Three-Dimensional Nonlinear Wave Computation by Desingularized Boundary Integral Method

William W. Schultz, Yusong Cao and Robert F. Beck

Program in Ship Hydrodynamics
University of Michigan
Ann Arbor, Michigan 48109

We present a continuation of the work presented at the last workshop. The mixed Eulerian-Lagrangian time marching procedure is combined with the desingularized boundary integral method[1,2] to find the nonlinear waves caused by a disturbance below the free surface.

To satisfy the fully nonlinear, unsteady free surface boundary condition we use the indirect method with desingularization[1]. The desingularization distance we use is

$$L_d = l_d (D_m)^\alpha, \qquad (1)$$

where l_d and α are constant parameters and D_m is the local mesh size. Our previous calculation used $\alpha = 1$. This ensured that the desingularization distance goes to zero with mesh refinement to ensure uniqueness and completeness. We have since found that $\alpha = 1$ does not allow convergence of the numerical integration with mesh refinement. A numerical test, where an integral of a constant source distribution over a square flat surface is evaluated at a point above the center of the square with a distance given by Eq.(1) is shown in Fig. 1. This shows that the numerical integration converges only when α is less than 1. The value of α should be greater than 0 for uniqueness and completeness properties. We choose $\alpha = 0.5$ because it has the same convergence rate as $\alpha = 0$ and produces algebraic systems with relatively good conditioning.

We now use a more accurate time integration scheme (a fourth order Runge-Kutta-Fehlberg method) to find the waves caused by a source-sink pair moving below the free surface starting from rest. The method is first applied to the linear case (small disturbance, i.e. small strength of the source-sink pair). In this case, the linear wave theory is valid. The nonlinear wave cuts along the plane of the disturbance movement ($y = 0$) computed by the present method compare well with "exact" linear results[3] in Fig. 2a. The method is then applied to a stronger disturbance as shown in Fig. 2b. Mesh and time step refinement showed that the difference between the nonlinear and linear results, even for the small differences in Fig. 2a, is mainly due to the nonlinearity of the free surface condition.

The effect of the truncated free surface is also studied. Two sizes of computational domains are used. Fig. 3 shows the wave cuts along the $y = 0$ plane for the linear case. It is found that as long as the disturbance, whose horizontal location is represented by the arrow moving from left to right, does not leave the computational domain, the solutions remain relatively unaffected. Hence the truncated boundaries pose little problem without any special treatment such as upwinding[4]. However, it is impossible to keep all waves within the computational domain for large time due to computational restriction of size and cost. This difficulty can be overcome by matching the linear solution at the edges of the domain,[5] but only after significant algebraic manipulation. We included a small number of time dependent Kelvin wave potentials to better satisfy the radiation condition. As expected, the result for our simple test problem was significantly improved and the results with the small computation domain using the Kelvin potentials are nearly identical to those for the large computation domain without using Kelvin potentials. However, we found that the cost for the evaluation of the Kelvin potential is too high for long time simulations because of the time convolution integral. Whether this idea is beneficial for problems with body and steady wave calculation remains to be studied.

Nevertheless, the indirect method without special treatment on computational edges is still superior to the direct method. The wave reflections on the edges were observed immediately after the start up using the direct method. Fig. 4 shows the prospective view and contour lines of the waves by a source-sink pair disturbance moving to the left using the indirect method. The waves have achieved the steady state. Our other wave calculations, such as waves created by a heaving disturbance with forward speed and waves in water of finite depth, indicate that the present method is very promising.

This work is supported under the Program in Ship Hydrodynamics at The University of Michigan, funded by The University Research Initiative of the Office of Naval Research, Contract Number N000184-86-K-0684. Computations are made in part using a CRAY Grant, University Research and Development Program at the San Diego Supercomputer Center.

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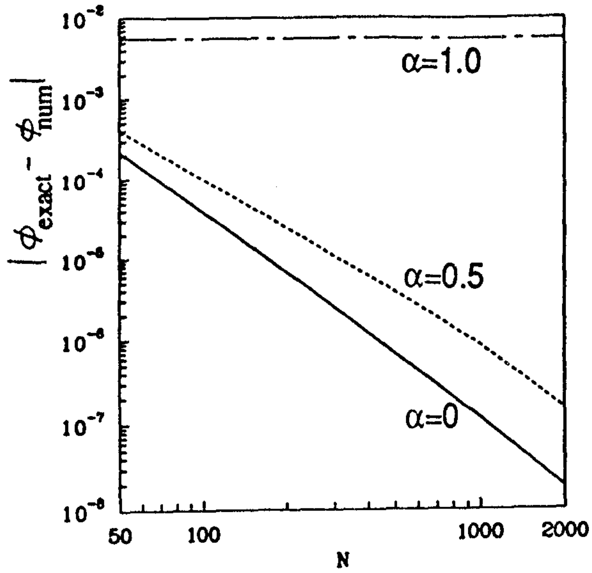


