THE INFLUENCE OF A SLOWLY OSCILLATING MOVEMENT ON THE VELOCITY POTENTIAL

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1. Introduction.

The estimation of a ship speed and power was usually based on still water performance. Assuming a potential flow the stationary problem of calculating the ship wave resistance is described by the Laplace equation and the conditions on the ship hull and the free surface. Numerous computer programs have been written to tackle this problem.
In order to be able to predict ship performance in seaway it is also desirable to be able to calculate the instationary problem of a sailing ship. For example the ship is sailing in waves or an oscillating vessel. The central problem in this paper will be the calculation of the added resistance of a ship when it is slowly oscillating. This problem will be divided into several subproblems:

- The treatment of the free surface condition.
- The treatment of the body boundary condition.
- The numerical treatment of the associated Green function.
- The numerical solution of the resultant integral equation.
- The calculation of the added resistance.

Every subproblem will be dealt with in this paper.

The research is carried out in cooperation with MARIN.

2. Mathematical formulation.

We consider an object moving horizontally with forward speed U in an infinitely extended fluid. The formulation will be done in a Cartesian coordinate system moving with the object, free surface elevation will be given by $z = \eta(x,y)$
The total velocity potential $\Phi_{total}(x,y,z,t)$ can be split in a steady and an unsteady part:

$$\Phi_{total}(x,y,z,t) = Ux + \overline{\varphi}(x,y,z) + \tilde{\varphi}(x,y,z,t) \tag{1}$$

We are especially interested in the influence of the steady part $\overline{\varphi}$ on the unsteady part $\tilde{\varphi}$.

Φ_{total} has to satisfy the following conditions:

Laplace's equation; $\Delta\Phi_{total} = 0$, and the boundary condition $\dfrac{\partial\Phi}{\partial n} = V_n$ at the hull.

At the free surface $\eta(x,y)$ we have the dynamic and kinematic boundary conditions, the fluid velocity normal to the surface has to be equal to the normal component of the velocity of the surface itself and along the free surface the pressure is a constant.

To derive an approximating solution of the problem we allow small oscillations in $\tilde{\varphi}$ and expand the free surface around $\eta = \eta_0$

Substitution of these expansions in the free surface boundary conditions leads to the next free surface condition:

$$g\left(\frac{\partial\tilde{\varphi}}{\partial z} - \frac{\partial\tilde{\varphi}}{\partial x}\frac{\partial\eta_0}{\partial x} - \frac{\partial\tilde{\varphi}}{\partial y}\frac{\partial\eta_0}{\partial y}\right) - \frac{\partial^2\overline{\varphi}}{\partial z^2}\left(\frac{\partial\tilde{\varphi}}{\partial t} + \nabla(Ux+\overline{\varphi})\cdot\nabla\tilde{\varphi}\right) + \frac{\partial^2\tilde{\varphi}}{\partial t^2} +$$
$$+ 2\nabla(Ux+\overline{\varphi})\cdot\nabla\frac{\partial\tilde{\varphi}}{\partial t} + \nabla(Ux+\overline{\varphi})\cdot\nabla(\nabla(Ux+\overline{\varphi})\cdot\nabla\tilde{\varphi}) = 0 \text{ at } z=\eta_0. \qquad (2)$$

With $\tilde{\varphi} = \hat{\varphi}\cdot e^{-i\omega t}$ this can be written in the following form:

$$-\omega^2\hat{\varphi} - 2i\omega U\hat{\varphi}_x + U^2\hat{\varphi}_{xx} + g\hat{\varphi}_z = \mathcal{L}(U;\overline{\varphi})\{\hat{\varphi}\} \quad \text{at } z=\eta_0. \qquad (3)$$

$\mathcal{L}(U;\overline{\varphi})$ denotes a linear differential operator (see also Hermans and Huijsmans[4]).

