ON THE AZIMUTHAL INTEGRATION FOR THE CALCULATION OF FIRST AND SECOND ORDER WAVE FORCES ON FLOATING PLATFORMS

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INTRODUCTION

The intricate shapes of the present floating platforms have motivated a lot of effort for the improvement of the numerical methods within the well established potential theory. In this context, any analytical improvement such as the one proposed here is welcomed. Going directly to the point, it is known (Wehausen 71) that for the calculation of the radiation damping and the horizontal mean drift forces, there are expressions that require azimuthal integrations of products of Kochin functions. It seems that these integrations have been performed numerically and this introduces an extra error to be controlled and requires unnecessary evaluations of the Kochin functions for intermediate azimuthal angles. However, inverting some sign of integrations, it is possible to express the azimuthal integrations in closed form. This, of course, improves precision and the speed of calculations.

BASIC EXPRESSIONS

Keeping in mind linear theory, the pertinent Kochin function may be expressed as (Wehausen 71):

$$H_j(\theta) = \frac{-k}{D} \iint_{S_B} \left[\frac{\partial \varphi_j}{\partial n} - \varphi_j \frac{\partial}{\partial n}\right] \frac{\cosh k(z+h)}{\cosh kh} e^{-ik(x\cos\theta + y\,\mathrm{sen}\,\theta)} dS \tag{1}$$

where k is the wave number, θ is the azimuthal angle, S_B is the mean body wetted surface, h is the water depth, φ_j is the complex velocity potential corresponding the unit amplitude body motion in direction j (j = 1,2,3,4,5,6) and

$$D = \mathrm{tgh}(kh) + kh\,\mathrm{sech}^2(kh) \text{ (a constant).}$$

By energy conservation, it can be shown that the radiation damping (B_{ij}) may be expressed as

$$B_{ij} = \frac{\rho}{4\pi w k} D \int_0^{2\pi} H_i(\theta)\, H_j^*(\theta) d\theta \tag{2}$$

(Wehausen 71) (* indicates complex conjugate).

By momentum conservation it is possible to express the mean drift horizontal forces ($\overline{F}_x$ and $\overline{F}_y$) and moments ($\overline{K}_z$) as (see Wehausen 71 and references there cited)

$$\overline{F}_x = \frac{\rho}{8\pi} \int_0^{2\pi} |H(\theta)|^2 (\cos\beta - \cos\theta) d\theta \tag{3}$$

$$\overline{F}_y = \frac{\rho}{8\pi} \int_0^{2\pi} |H(\theta)|^2 (\mathrm{sen}\beta - \mathrm{sen}\,\theta) d\theta \tag{4}$$

$$\overline{K}_z = \frac{-\rho}{8\pi k} \mathrm{Im}\left\{ \int_0^{2\pi} H(\theta)\, D^*(\theta) d\theta \right\} + \frac{1}{2k^2} \rho\omega\, |A|\, \mathrm{Re}\{D(\beta)\} \tag{5}$$

(x and y are on the horizontal plane and z is vertical) where

$$D(\theta) = \frac{d\, H(\theta)}{d\theta} \tag{6}$$

and since the total velocity potential may be decomposed as

$$\varphi_t = \varphi_w + \varphi_7 + \sum_{j=1}^{6} \eta_j \varphi_j \tag{7}$$

where φ_w corresponds to the incoming wave of amplitude A, $A\varphi_7$ to the scattering and φ_j to the radiation (η_j is the displacement in generalized direction j) then

$$H(\theta) = \sum_{j=1}^{7} \eta_j H_j(\theta). \tag{8}$$

AZIMUTHAL INTEGRATION

Introducing now

$$I_{ij} \equiv \int_o^{2\pi} H_i(\theta)\, H_j^*(\theta) d\theta \tag{9}$$

it possible to show (Fernandes and Levy 90) that (observe the integration over S_B in (1))

$$I_{ij} = \left(\frac{-k}{D}\right)^2 \iint_{S_B} dS_p \iint_{S_B} dS_Q\, T_{ij}(P_j,Q) \tag{10}$$

where $P \equiv (x,y,z)$ e $Q \equiv (x',y',z')$ are points on the body and $T_{ij}(P,Q)$ does not depend on θ and may be expressed as

