(For the 6th International Workshop on Water Waves and Floating Bodies, April 1991)

Free Surface Flow Past a Cylinder

by
S.L. Cole and T.D. Strayer
Rensselaer Polytechnic Institute
Mathematical Sciences Department
Troy, NY 12180-3590

This paper uses a three part approach of experimentation, theory and numerical computation to study the flow past a semi-submerged circular cylinder. It is hoped that the qualitative features of this simple model will shed some insignt into the more complicated problem of the bow wave flow in front of a blunt ship.

Observations

Experiments were conducted by towing an 11 inch cylinder in an 80 ft. by 4 ft. by 4 ft. tank. A capacitance type wave gauge located in the middle of the tank recorded the wave height as the cylinder was towed toward it. A typical wave profile is plotted in Figure 1. Note that since the wave gauge remains fixed as the cylinder is towed toward it, the wave profile does not represent the free surface height at a fixed time; measurements recorded at larger distances from the cylinder were taken earlier than those at shorter distances. Note also that the vertical scale is greatly stretched. All lengths have been nondimensionalized using the radius of the cylinder.

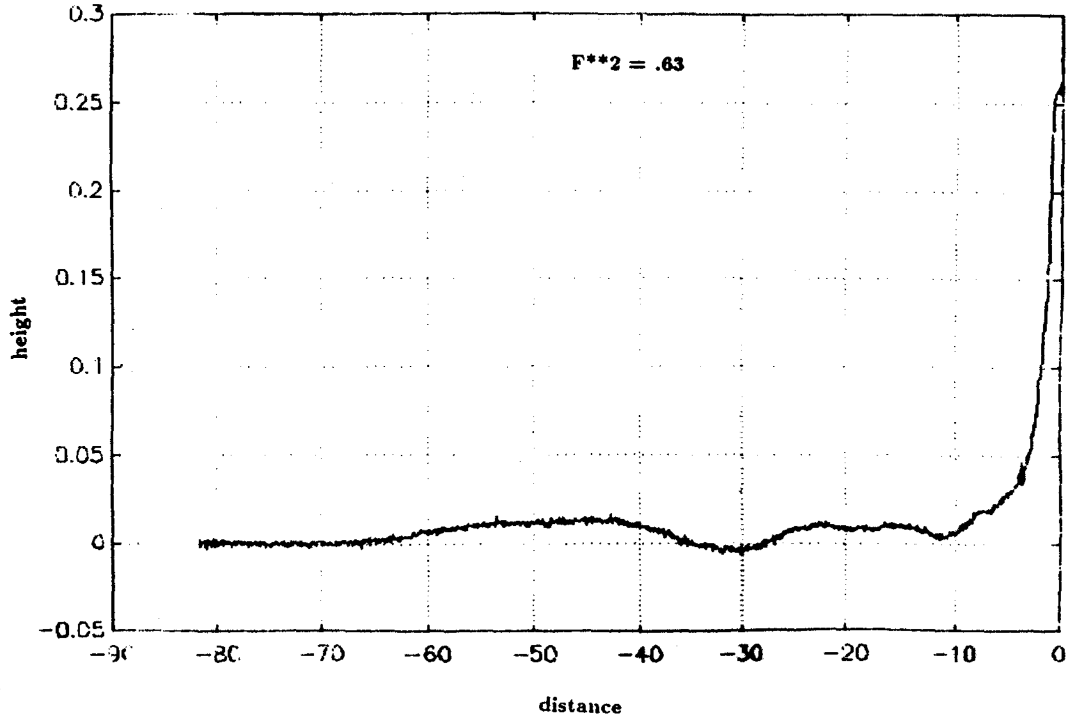

Figure 1: The free surface height in front of the cylinder.

Waves are shed from the cylinder giving an unsteady profile even for large times. The qualitative flow features, however, remain the same from run to run. In all runs, the leading wave runs out at the speed $\sqrt{gh}$ where h is the undisturbed free surface height. This corresponds to the speed of an infinitely long linear wave produced by a line of pressure points impulsively set into motion. There is a sharp rise in the free surface beginning approximately 2 to 3 radii out from the cylinder and running up to a relatively flat plateau region which extends from about 1 radius out to the cylinder. Just upstream of the sharp rise is another relatively flat region preceeded by a wavy region. The frequency of the waves between the leading surge and the sharp rise increases with decreasing Froude number.

