^{\infty} A_m(t)\; R_0\; (\frac{R}{R_0})^m \;\cos m\theta$$

$$\Phi^e(R,\theta,t) = \sum_{m=0}^{\infty} B_m(t)\; R_0\; (\frac{R_0}{R})^m \;\cos m\theta$$

with : $B_m(t) = -A_m(t)$ in order to ensure continuity of the radial velocity, R_0 being the cylinder radius.

Restricting ourselves to the $\cos\theta$ components we can write the radial velocity as :

$$\Phi_R|_{R=R_0} = A(t)\;\cos\theta$$

The pressure drop being :

$$p^i - p^e = -2\rho\; A'(t)\; R_0\;\cos\theta$$

so that the discharge equation takes the form :

$$-2\rho\; A'(t)\; R_0\;\cos\theta = \frac{\rho}{2\mu\tau^2}[A(t)\cos\theta - U(t)\cos\theta]\;|A(t)\cos\theta - U(t)\cos\theta|$$

(We recall that μ is a discharge coefficient and τ the porosity.)

Applying Lorenz linearization to $\cos\theta\;|\cos\theta|$ we obtain :

$$A'(t) = -\frac{2}{3\pi\mu\tau^2 R_0}[A(t) - U(t)]\;|A(t) - U(t)|$$

that is we obtain an evolution equation for $A(t)$. This equation, for a given forced velocity $U(t)$, is integrated in the time domain.

Results have first been obtained in the case of forced sinusoidal motion and compared to the analytical ones. Excellent agreement has been obtained (fig. 1) with some slight deviations which are to be linked to the Lorenz linearization in time used in the analytical approach.

Next the case of biharmonic motion has been investigated : a 1 meter amplitude 35 seconds period harmonic motion is superimposed to a 10 seconds period one, with amplitude ranging from 0 to 2 meters. This grossly models a combination of low frequency motion (at the first natural period of the ROSEAU tower) and wave frequency motion. Figure 2 shows the low frequency motion added mass and damping coefficients, as functions of the wave frequency motion amplitude.

3 D POROUS CYLINDER

We bound the fluid domain by two horizontal planes at $z = 0$ and $z = h$, with the cylinder extending vertically from $z = z_b$ to $z = z_t = h - z_b$ (see Fig. 3). Retaining only their $\cos\theta$ components we can write the interior and exterior expansions of the potential as :

$$\Phi^i(R,z,\theta,t) = \{A_0(t)\frac{R}{R_0} + \sum_{n=1}^{N} A_n(t)\ \cos k_n z\ I_1(k_n R)\}\ \cos\theta$$

$$\Phi^e(R,z,\theta,t) = \{B_0(t)\frac{R_0}{R} + \sum_{n=1}^{N} B_n(t)\ \cos k_n z\ K_1(k_n R)\}\ \cos\theta$$

with $k_n = 2n\pi/h$.

Equality of the radial velocities permits us to introduce as new unknowns the $a_n(t)$ defined by :

$$\Phi^i - \Phi^e|_{R=R_0} = \sum_{n=0}^{N} a_n(t)\ \cos k_n z\ \cos\theta$$

$$\Phi_R|_{R=R_0} = \sum_{n=0}^{N} a_n(t)\ \alpha_n\ \cos k_n z\ \cos\theta$$

with $\alpha_0 = (2R_0)^{-1}$ $\qquad \alpha_n = k_n\ [\frac{I_1(k_n R_0)}{I_1'(k_n R_0)} - \frac{K_1(k_n R_0)}{K_1'(k_n R_0)}]^{-1}$

The following set of equations has then to be verified :

$$\sum_{n=0}^{N} a_n(t)\ \cos k_n z = 0 \qquad 0 \le z \le z_b \qquad z_t \le z \le h$$

$$\sum_{n=0}^{N} a_n'(t)\ \cos k_n z = f(z,t) \qquad z_b \le z \le z_t$$

with :

$$f(z,t) = -\frac{4}{3\pi\mu\tau^2}\ [\sum_{n=0}^{N} a_n(t)\ \alpha_n\ \cos k_n z - U(t)]\ |\sum_{n=0}^{N} a_n(t)\ \alpha_n\ \cos k_n z - U(t)|$$