$$T_{ij}(P,Q) = \frac{\cosh k(z+h)}{\cosh kh} \cdot \frac{\cosh k(z'+h)}{\cosh kh} \{\alpha_j(P)\alpha_j^*(Q) I(P,Q) +$$
$$+ [\alpha_i(P)\alpha_j^{c^*}(Q) + \alpha_i^c(P)\alpha_j^*(\theta)]\, I^c(P,Q) + [\alpha_i^c(P)\alpha_j^{s^*}(\theta) + \alpha_i^s(P)\alpha_j^{ccs}(P,Q) +$$
$$+ \alpha_i^c(P)\, \alpha_j^{c^*}(Q)\, I^{cc}(P,Q) + \alpha_i^s(P)\, \alpha_j^{s^*}(\theta)\, I^{ss}(P,Q)\ \} \tag{11}$$

where

$$\alpha_j(P) \equiv \frac{\partial \varphi j(P)}{\partial n_p} - kN_3(P)\varphi_j(P)$$

$$N_3(P) = tghk(z+h)\, n_3(P)$$

$$\overset{c}{\alpha}_j(P) \equiv ikn_1(P)\varphi_j(P); \qquad \overset{s}{\alpha}_j(P) \equiv ikn_2(P)\varphi_j(P)$$

With $\rho \equiv (\xi^2 + \eta^2)^{1/2}$; $\xi \equiv x-x'$; $\eta \equiv y-y'$ the I's in expression (11) may be expressed in terms of Bessel functions (J_o, J_1 e J_2 as defined in Abramowitz and Stegun 72) such that

$$I(P,Q) = 2\pi\, J_o(k\rho) \tag{12}$$

$$I^c(P,Q) = i2\pi\, k\xi \frac{J_1(k\rho)}{k\rho} \tag{13}$$

$$I^s(P,Q) = i2\pi\, k\eta \frac{J_1(k\rho)}{k\rho} \tag{14}$$

$$I^{cc}(P,Q) = 2\pi\left[-\left(\frac{k\xi}{\rho}\right)^2 J_2(k\rho) + \frac{J_1(k\rho)}{k\rho}\right] \tag{15}$$

$$I^{cs}(P,Q) = -2P\left(\frac{k\xi}{k\rho}\right)\left(\frac{k\eta}{k\rho}\right) J_2(k\rho) \tag{16}$$

$$I^{ss}(P,Q) = 2\pi\left[-\left(\frac{k\eta}{k\rho}\right)^2 J_2(k\rho) + \frac{J_1(k\rho)}{k\rho}\right] \tag{17}$$

Hence, with (12)–(17) for any two points on S_B, one may calculate $T_{ij}(P,Q)$ via (11) and perform the integration shown in (10) in order to calculate B_{ij}. Note that the precision now is the same as the one imposed by the body description.

For the mean drift forces the procedure is analogous. From (3), (4) and (5)

$$\overline{F}_x = -\frac{\rho}{8\pi}\sum_{i=1}^{7}\sum_{j=1}^{7} \eta_i\eta_j^*[\cos\beta\, I_{ij} - I_{ij}^c] \tag{18}$$

$$\overline{F}_y = -\frac{\rho}{8\pi}\sum_{i=1}^{7}\sum_{j=1}^{7} \eta_i\eta_j^*[\mathrm{sen}\beta\, I_{ij} - I_{ij}^s] \tag{19}$$

$$\overline{K}_z = \frac{-\rho}{8\pi}\,\mathrm{Im}\{\sum_{i=1}^{7}\sum_{j=1}^{7} \eta_i\eta_j^* I_{ij}^d\} + \frac{\rho\omega|A|}{2k^2}\mathrm{Re}\{D(\beta)\} \tag{20}$$

where

$$I_{ij}^c \equiv \int_0^{2\pi} H_i(\theta)H_j(\theta)\cos\theta d\theta \tag{21}$$

$$I_{ij}^s \equiv \int_0^{2\pi} H_i(\theta)H_j(\theta)\mathrm{sen}\theta d\theta \tag{22}$$

$$I_{ij}^d \equiv \int_0^{2\pi} H_i(\theta)D_j^*(\theta)d\theta \tag{23}$$

which leads to

$$I_{ij}^c = (-\frac{k}{D})^2 \iint_{S_B} dS_P \iint_{S_B} dS_Q\ T_{ij}^c(P,Q) \tag{24}$$

$$I_{ij}^s = (-\frac{k}{D})^2 \iint_{S_B} dS_P \iint_{S_B} dS_Q\ T_{ij}^c(P,Q) \tag{25}$$

$$I_{ij}^d = (\frac{-k}{D})^2 ik \iint_{S_B} dS_P \iint_{S_B} dS_Q\ T_{ij}^d(P,Q) \tag{26}$$