Fig.1 Convergence of numerical integration
(N: number of panels)

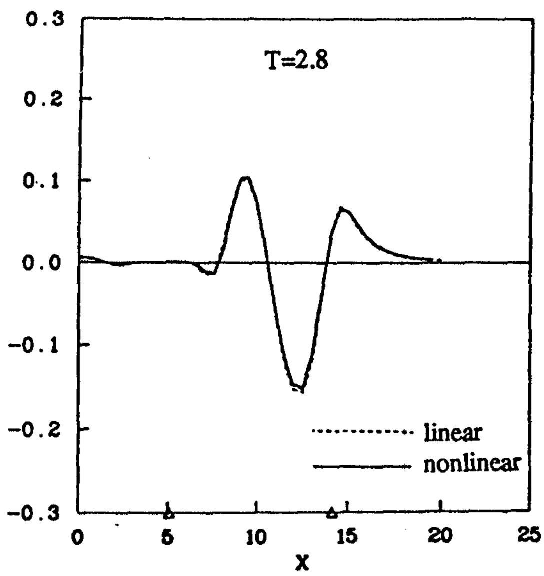


Fig. 2a Linear case

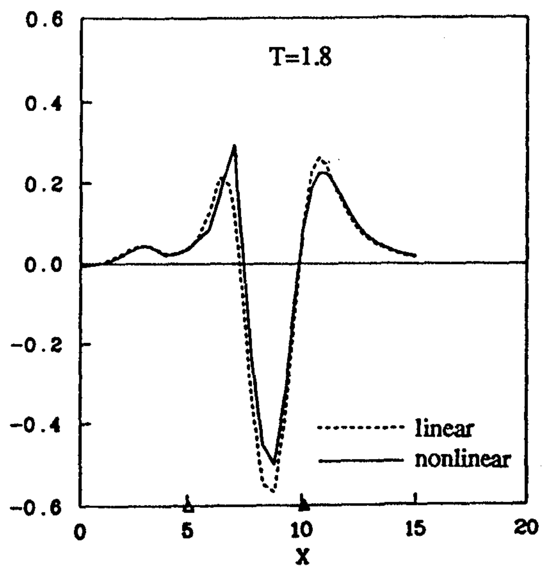
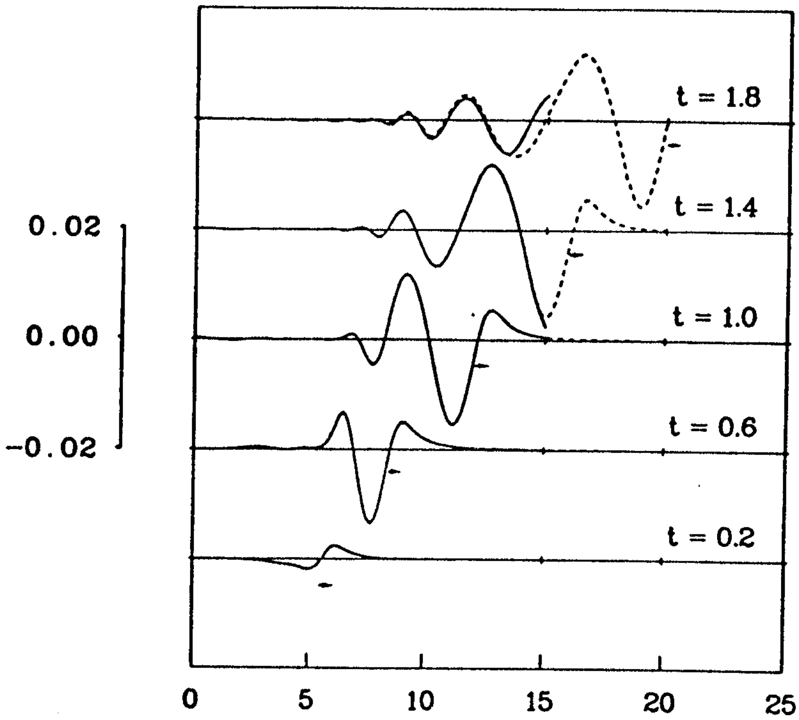