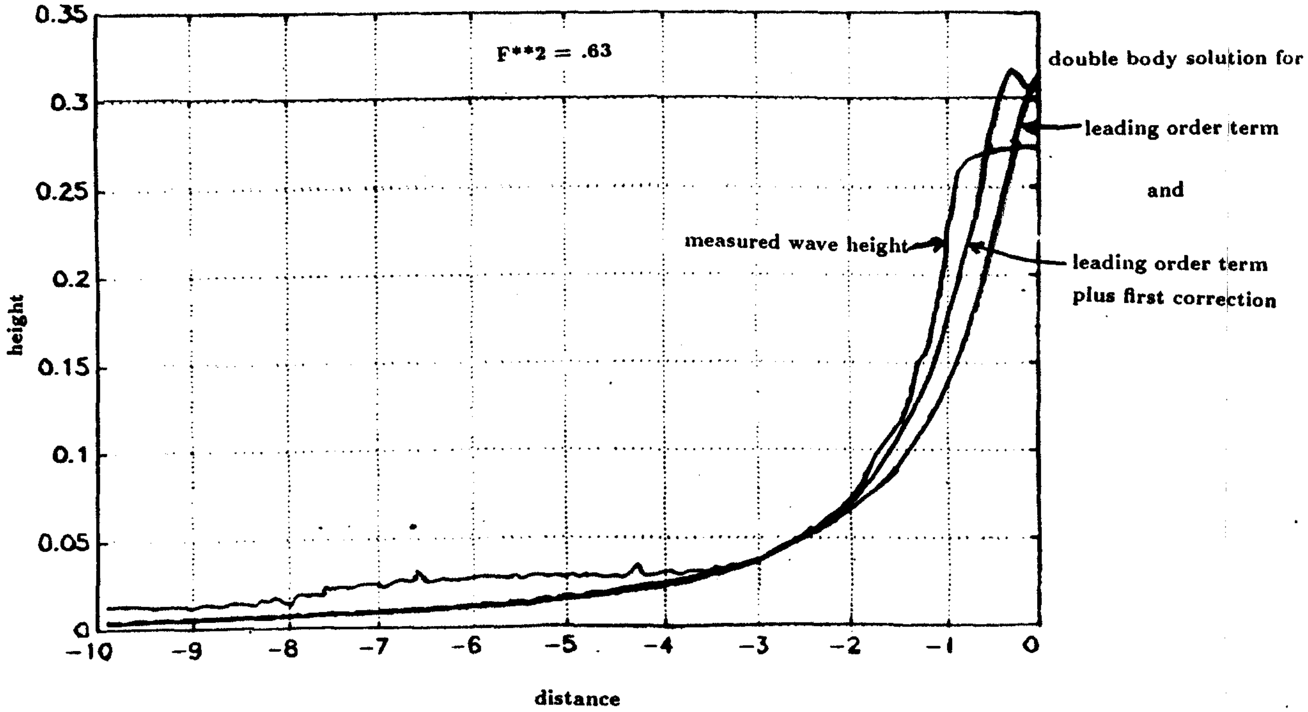

Figure 2: The measured free surface height and computed double body height in front of the cylinder.

Figure 2 shows a plot of the measured wave height for the same run as in Figure 1 plotted out to a distance of 10 radii upstream overlayed on a plot of the double body solution. This shows that the measured height is above the double body height from 4 radii to 10 radii out. The beginning of the sharp rise at 2 to 3 radii out for both the experimental measurements and the double body calculations are very similar in form though the measured free surface rises more sharply. The calculated double body height is higher than the measured height close to the bow.

A stagnant layer of fluid residues along the free surface beginning at the plateau. Because the experimental Reynold's numbers are quite large the interaction of the potential flow with the stagnant layer causes the potential flow to become unstable and separate almost immediately after passing under the stagnant layer. A nearly triangular shaped viscous region is enclosed bounded by the free surface, the cylinder and the separated potential flow.

Interpretations

It is believed by the authors that the sharp rise in the experimental free surface profile is due to its similarity to the double body flow and not to flow separation as previously suggested since underneath the free surface, the flow is found to be nearly potential from far upstream through the region of the sharp rise until it reaches the plateau near the top of the experimental wave profile. The mean experimental flow before the plateau appears to be very similar to double body flow past a cylinder with a triangular extension in front. This extension accounts for the displacement upstream of the sharp rise. The free surface in the plateau region is lower than that predicted by stagnation point flow as one would expect due to viscous losses in the triangular region beneath it.

