THE DIFFRACTION OF A SOLITARY WAVE BY A FREE-SURFACE PIERCING CYLINDER

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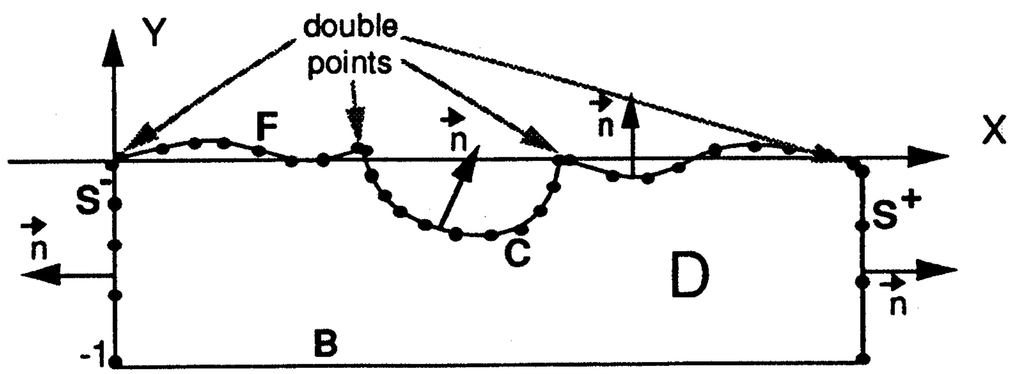
We report herein on numerical simulations of the nonlinear interaction between a solitary wave and a fixed free-surface piercing body in a bidimensionnal *numerical wave basin*. This kind of interaction has already been studied in the case of fixed obstacles lying on the sea bed by different approaches. The case of a triangular obstacle was treated by Seabra-Santos & al. (1987) in the general frame of shallow water theory; the diffraction by a semicircular bump was modelized more recently by Cooker & al. (1990) with a scheme deduced from Dold & Peregrine (1986). Some of the new physical phenomena they described, especially the backward breaking of the secondary crest, were also observed in the case of a solitary wave passing over a fixed circular cylinder by Clement (1990) using the present numerical method. We are interested here by the interaction with free-surface piercing bodies for which the flow evolution will be obviously different. As in the case of completely submerged bodies, some new unexpected features of the flow history have been highlighted by solving the complete nonlinear problem.

The mathematical formulation we have developped belongs to the family of M.E.L methods derived from the pioneer work of Longuet-Higgins & Cokelet (1976). The main features of our computing code are first briefly exposed. Then, some results obtained with a fixed cylinder of 0.4h total draught are shown. Some other cases with different values of the draught parameter will be presented at the conference and in the revised abstract.

Mathematical formulation - Numerical resolution



Definition sketch

The fluid is supposed incompressible, inviscid and at rest at t=0; the fluid particles velocity then derives from a scalar potential Φ which satisfies Laplace equation in the time-varying fluid domain D at every time t>0 . The third Green's identity then leads to a Fredholm integral equation which is solved by a standard boundary elements method. The unknowns of the mathematical problem (*i.e* the velocity

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potential on the solid surfaces and its normal derivatives on the free surface) are represented by Rankine sources and normal dipoles distributions over the moving boundaries.
The main features of the Laplace solver are the following:

- symetry about the sea bottom to save memory allocation
- discretization of the boundaries into linear segments
- approximation of Φ and $\partial\Phi/\partial n$ by piecewise linear functions
- double points at the interface free-surface/solid surfaces ; at these points the continuity of the potential is explicitly imposed ; hence, the normal velocity on the free-surface remains the only unknown at the corner (see e.g Grilli & al. 1989).
- collocation points are located at the vertices of the polygon, including the double points

Following the free-surface particles in a lagrangian manner, the kinematic and dynamic conditions at the free surface reduce to the usual set of ordinary differential equations; they are integrated by a 4th order Runge-Kutta method. This method was preferred here for its time step dynamic adaptation ability.
During the time-stepping process, we proceed to the following operations:

- the tangential velocities on the domain boundaries are computed by a weighted-arctangent method due to Halsey (1977).
- the solid surfaces (the vertical walls and the body) are regridded every time step in order to give the last segment on these surfaces the same length as the adjacent segment on the free-surface. As a matter of fact, after a thorough parametric study, we have found that this aspect ratio (≈1) of the meshes surrounding the corners minimizes the relative resolution error of our Laplace solver.

Neither smoothing nor regridding of the free surface were found necessary for we did not observe any instability.