Fig. 2b Nonlinear case



- - - - - - - large computation domain ($0 < x < 20$, $0 < y < 7$)
——————- small computation domain ($0 < x < 15$, $0 < y < 7$)

Fig. 3 Effect of computation domain size

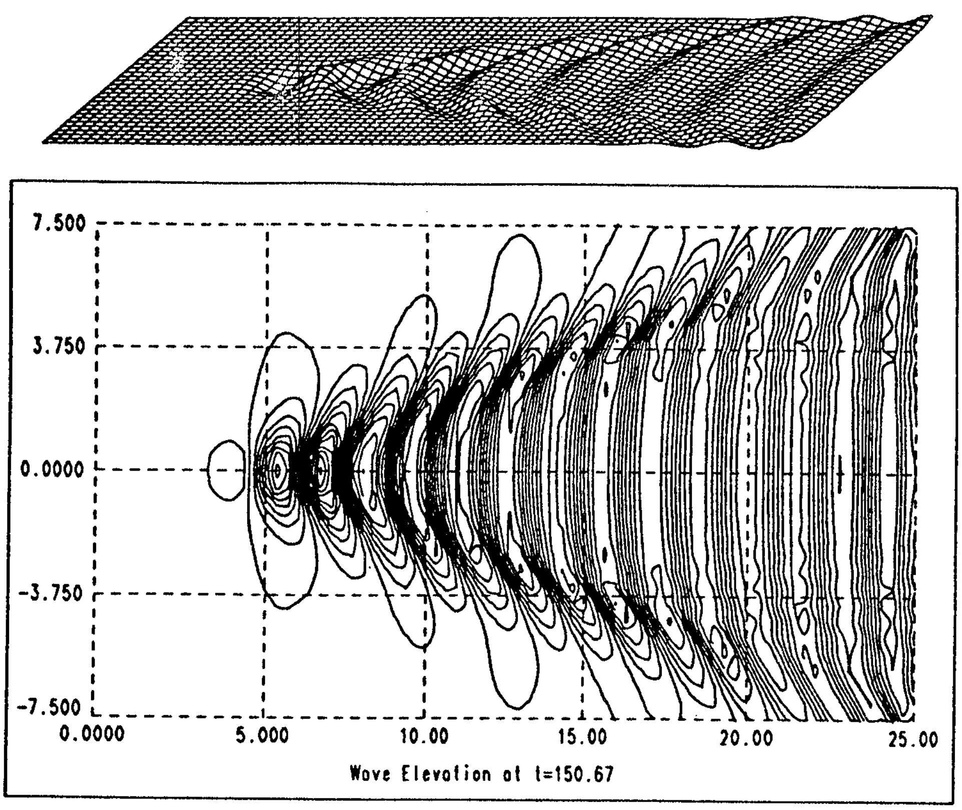


Fig. 4 Wave pattern of a moving source-sink disturbance at Froude number $Fr = V/\sqrt{gh} = 0.7$, where h is the depth of the disturbance

DISCUSSION

Bertram: (i) Desingularization works fine for 'smooth' surfaces. Near wedges (such as ship bows, etc.) it will introduce difficulties. (ii) Nagatake found that the radiation condition can be enforced by shifting, even if singularities are used on the free surface.

Schultz: (i) Our experience in 2-D with $\frac{\pi}{2}$ corners shows that desingularization does not degrade the numerical accuracy. For sharper angles it may be necessary to reduce the desingularization distance near the corner. (ii) We enforce the zero values of the elevation and the potential of the new added material points at the upstream truncation line but we do not require a radiation condition for the other truncation lines because no spatial derivatives are required on the free surface. There are many ways of 'enforcing' radiation conditions when they are required.

Chau: Does the Desingularization Boundary Integral Method also suffer from irregular frequencies?

Schultz: We have not used this method in the frequency domain, but we suspect that it would.

Eatock Taylor: You showed results associated with an oscillating dipole. This suggests an interest in bodies excited into oscillating motions by incident waves. Are you planning to use your method for such problems, and, if so, how will you impose the appropriate boundary conditions on the outer region of the computational domain?

Schultz: Eventually we plan to study seakeeping problems. We have investigated the forced oscillation of a source-sink pair in order to study our unsteady linear Neumann-Kelvin solutions that become unbounded at $\tau = 1/4$. We could, however, impose incident waves other than $\phi = \eta = 0$ at the upstream truncation line for the seakeeping problem.

