WAVE GROUPS AND SLOW MOTION OF AN OCEAN STRUCTURE

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1. INTRODUCTION

Consider a two-dimensional ocean structure exposed to an irregular sea. The low frequency sway equation can be written in the form

$$(m+m_a).\ddot{x} + c(\dot{x}).\dot{x} + k(x).x= D(t) \tag{1}$$

The purpose of the paper is to indicate how simple expressions for the hydrodynamic coefficients m_a, $c(\dot{x})$ and $D(t)$ can be obtained from well known concepts about "wave groups". In this way, one considers the realization of an irregular wave with duration τ, of order 15 minutes for slow motion simulation, given by the series:

$$\eta(x,t) = \sum_{j=1}^{n_c} A_j \cos(k_j x-\omega_j t+\gamma_j) \; ; \quad \omega_j=j.\frac{2\pi}{\tau} \tag{2}$$

If $\zeta(x,t)$ represents an analogous time series, with phase lag $\pi/2$, the "wave envelope" $a(0,t)$ and the "local frequency" $\omega_L(0,t)$ can be defined by the expressions:

$$a(0,t) = (\eta^2(0,t) + \zeta^2(0,t))^{1/2} ,$$
$$\omega_L(0,t) = (\zeta(0,t).\eta_t(0,t)-\zeta_t(0,t).\eta(0,t))/a^2(0,t) . \tag{3}$$

Defining m_r the r-moment of the related energy spectrum, the variation in time and space of $(a(x,t); \omega_L(x,t))$ can be gauged by $\nu=((m_2 m_0/m_1^2)-1)^{1/2}$. The upper bound n_c of the finite summ (2) is related with the assumed cut-off frequency and ν is a function of n_c. In JONSWAP spectrum, with γ=3.0, the error in energy is about 5% when the cut-off frequency is twice the peak frequency. In this case $\nu\cong 0.225$

and so both $a(x,t)$ and $\omega_L(x,t)$ are slowly varying in time and space. This fact can be used to derive an asymptotic theory, correct with error factor of the form $[1+0(\nu^2)]$. Notice that $\nu^2 \simeq 5\%$, compatible with the energy error.

2. ADDED MASS AND EXCITING FORCE

With a relative error $0(\nu^2)$ one can assume a rigid flat plate at free-surface. If $\psi_2(x,z)$ is the related sway potential, the added mass is given by:

$$m_a = \int_{\partial B} \psi_2 \, \eta_x \, d\,\partial B \tag{4)a}$$

The exciting force can be written in the form $D(t)=D_1(t)+D_2(t)$, where the first term is due to the quadratic effect of the linear potential and $D_2(t)$ is the effect of the second order potential. From conservation of "wave group" momentuum one obtains, with error $0(\nu^2)$,

$$D_1(t)= \frac{1}{2}\,\rho g a^2(0,t)\,|R(\omega_L)|^2 + \frac{1}{2}\,\rho g a(0,t)\ .\ \frac{\partial a}{\partial t}(0,t)\ .$$

$$.\,[1-2|R(\omega_L)|^2]\,.\,\frac{d\gamma}{d\omega}(\omega_L)\ , \tag{4)b}$$

where $R(\omega)=|R(\omega)|\,\exp(i\,\gamma(\omega))$ is the total reflection coefficient. The first parcel of (4)b is equal to Marthinsen's expression (1983) and it is consistent with Newman's approximation (1974); the second parcel corrects Marthinsen's expression with an order ν effect.

Let $\{N_{20}(x;\omega);\ B_{20}(x,z;\omega)\}$ be the free-surface and body zero frequency exciting terms of the second order potential due to an harmonic wave with frequency ω. They depend only on the linear solution and following Aranha & Pesce (1986) one can writes, with an overall error $0(\nu^2)$, that

$$D_2(t) = \frac{1}{2}\rho g a(0,t) \cdot \frac{\partial a}{\partial t}(0,t) \cdot Q_{20}(\omega_L) ,$$

$$Q_{20}(\omega) = -\frac{1}{g}\left[\int_F N_{20}(x;\omega).\psi_2(x,0)dx + \right. \tag{4)c$$

$$\left. + \int_{\partial B} B_{20}(x,z;\omega).\psi_2(x,0).d\partial B\right],$$

where $\psi_2(x,z)$ is the sway potential at zero frequency; see (4)a.

3. DAMPING COEFFICIENT

Writing $c(\dot{x})=c_1(\dot{x})+c_2$, the first parcel is due to viscous dissipation and it has the standard expression,

$$c_1(\dot{x}) = \frac{1}{4}\rho\, C_D\, B\, |\dot{x}| , \tag{5)a$$

where B is the typical dimension of the body and C_D is the drag coefficient of the double-body immersed in infinite fluid. The second parcel is due to "wave damping" and using the "dynamical equation" of the wave-group theory (conservation of "wave action") one obtains:

$$c_2 = \frac{1}{2}\rho g a^2(0,t) \cdot \frac{\omega_L}{g}\left[4 + \omega_L \cdot \frac{d|R|^2}{d\omega}\bigg|_{\omega=\omega_L}\right] \tag{5)b$$

The above expression is exact within the context of the accepted theory, namely: potential flow correct to leading order in the drift velocity and to second order in wave amplitude; see Aranha (1990).

Besides the simplicity, all force coefficients (4), (5) are variational, with exception to (5)a. This makes even simpler theirs computations.

4. REFERENCES

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Grue: Your results compare well with results by Faltinsen (1988) where the wave drift force at $U = 0$ is roughly an increasing function of wave frequency. However, there also exist examples where the wave drift force at $U = 0$ is a decreasing function of wave frequency, and furthermore, examples when the wave drift damping for 2D examples may become negative. (See Grue, 1988, Applied Ocean. Res.) What will your theory say to these examples?

Aranha: The only result that I was aware of while doing this work was Faltinsen's. For this reason, I used his result to compare with my wave drift damping formula. From the general expression we can deduce, by derivation with respect to U, the damping coefficient. This coefficient can indeed become negative.

Tulin: Are the ocean data from Rice's paper showing separation of wave groups typical of deep ocean waves?

Aranha: The data shown was taken from Rice's Thesis (1982) and I am unable to say whether it is typical or not. However, I can say that modulation of the wave envelope does seem to be typical of actual sea surface time series, though it is somewhat obscurred in the time series simulation, which is based on a variance spectrum and random phase.

Faltinsen: Your expressions for drift forces are based on conservation of momentum and energy according to potential theory. In special cases where roll resonance occurs, it is well known that roll amplitude is influenced by viscous effects. The roll motion will affect the drift forces. How would you propose to deal with this special problem?

Aranha: The formula is based on potential flow. However, if we assume that viscosity is important only in the vicinity of the body, and that the far field waves, although influenced by energy dissipation on body surface, propagate without any further dissipation, then the formula can be extended to cover this situation.