Hydrodynamic Forces on a Submerged Cylinder Advancing in Waves of Finite Water Depth

G.X. Wu

Department of Mechanical Engineering, University College London, Torrington Place, London WC1E 7JE, U.K.

1. INTRODUCTION

There is increasing interest recently to analyse the potential flow of a two dimensional cylinder advancing in regular waves at constant forward speed. Grue & palm (1985) obtained the solution for a submerged circular cylinder. A more general case of an elliptical cylinder was solved by Mo & Palm (1987) using a method of high order source distribution. The solution of an arbitrary non-lifting submerged cylinder was obtained by Wu & Eatock Tylor (1987) using the coupled finite element method.

In this work, we will use the coupled finite element method to analyse the problem of a cylinder advancing in regular waves of finite water depth. When there is no incoming wave, the water depth has marked effects on the wave resistance. In particular there is critical point at Froude number equal to one. When there is an incoming wave, the situation becomes more complex. The above critical point and its effects on the steady potential still exist. In addition to that, there is another critical point associated with the interaction bewteen steady and unsteady potential. In infinite water depth, this critical point is at $\tau=\omega U/g=0.25$ (U forward speed, ω encounter frequency and g gravitational acceleration). In finite water depth, the determination of this point is not straightforward. Its effect is more complicated. We shall discuss this problem by first considering the potential due to a single source.

2. GREEN FUNCTION

The Green function is defined as the potential of a unit source undergoing the same motion as the cylinder. It is essential to the coupled finite element method. Brief discussion has been given by Becker (1956). It can be written as

$$G(x,z,\xi,\zeta)=\ln(r/d) + \ln(r_2/d) + H(x,z,\xi,\zeta) \tag{1}$$

where

$$r=\sqrt{[(x-\xi)^2 + (z-\zeta)^2]} \tag{2}$$

$$r_2=\sqrt{[(x-\xi)^2 + (z+\zeta+2d)^2]} \tag{3}$$

and d is the water depth. We use the Fourier transform of lnr and write H in the following form

$$H = \int_0^\infty \{ \cosh m(z+d) \; [A(m) \, e^{-im(x-\xi)} + B(m) \, e^{im(x-\xi)}] + C(m) \} \, dm \tag{4}$$

By imposing the boundary conditions on G, we obtain

$$A(m) = \frac{-[m\upsilon+(\tau m)^2+2\tau m\upsilon+\upsilon^2]}{m\nu[m\tanh(md)-(\tau m)^2/\nu-2\tau m-\nu]} \; \frac{e^{-md}\cosh m(\zeta+d)}{\cosh m(\zeta+d)} \tag{5a}$$

$$B(m) = \frac{-[m\upsilon+(\tau m)^2-2\tau m\upsilon+\upsilon^2]}{m\nu[m\tanh(md)-(\tau m)^2/\nu+2\tau m-\nu]} \; \frac{e^{-md}\cosh m(\zeta+d)}{\cosh m(\zeta+d)} \tag{5b}$$

$$C(m) = -2e^{-md}/m \tag{5c}$$

where $\nu=\omega^2/g$.

It can be seen that A(m) and B(m) are singular when $m\nu\tanh(md)=(\tau m+\nu)^2$ and $m\nu\tanh(md)=(\tau m-\nu)^2$ respectively. The second equation always has two solutions k_3 and k_4 with $k_3>k_4$; but care is needed in the first equation. We write it as

$$\sigma = Um+\omega = \sqrt{[mg\tanh(md)]} \tag{6a}$$

where σ is in fact the frequency in the coordinate system fixed in space. It is apparent that for a sufficiently large U there is no solution. When U decreases there will be one solution at which the derivatives of both sides of equation (6a) is identical, or

$$U = C_g = \frac{\sigma}{2m} \left[1 + \frac{2md}{\sinh(2md)}\right] \tag{6b}$$

where $G_g=d\sigma/dm$ is the group velocity in the fixed system. When U further decreases there will be two solutions k_1 and k_2 with $k_1>k_2$.