The $a_n(t)$ being chosen as zero at $t = 0$, we can differentiate with time both sides of the first equation. By integrations in z the system is then transformed into :

$$a_n'(t) = \frac{2-\delta_{n0}}{h} \int_{z_b}^{z_t} f(z,t)\ \cos k_n z\ dz \qquad \text{for } n = 0, 1, \ldots, N$$

Thus we obtain coupled evolution equations for the $a_n(t)$, which are solved in the time domain. Fig. 4 shows some results for the case of forced harmonic motion, as compared to the experimental ones and to previous numerical results based on Lorenz linearization in time.

The next step is to solve the motion of a porous stabilizer under irregular waves. It may seem that the present method does not apply since it is based on "infinite fluid" wave numbers. However, owing to the fact that the stabilizer is deeply submerged, we intend to combine free surface incident flow with infinite fluid diffraction and radiation flows – see for instance Ogilvie's approximate solution (1963, part 7) or Chaplin (1981) for the case of a horizontal cylinder under waves.

REFERENCES

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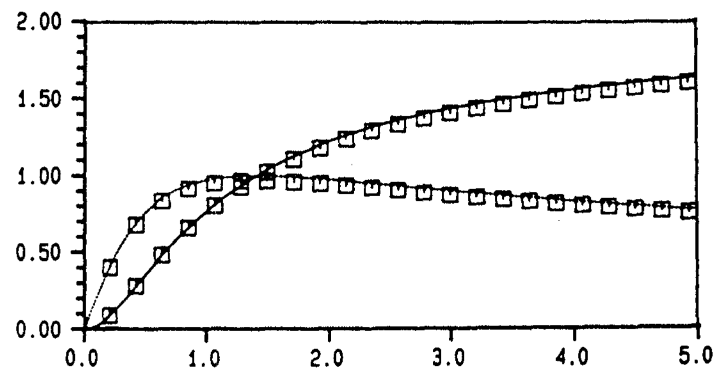

Figure 1 : Two dimensional porous cylinder - harmonic motion
Added mass and damping coefficients as functions of $(\frac{4}{3\pi})^2 \frac{1}{\mu\tau^2} \frac{a}{R_0}$
Comparison between analytical results (added mass : full line ; damping : dotted line) and numerical results based on time domain simulation.

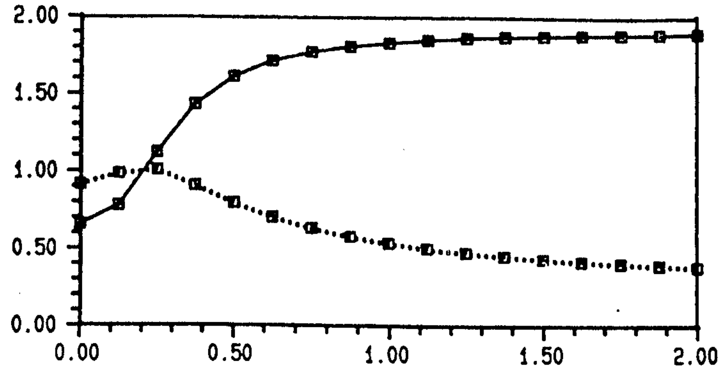

Figure 2 : Two dimensional porous cylinder - biharmonic motion
Added mass (full line) and damping (dotted line) coefficients
of the low frequency motion as functions of the amplitude
of the wave frequency motion.