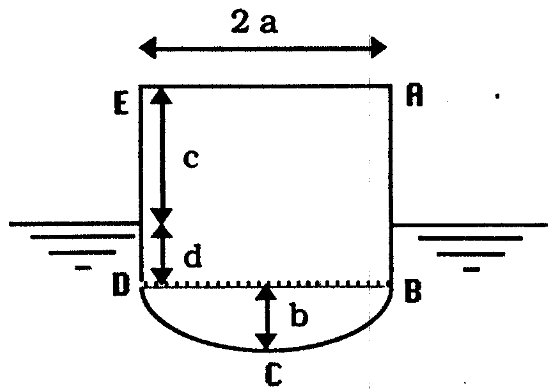
Diffraction of a solitary wave by a fixed cylinder

The solitary wave is generated by an appropriate motion of the left vertical surface S^- acting as a wavemaker. The velocity law U(t) of this translation motion of the paddle is computed from the solitary wave solution of the KdV equation (see Seabra-Santos & al. 1987):

$$U(t) = \frac{C\,\eta(t)}{1+\eta(t)} \qquad \eta = A\,\mathrm{sech}^2\left[\sqrt{\frac{3A}{4(1+A)}}\,(X + X_0 - Ct)\right] \qquad C = \sqrt{(1+A)}$$

The fixed parameters for the reported example are as follow:

- solitary wave amplitude	A=0.25h
- spatial wave shift	X0=5.h
- half-breadth	a=0.4h
- elliptical bottom draught	b=0.2h
- total draught= c+d	b+d=0.4h
- free-board	c=0.7h
- overall segments number	N=260
- time-step	0.2<δt<0.001



the free-surface piercing cylinder

The figure 1 below shows a waterfall view of the diffraction process. The time is growing from bottom to top; the soliton propagates from left to right; the vertical to horizontal scale ratio is amplified.
In the first phase of the sequence, the phenomenon looks like the reflection of a solitary wave approaching a wall: the wave grows on the left side of the body up to a maximum run-up value of 0.425h (i.e 1.7 times the initial soliton amplitude).

Then, the wave system splits into a transmitted and a reflected part. The transmitted wave consists in a solitary wave of lower amplitude than the initial one. The question of deciding if a time shift between the incident and the transmitted crest lines in the (X,t) plane exists or not may not be definitely answered. In the case reported here, this shift is practically nul.

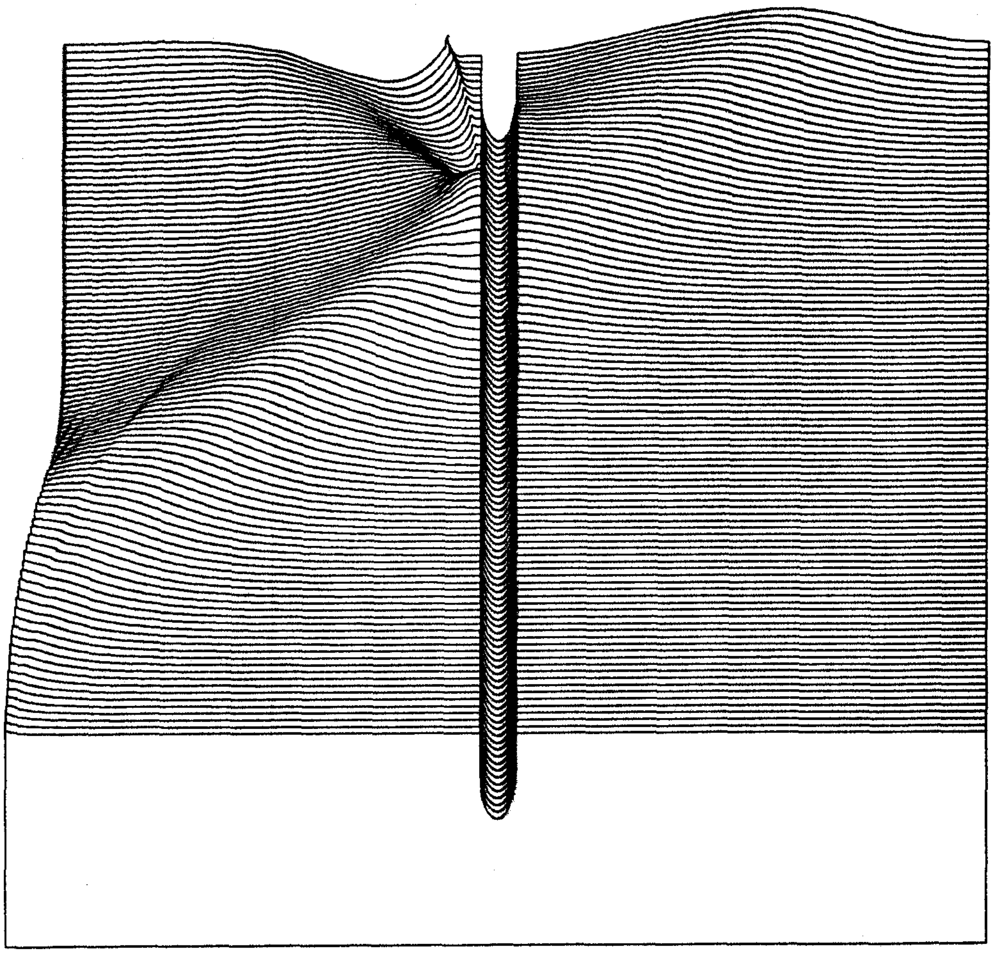


Figure 1: Waterfall view of the interaction sequence

On the weather side of the body, the behaviour of the wave system is completely different from what was expected. Instead of a positive free-surface deformation, a dispersive depression appears in the vicinity of the cylinder wall, and propagates upstream while broadening. At the downstream edge of this depression, a very short wavelenth protuberance sharpens and finally expels a backward jet. In the same time, the free-surface between this sharp wavelet and the cylinder remains practically at rest and at its equilibrium level! Finally, the backward breaking of the wavelet stops the numerical simulation.
At the moment, we are not able to propose any explanation of this unexpected behaviour; first of all, we need an experimental validation of it. The phenomenon is probably very sensitive to the draught of the body, (or to the draught/amplitude ratio ?).