The body boundary condition for $\hat{\varphi}$ is written as:

$$\frac{\partial\hat{\varphi}}{\partial n} = -\underline{n}\cdot(i\omega\underline{\alpha} + \nabla(\underline{\alpha}\cdot\nabla(Ux + \overline{\varphi}))) \qquad (4)$$

with $\alpha(x,t) = \alpha(x)\cdot e^{-i\omega t}$

similar as the one found by Timman et al.[8]

Using Green's theorem and combining the formulation inside and outside the ship we may obtain a description of the potential function $\hat{\varphi}$ by means of a singularity distributions as for instance done by Brard [3] and Hermans [4].

3. Calculation of the source strength and potential function.

The use of an expansion of $\sigma,\hat{\varphi}$ and G in the small parameter ω and substitution of these expansions in the equations as obtained by the singularity distributions leads for the first order problem to:

$$-\frac{1}{2}\sigma_0(\underline{x}) - \iint_{\Sigma}\sigma_0(\xi)\,\frac{\partial G_0(x,\xi)}{\partial n_x}\,dS_\xi + \frac{U^2}{g}\int_{C_f}\alpha_n\,\sigma_0(\xi)\,\frac{\partial G_0(x,\xi)}{\partial n_x}\,d\eta =$$
$$-\underline{n}\cdot\nabla(\underline{\alpha}\cdot\nabla(Ux + \overline{\varphi})) + \frac{1}{g}\iint_{F.S.}\mathcal{L}_0(\hat{\varphi})\,\frac{\partial G_0(x,\xi)}{\partial n_x}\,dS_\xi \qquad (5)$$

with $\hat{\varphi}_0$ is given by:

$$\hat{\varphi}_0(x) = -\iint_{\Sigma} \sigma_0(\xi)\, G_0(x,\xi)\, dS_{\xi} + \frac{U^2}{g}\int_{C_f} \alpha_n\, \sigma_0(\xi)\, G_0(x,\xi)\, d\eta + -\frac{1}{g}\iint_{F.S} \mathcal{L}_0(\hat{\varphi})\, G_0(x,\xi)\, dS_{\xi} \qquad (6)$$

The second order problem shows resemblance with the first order. Combined terms like $\sigma_1 \cdot G_0$ and $\sigma_0 \cdot G_1$ occur in the equations.
The solution obtained by singularity distributions are two coupled integral equations. Equations (5) and (6) will be solved using an iterative scheme. In this scheme use will be made of the numerical evaluation of the wave resistance Green function as done by Newman ([6],[7]).The algorithm described by Newman has slightly been adjusted to be able to calculate the single integral outside the centerplane. This will be done with the use of Padé–approximants ([1],[2]). Results for the steady wave potential of a Wigley hull can be seen in Fig. 1. The result can be compared with the measured value of Kitazawa and Kajitani [10].

4. Calculation of the Added Resistance.

Once all the characteristic quantities are known, the pressure can be determined from Bernoulli's equation. (added) Resistance can be calculated directly by integration of first and second order pressure, or by means of conservation of momentum, derived in a similar way as done by Huijsmans [9].