A vortical flow exists inside the nearly triangular shaped viscous region. It is postulated that the vortical flow is driven by the separated potential flow and not the free surface breaking waves as previously conjectured. This hypothesis is confirmed by modeling the flow inside the triangular region as a type of driven cavity flow with the velocity constant along the boundary adjacent to the separated flow and no flow along the cylinder or the free surface. Such a calculation can be expected to be reasonable only for low Reynold's number flows. The computed flow for a Reynold's number of 500 is given in Figure 3. Smaller vortices exist near the corners of this flow. The experimental flow for significantly higher Reynold's numbers is three-dimensional and exhibits breaking waves. It is expected that a more sophisticated driven cavity type calculation that includes a free surface would also exhibit this effect.

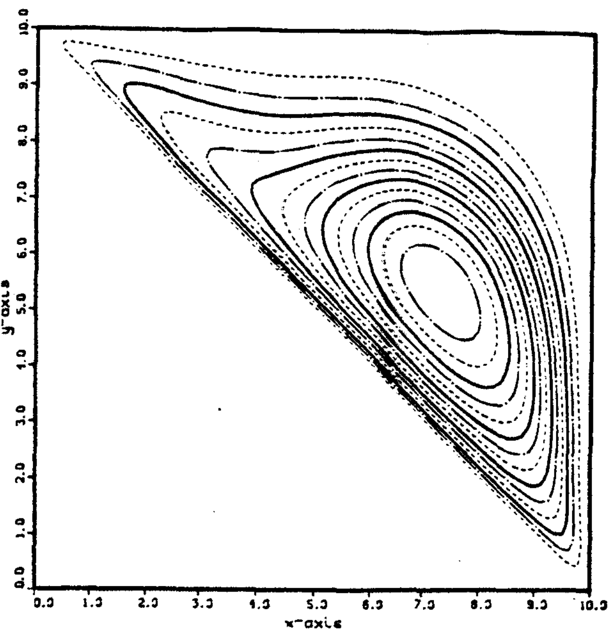

Figure 3: Driven triangular cavity calculation for R = 500.

The upstream waves between the leading surge and the sharp rise cannot be modeled using a regular perturbation expansion alone (double body expansion) since this will not give rise to any waves. Instead a simple steady model of the potential flow can be derived from a singular perturbation about the double body flow. The potential and free surface are represented respectively as

$$\Phi = \varphi(x,y) + F^2\varphi_1(x,y) + Re\left\{F^3 A(x,y)e^{iS(x,y)/F^2}\right\} + ...$$

$$\zeta = F^2\eta(x) + Re\left\{F^3 B(x)e^{iS(x,0)/F^2}\right\} + F^4\eta_1(x) + ...$$

where φ, φ_1 and η, η_1 etc. satisfy the regular perturbation problem while the terms involving e^{iS/F^2} satisfy a singular problem and yield upstream waves whose frequency increases with decreasing Froude number. Substituting these expressions into the equations for potential flow past a cylinder with a triangular extension in front (see Fig. 4) yields

$$S_x(x,0) = 1/\varphi_x^2$$

with

$$\varphi_x \underset{x \to 0^-}{\sim} 0\left(|\, x\,|^{\frac{\alpha/\pi}{1-\alpha/\pi}}\right).$$

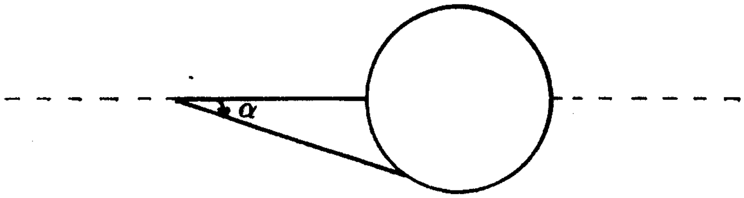

Figure 4: Potential flow calculation past a cylinder with a triangular extension in front.