On the figure 2 we have plotted the horizontal and vertical component of the forces experienced by the body during the simulation. The body being held motionless, the forces are computed at the end of the run from the time history of the flow which is stored on permanent data files. We can notice that the maximum value of the horizontal component is larger than the hydrostatic vertical component, and negative (*i.e* in a direction opposite to the direction of the soliton propagation). This constitutes another unexpected feature of our results.

Finally, the power of the MEL method is one more time clearly demonstrated by this new application. The new phenomena we have highlighted will be precisely investigated in a further parametric study using the present numerical approach.

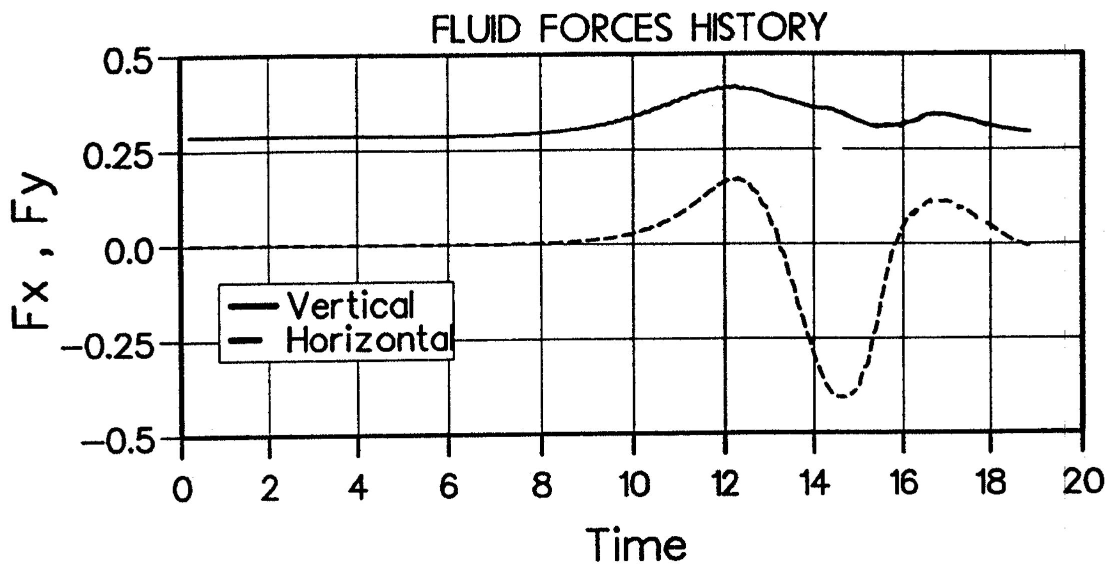


Figure 2

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Wehausen: The definition of a solitary wave depends upon the particular shallow-water approximation with which one starts, but no particular choice can provide a steady wave for Laplace's equation, except approximately. On the other hand, I don't believe that any of the standard shallow-water equations could predict the behavior that your calculations have disclosed. An exception might be a higher-level Green-Naghdi equation.

Clement: First of all I thank you for your comment and for bringing this last equation to my attention. Nevertheless, the major advantage of this numerical method is that it is free from any of the assumptions leading to the shallow-water formulations of the problem. So, it is not surprising to find a flow behavior unpredictable by these theoretical models.

Grue: Are you claiming that there are no reflected solitons, regardless of the height of the gap between the bottom and the body? Obviously, there is total reflection when the body extends down to the bottom!

Clement: The interaction must reduce to a total reflection of the incident soliton when the gap height goes to zero. We intend to perform a series of simulations with gradually increasing values of the parameter, g, while varying the amplitude parameter, a, as well. After this systematic study, we will be able to locate in the (a, g) plane the region where the reflection has the present feature. We are now awaiting a more powerful computer to proceed to these massive computations, since one run takes approximately ten hours of CPU time on our present scalar machine.

Tulin: It can be imagined that the jet which forms at reflection represents a scale of the motion much smaller than the long wave scale of the soliton. Therefore the jet may be considered as an inner flow driven by the soliton outer flow. Have you considered this as a possible means of explaining the formation of the jet?

Clement: You are probably right that the scale of the jet is different from that of the soliton; but the model we are using is entirely numerical and thus prevents us from separating the two phenomena, or studying their co-influence.