5. References.

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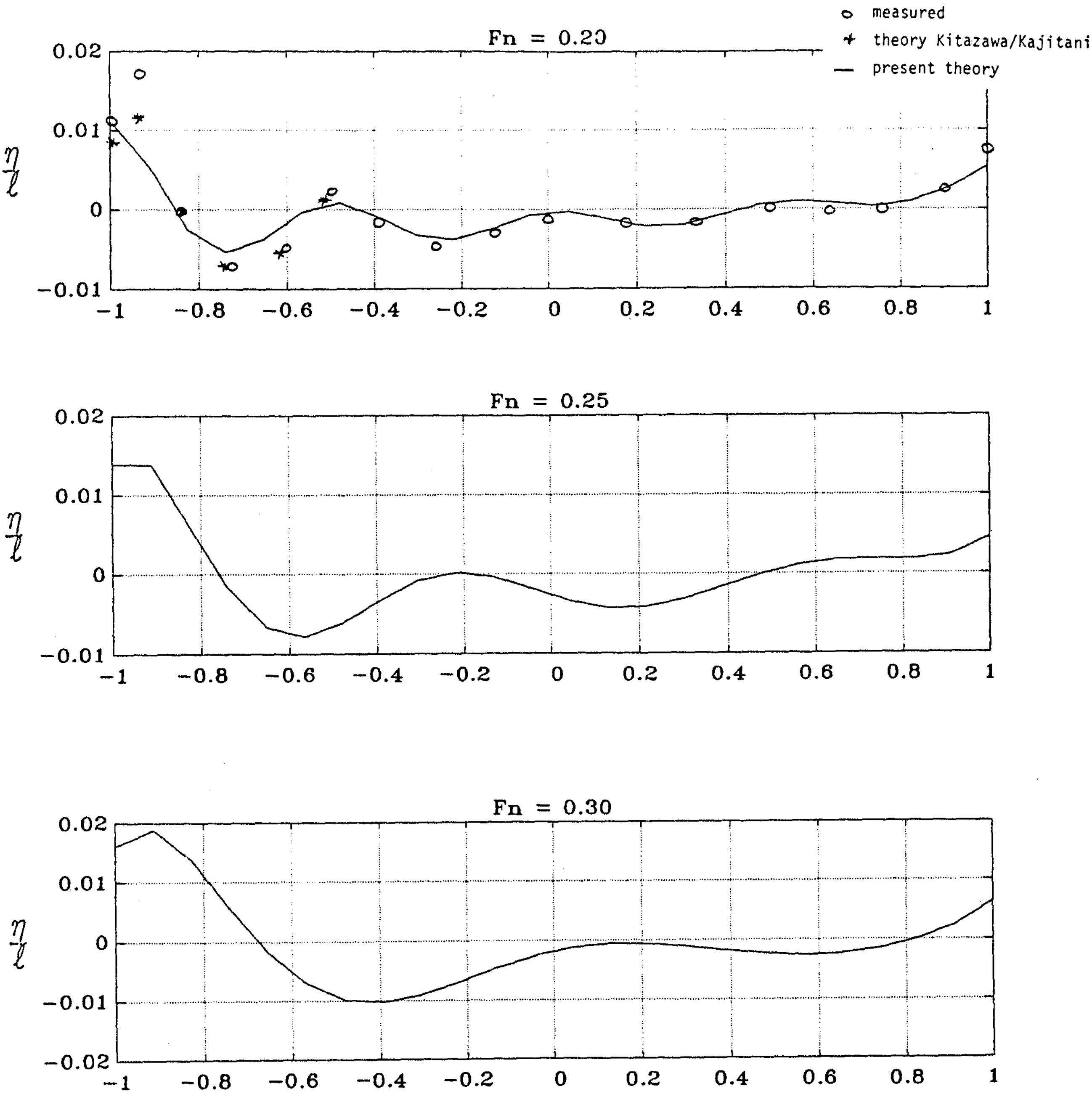

Fig. 1. Hull side wave profiles of WIGLEY

DISCUSSION

Chun: The central problem here is the influence of the steady potential on the unsteady part. What is your conclusion on this influence, in terms of its effect on the first-order forces and the second-order forces (added resistance)?

van der Stoep: So far, we only have some preliminary results for the unsteady wave. Eventually, we hope to draw some conclusions about the first-order and second-order forces.

Bertram: I think that there are two terms missing from your equation (2):

(i) Shouldn't there be a term

$$\frac{\partial\tilde{\varphi}}{\partial z}\frac{\partial\eta_0}{\partial z}$$

for the sake of completeness? (If you substitute for η_0 using a Bernoulli expression, this is important.)

(ii) Shouldn't there be a term of the form $\eta_1 f(Ux+\overline{\varphi})$ (where $\eta_0+\eta_1$ is the actual free surface) in the Taylor expansion?

van der Stoep:

(i) The free-surface elevation η_0 is a function of x and y only.

(ii) The dynamic boundary condition has been used to eliminate η_1 from the kinematic boundary condition. So, up to order ϵ^2, no term in η_1 can appear.