Kleinman: The 'desingularized' integral equation has been used in other contexts. I believe Fairweather examined simpler boundary value problems for the Laplacian in a closed domain with Dirichlet boundary conditions, and Kress and co-workers looked at corresponding problems for the Helmholtz equation. Both found that the results improved as the fictitious surface approached the actual surface, and I think they concluded that they might just as well (or even preferably) use the non-desingularized (!) or weakly-singular equation. Have you (or anyone) used the iterative scheme GMRES on that equation and, if so, how do results compare with those from the 'desingularized' equation?

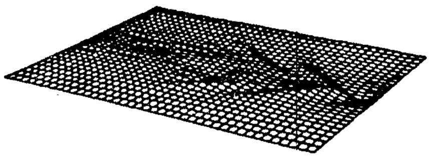
Schultz: We believe that their conclusions could be correct for the direct method (derived from Green's theorem) especially if the solution contains a corner singularity. We did some numerical tests with the direct method and arrived at a conclusion similar to their's. There is a trade off between the accuracy and the computational effort (especially the evaluation of the singular integrals). However, this is not true for the indirect method. The 'desingularized' equation can give better solutions than the singular equation; see Ref. 1, [1] and [2]. At the last workshop, we also showed that the desingularized equation gave better results for some problems with open boundaries. We have found that the condition number of the resulting system from the desingularized equation increases as the desingularization distance increases, hence it takes slightly longer for GMRES to converge. However, a larger condition number does not necessarily imply a poorer solution of the problem. When we desingularize on the order of the mesh size the condition number appears to be adequate.

Miloh: For simple 3-D shapes, like spheres, spheroids or ellipsoids, the exterior field may be continued analytically inside the bodies and there exists an 'ultimate' singularity system (the centre of the sphere, the line between the foci for the spheroid and the fundamental elliptic disc for the ellipsoid). For these particular shapes, I agree with you that the indirect method has some advantages over the direct one. The two methods may be comparable for smooth convex bodies, but for bodies with corners (a sharp keel for example) I think that the direct method renders more accurate results and should be superior to the indirect one.

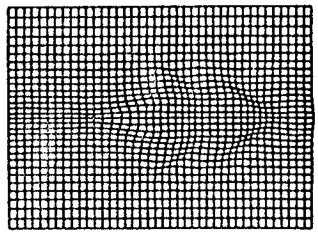
Schultz: We agree that the indirect method is most beneficial for smooth boundaries (like those in our present research). However, in that case, the two methods are not comparable — the indirect method is superior. Near corners, the desingularization distance may need to be reduced. As it $\rightarrow 0$, the direct and indirect methods have been shown to be equivalent by Brebbia & Butterfield [3].

Peregrine: Please comment on the spatial discretization grid and how much it distorts.

Schultz: The initial grid (at t=0) is equally spaced in the longitudinal direction and the spacing is increased algebraically in the transverse direction. Since we use a fixed coordinate system with a computational window that follows the disturbance, it would be difficult to have a non-uniform grid with finer spacing near the disturbance. The figures below give an example of how the grid is distorted. It is clear that the node points (material points) tend to get closer at the crests.



Perspective view



Top view

Raven: (i) Could you give an indication of the computation time needed for a steady problem? (ii) For surface piercing bodies there is a corner in the domain at the intersection of the hull and free surface. The solution is singular or at least rapidly varying there. For ordinary boundary integral methods there are indications that this may be 'discretized away' without much effect on the rest of the domain. Will not your 'indirect method' give additional problems in modelling this flow behaviour, perhaps due to the impossibility of analytic continuation up to the singularity location? (iii) I should expect $\alpha = 1.0$ to be correct from dimensional arguments. Can you explain how your results can be different from that?

Schultz: (i) For $N = 1200$, we require 2 seconds per time step on a Cray Y-MP. The computation we use here requires approximately 100 time steps to approach steady state. (ii) Again, we expect no additional difficulties over the non-desingularized methods at corners. (iii) Equation (1) is dimensionless. If it were not, the dimension of l_d would need to be adjusted with α.

Yeung: Please clarify how you close your computational domain, i.e., what do you do along the four extreme edges of the free surface? In particular, are you assuming that the velocities of the edge (material) points are unaffected by singularities beyond the computational mesh?

Schultz: There is no special treatment at the domain edges. We assume that there are no singularities beyond the computational domain, even though there should be when waves travel outside it.

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