with the expression for $T_{ij}^c(P,Q)$ [$T_{ij}^s(P,Q)$] following from (11) by taking the $I^{\cdot\cdot c}$ [$I^{\cdot\cdot s}$] instead of $I^{\cdot\cdot}$. The expression for $T_{ij}^d(P,Q)$ is more complicated and given by

$$\begin{aligned} T_{ij}^d(P,Q) = \frac{\cosh k(z+h)}{\cosh kh}\frac{\cosh k(z'+h)}{\cosh kh}\{&\alpha_i(P)\beta_j^{s*}(Q)I^s(P,Q) + \alpha_i(P)\beta_j^{c*}(Q)I^c(P,Q)\\ &+ [\alpha_i(P)\beta_j^{cs*}(Q) + \alpha_i^c(P)\beta_j^{s*}(Q) + \alpha_i^s(P)\beta_j^{s*}(Q)]\, I^{sc}(P,Q)\\ &+ [\alpha_i(P)\beta_j^{cc*}(Q) + \alpha_i^c(P)\beta_j^{c*}(Q)]\, I^{cc}(P,Q)\\ &+ [\alpha_i(P)\beta_j^{ss*}(Q) + \alpha_i^s(P)\beta_j^{s*}(Q)]\, I^{ss}(P,Q)\\ &+ [\alpha_i^c(P)\beta_j^{cs*}(Q) + \alpha_i^s(P)\beta_j^{cc*}(Q)]\, I^{ccs}(P,Q)\\ &+ [\alpha_i^c(P)\beta_j^{ss*}(Q) + \alpha_i^s(P)\beta_j^{cs*}(Q)]\, I^{css}(P,Q)\\ &+ \alpha_i^c(P)\beta_j^{cc*}(Q)\, I^{ccc}(P,Q) + \alpha_i^s(P)\beta_j^{ss*}(Q)\, I^{sss}(P,Q)\} \end{aligned} \tag{27}$$

where

$$\beta_j^s(P) \equiv -x\frac{\partial\varphi_j}{\partial n} - \varphi_j(P)(-n_1 - kx\, N_3); \qquad \beta_j^c(P) \equiv -y\frac{\partial\varphi_j}{\partial n} - \varphi_j(P)(-n_2 - ky\, N_3)$$

$\beta_j^{ss}(P) \equiv ik\, n_2\, \varphi_j(P)$; $\quad \beta_j^{cc}(P) \equiv ik\, n_1\, \varphi_j(P)$ and $\quad \beta_j^{cs}(P) \equiv ik\, (xn_1 - yn_2)\varphi_j(P)$.

The remaining I functions are given by

$$I^{ccc}(P,Q) = 2\pi i \left(\frac{k\xi}{k\rho}\right)\left\{-\left[3-4\left(\frac{k\xi}{k\rho}\right)^2\right]\frac{J_2(k\rho)}{k\rho} - k\xi\left(\frac{k\xi}{k\rho}\right)\frac{J_1(k\rho)}{k\rho}\right\} \tag{28}$$

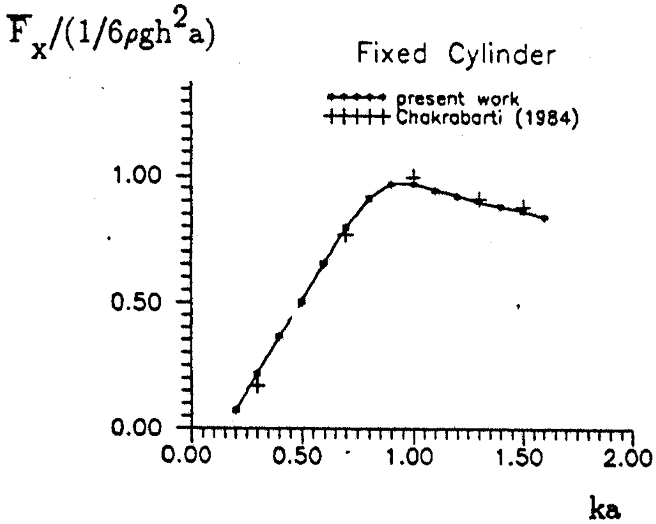
$$I^{ccs}(P,Q) = 2\pi i \left(\frac{k\eta}{k\rho}\right)\left\{-\left[1-4\left(\frac{k\xi}{k\rho}\right)^2\right]\frac{J_2(k\rho)}{k\rho} - k\eta\left(\frac{k\eta}{k\rho}\right)\frac{J_1(k\rho)}{k\rho}\right\} \tag{29}$$