Invoking the radiation condition, we can write the Green function as

$$
\begin{aligned}
G = & \ln(r/d) + \ln(r_2/d) \\
& + 2 \int_0^\infty \frac{e^{-md}}{m} \left\{ \frac{\cosh m(\zeta+d) \cosh m(z+d)}{\cosh md} \cos m(x-\xi) - 1 \right\} dm \\
& + pv \int_0^\infty \frac{\nu[1+\tanh(md)]}{(\tau m)^2 + 2\tau\nu m - m\nu\tanh(md) + \nu^2} \frac{e^{-md}\cosh m(\zeta+d)}{\cosh(md)} \cosh m(z+d) \\
& \quad e^{-im(x-\xi)} dm \\
& + pv \int_0^\infty \frac{\nu[1+\tanh(md)]}{(\tau m)^2 - 2\tau\nu m - m\nu\tanh(md) + \nu^2} \frac{e^{-md}\cosh m(\zeta+d)}{\cosh(md)} \cosh m(z+d) \\
& \quad e^{im(x-\xi)} dm \\
& + \pi i \frac{\nu[1+\tanh(k_1 d)]}{2\tau^2 k_1 + 2\tau\nu - \nu\tanh(k_1 d) - k_1 \nu d \operatorname{sech}^2(k_1 d)} \frac{e^{-k_1 d}\cosh k_1(\zeta+d)}{\cosh(k_1 d)} \\
& \quad \cosh k_1(z+d) e^{-ik_1(x-\xi)} \\
& - \pi i \frac{\nu[1+\tanh(k_2 d)]}{2\tau^2 k_2 + 2\tau\nu - \nu\tanh(k_2 d) - k_2 \nu d \operatorname{sech}^2(k_2 d)} \frac{e^{-k_2 d}\cosh k_2(\zeta+d)}{\cosh(k_2 d)} \\
& \quad \cosh k_2(z+d) e^{-ik_2(x-\xi)} \\
& - \pi i \frac{\nu[1+\tanh(k_3 d)]}{2\tau^2 k_3 - 2\tau\nu - \nu\tanh(k_3 d) - k_3 \nu d \operatorname{sech}^2(k_3 d)} \frac{e^{-k_3 d}\cosh k_3(\zeta+d)}{\cosh(k_3 d)} \\
& \quad \cosh k_3(z+d) e^{ik_3(x-\xi)} \\
& - \pi i \frac{\nu[1+\tanh(k_4 d)]}{2\tau^2 k_4 - 2\tau\nu - \nu\tanh(k_4 d) - k_4 \nu d \operatorname{sech}^2(k_4 d)} \frac{e^{-k_4 d}\cosh k_4(\zeta+d)}{\cosh(k_4 d)} \\
& \quad \cosh k_4(z+d) e^{ik_4(x-\xi)}
\end{aligned} \tag{7}
$$

When there is no solution from equation (6a) the terms of k_1 and k_2 should be deleted from equation (7). The critical point is defined as that at which both equations (6a) and (6b) are satisfied or forward speed equal to the group velocity. Unlike in the infinite water depth where the critical point only depends on τ, it not only depends on U and ω separately here but also the water depth d.

Even though it is not possible to give an explicit equation for the critical point, there are some sufficient conditions for supercritical flow. We may write equation (6a) as

$$\frac{U}{\sqrt{(gd)}} + \frac{\omega}{m\sqrt{(gd)}} = \sqrt{[\tanh(md)/(md)]} \tag{8}$$

It is apparent when $Fn=U/\sqrt{(gd)}>1$ there is no solution in this equation. Thus we conclude the steady supercritical flow must be accompanied by the unsteady supercritical flow.

We may also write equation (6a) as

$$[U\sqrt{(m/g)}+\omega/\sqrt{(gm)}]^2 = \tanh(md) \tag{9}$$

We notice that the left hand side of this equation has a minimum equal to 4τ at $m=\omega/U$. It is apparent that the minimum at least must not be larger than one (or $4\tau<1$) if the equation has a solution. Thus we conclude that if the flow is supercritical in the infinite water depth it is also supercritical in finite water depth.

Look equation (6a) more closely. If m_i $(i=1,...,4)$ are wave numbers corresponding to water depth d_1 and k_i $(i=1,...,4)$ to d_2 with $d_1>d_2$, we have

$$m_1>k_1>k_2>m_2 \;, \qquad m_3>k_3>k_4>m_4 \tag{10}$$

Thus if m_1 and m_2 do not exist, k_1 and k_2 do not exist either. This generalizes the conclusion from equation (9).

Various mathematical identities will be presented in the workshop. Results will also be provided to show the effect of the water dpeth on the hydrodynamic coeffcients and exciting forces on various cylinders.

References

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DISCUSSION

Palm: For small U, you obtain two critical values of $\tau \equiv \omega U/g$. That means that two waves of different wavenumbers are propagating upstream with the same group velocity, U. Could you show this by considering the group velocity (for $U = 0$) as a function of the wavenumber k?

Wu: I should have explained clearly in my abstract that, at low forward speed, there are k_1 and k_2 waves as well as k_3 and k_4 waves, but the first pair are not critical points.