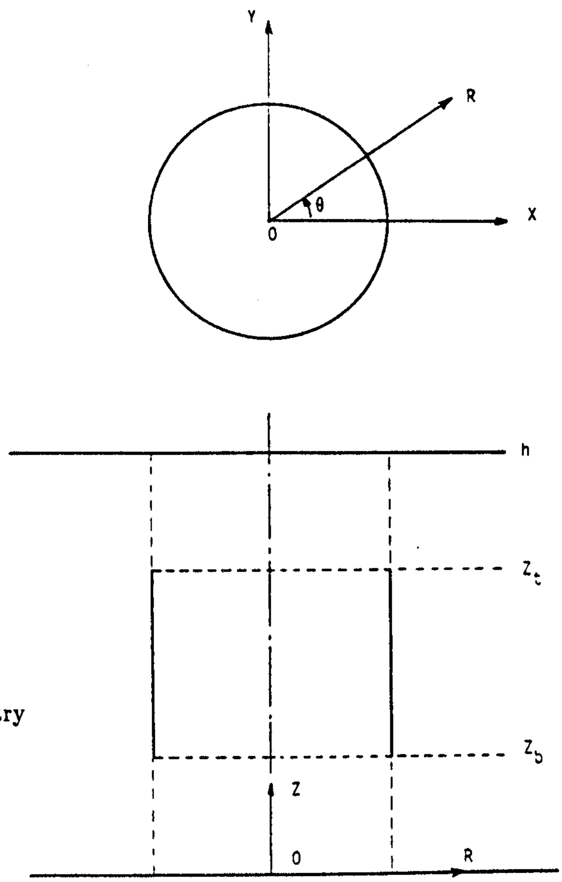

Figure 3 : Three dimensional porous cylinder : geometry

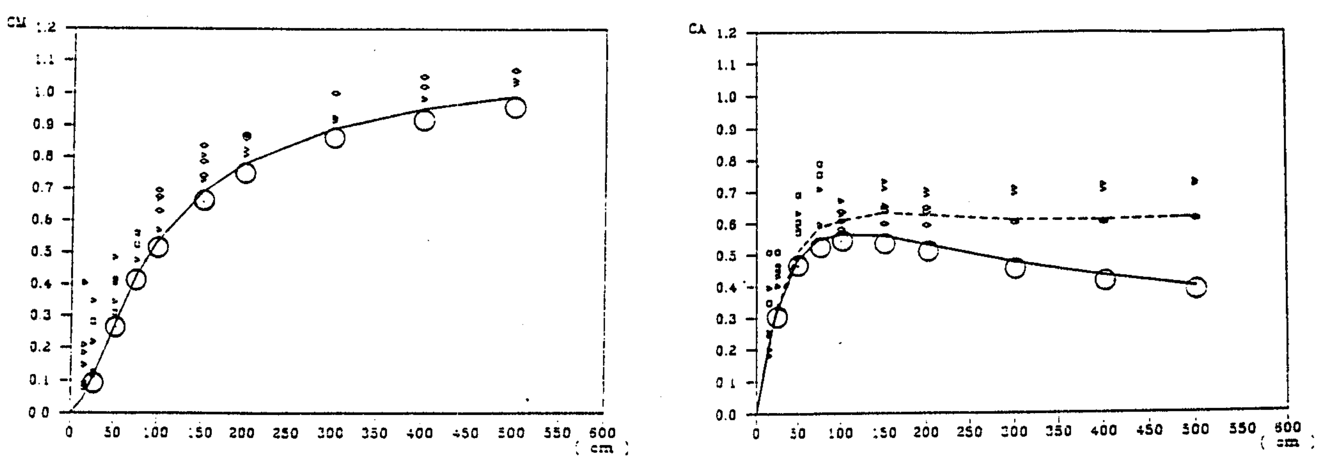

Figure 4 : Three dimensional porous cylinder - harmonic motion
Added mass (left) and damping (right) coefficients as functions of the motion amplitude :
- square, triangle and diamond symbols : experimental values
- full lines : frequency domain numerical values
- dotted line : idem with correction for drag
- circles : time domain numerical values.

DISCUSSION

Tyvand: Have you investigated the problem with Darcy's law (normal velocity proportional to the pressure drop) instead of the quadratic law?

Molin: The amplitude dependence of the added-mass and damping coefficients can only be accounted for by a nonlinear discharge law at the cylinder wall. Therefore, I did not investigate the case of a linear law.

Miloh: Is it necessary to use the Lorenz linearization for the product $|\cos\theta|\cos\theta$ instead of using the full Fourier expansion, and what is the error introduced by this approximation?

Molin: In the two-dimensional case, it is quite easy to use the full Fourier expansion; in fact, this has been done. The differences were found to be less than 5% for the added-mass and damping coefficients.