$$I^{ssc}(P,Q) = 2\pi i \left(\frac{k\xi}{k\rho}\right)\left\{-\left[1-4\left(\frac{k\eta}{k\rho}\right)^2\right]\frac{J_2(k\rho)}{k\rho} - k\eta\left(\frac{k\eta}{k\rho}\right)\frac{J_1(k\rho)}{k\rho}\right\} \tag{30}$$

$$I^{sss}(P,Q) = 2\pi i \left(\frac{k\eta}{k\rho}\right)\left\{-\left[3-4\left(\frac{k\eta}{k\rho}\right)^2\right]\frac{J_2(k\rho)}{k\rho} - k\eta\left(\frac{k\eta}{k\rho}\right)\frac{J_1(k\rho)}{k\rho}\right\} \tag{31}$$

SIMPLE APPLICATIONS

The expressions above are easily programmable from any code that calculate the velocity potential φ_j, j=1,2,...7 at points though the body. This has been done for the present work from a code developed by Levy 89. As examples of applications two classical cases are considered: the MacCamy and Fuchs slender vertical cylinder (see for instance Chakrabarti 84) and the hemisphere as discussed in Kokkinowrachos 82 both with radius a. In the Figures 1 and 2 the mean drift horizontal force are presented respectively.



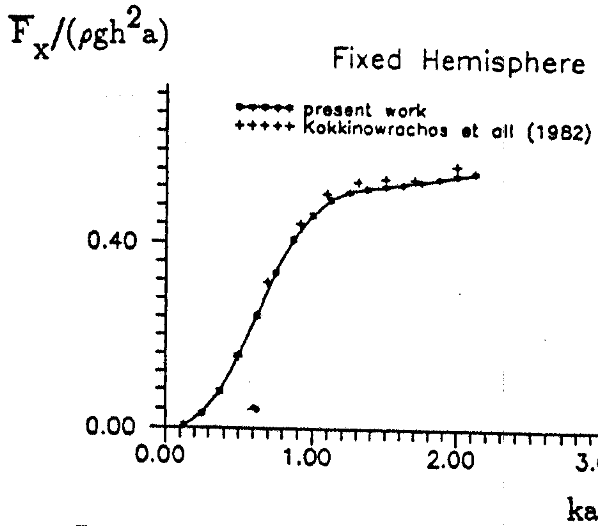


Fig. 1 Mean drift force, cylinder (h/a=5) Fig. 2 Mean drift force, hemisphere

REFERENCES

Abramowitz, M. and Stegun, I.A. 1972 Handbook of Mathematical, Dover Publications, 1972

Chakrabarti, S.K. 1984 Steady Drift Force on Vertical Cylinder — Viscous vs. Potential; Applied Ocean Research, Vol. 6, No. 2.

Fernandes, A.C. and Levy, L.A.P. 1990 Cálculo de esforços de onda de primeira e segunda ordem em navios e plataformas flutuantes através de integração azimutal analítica, Congresso da SOBENA Rio de Janeiro, RJ, Brazil

Kokkinowrachos, K., Bardis, L., Mavrakos, S. 1972 Drift Forces on One– and Two– body Structures in Regular Waves, International Conference on Behavior of Offshore Structures (BOSS).

Levy, L.A.P. 1989 Análise de Movimentos de Corpos Flutuantes no Domínio do Tempo, MSc thesis; Engenharia Oceânica; UFRJ, COPPE.

Wehausen, J.V. 1971 The Motion of Floating Bodies, Annual Review of Fluid Mechanics, Vol. 3 pp.237–268.

Yue: You treat integrals of periodic functions given (presumably) discrete data. It seems that computationally you cannot hope to do better than equal spacing quadratures which gives exponential convergence. Do I misunderstand your objective?

Fernandes: I guess you do. It is difficult to find a quadrature that is better than an analytical, closed-form result when the latter is made up of simple functions such as Bessel's.

Ohkusu: You did not mention the merits of your method in terms of practical applications and the accuracy of numerical computations. I wonder if you could give any specific examples.

Fernandes: I cannot give you a direct answer to that question because I have never tried the quadrature approach. Certainly the formula will improve the precision up to the precision of the calculation of the Bessel functions. But I am not sure about speed since it depends on the stategy used for implementation of the formula.