This implies that for $\alpha < \pi/3$, S tends to zero and for $\alpha \geq \pi/3$, S tends to infinity as x tends to 0^-. In these experiments, the viscous triangular region corresponds to $\alpha < \pi/4$ giving the phase function S tending to zero and the free surface term B increasing monotonically as the plateau is approached as is observed experimentally. As x tends to $-\infty$, S tends to a constant. It is postulated that if these runs were made in a much longer tank that free waves of constant frequency would travel upstream of the cylinder.

Tuck: The theoretical model assumes that the free surface at the top of the triangular vortical region is flat and horizontal. At these rather high Froude numbers is there not a substantial distortion of this flat boundary?

Cole & Strayer: The free surface height of a stagnation point in potential flow is only $F^2/2$, in dimensionless units. Our heights along the cylinder are somewhat lower than the stagnation point heights. The vortical flow calculation presented is for a very low Froude number case and the surface is quite flat.

Newman: In similar experiments (with a relatively shallow cylinder) we observed stationary capillary waves in the upstream separated domain. Have you observed this?

Cole & Strayer: We have observed stationary capillary waves upstream of the separated region.

Beck: At Michigan we have been doing many experiments involving vortex/free surface interactions. We have found that the condition of the free surface (*i.e.*, amount of surfactants) is of critical importance. In your experiments, what have you done to monitor the condition of the free surface?

Cole &Streyer: We agree with you that surface contaminants play a critical role in low Froude number experiments. As we towed our cylinder down the length of the tank, contaminants piled up in front of the cylinder and were carried along to the end of the tank. We found that even after filtering the water surface for several hours and then letting the tank settle (as was necessary to get reasonable results), our low Froude number runs had a separation layer of one length at the "clean" end of the tank and a substantially longer one at the "dirty" end. Therefore, we chose to avoid these problems entirely by presenting results only for sufficiently high Froude numbers that the recorded features were relatively insensitive to surface tension effects.

Yeung: We are pleased to see that the authors' experimental observations are in no fundamental disagreement with those reported in Grosenbaugh & Yeung (J. Ship Res., 1989). These are difficult experiments to carry out and the authors' care in doing so is to be complimented. The main point put forward in the abstract is that the observed sharp rise in wave elevation, as well as its location, is due to the double-body flow alone. We believe however, that this point cannot be determined without consideration of the effects of stagnant or trapped contaminants. As such surface tension cannot be completely neglected. In fact, capillary waves are always evident in these low Froude number runs. There seems to be general agreement that vorticity in the trapped region is driven by the predominantly inviscid stream around it, so the real issue is the generation of white water around the hull. not the issue here. Is this white water due to the trapped vorticity, or is it caused by breaking waves in front of the ship.

Cole & Strayer: We agree that surface tension plays a critical role in low Froude number runs. The conclusions we drew concerning the location of the potential flow region and the stagnation layer in this talk were based on our higher Froude number runs which were very repeatable over a variety of surface conditions. These runs clearly indicated that the stagnant layer began well after the sharp rise in the free surface. For $F^2 = .63$ this rise begins approximately 2 or 3 radii out from the cylinder, whereas the stagnant layer lies on the plateau approximately 1 radii out. In addition, this rise looks very similar to that predicted by potential flow. Thus we conclude that the rise is a potential flow effect. We noticed also that small capillary waves are present out in front of the plateau. We do not believe that they significantly alter the flow in the higher Froude number cases. It is perhaps useful to point out that our experiments were performed in a towing tank while yours were performed in a flume. The very nature of flume experiments allows contaminants to build up primarily at the cylinder so as to give a steady effect after a long enough time. Our cylinder collected contaminants in front of it as it progressed down the tank giving very different results at the "clean" end of the tank versus the "dirty" end for low Froude number runs. Thus, for these runs the importance of surface tension effects was quite obvious. The higher Froude number runs, however, did not change significantly from one end of the tank to the other. Although our triangular computations were made with a rigid surface and for low Reynold's numbers, it is our belief that similar computations made with a free surface at higher Reynold's numbers would exhibit breaking waves. Thus, we believe that the white water present around the ship's hull is produced by breaking waves which are produced by the driven vortical flow. This idea is consistent with bulbous bow theory. A bulbous bow accelerates the outer potential flow as it passes around the bulk causing the surface potential flow to remain stable closer into the ship and thus reducing the size of the viscous region. This in turn reduces the white water produced by the breaking waves.