A Non-Linear Discrete Time Model of The Drift Force

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A novel non-linear discrete time system identification technique has recently been applied to characterise TLP surge motion data, measured in experiments. This note outlines the model formulation and compares it with the traditional frequency domain quadratic transfer function and time domain convolution models. The results are fairly encouraging and point to a number of experimental issues in need of further study.

The traditional quadratic frequency domain model of response in one or more degrees of freedom is:

$$X(jw) = G(jw)\, W(jw) \quad + \quad \int_{-\infty}^{\infty} H(w_1, w-w_1)\; W(jw_1) W(j(w-w_1)) dw_1$$

This has a time domain counterpart known as the Volterra model:

$$x(t) = \quad \int_{-\infty}^{\infty} g(T)\; w(t-T)\; dT \;+\; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(T_1,T_2) w(t-T_1) w(t-T_2) dT_1\; dT_2$$

$G(jw)$ and $g(t)$, $H(w_1,w_2)$ and $h(t_1,t_2)$ are connected by standard Fourier transform relations (ref.1). It is straightforward to estimate $G(jw)$ and $H(w_1,w_2)$ from experimental data, so long as the input signal satisfies certain conditions (ref.1); a Gaussian signal is sufficient but not necessary and small departures from the criteria can be significant. Under these conditions, the estimation of G and H from experimental measurements is coupled and requires accumulation of higher order spectral moments. If higher order nonlinearities are present, these will bias the estimates of G and H derived by conventional FFT techniques. Numerically, the process is somewhat sensitive and long runs of stationary data are required to ensure statistical stability. Unfortunately the leading diagonal of the transfer function cannot be estimated from the data of a single random sea experiment as all frequencies contribute to the mean offset. The statistics of the numerical evaluation of the quadratic convolution kernel from the measured frequency response function does not appear to be well understood. Note that both G and H are dependent on the linear dynamics of the vessel; it would be more convenient to determine the pure quadratic and linear transfer functions between wave elevation and force but this is not possible unless the model is restrained; it is then difficult to pick the second order force components out from the first order forces and motion dependent forces such as wave drift damping cannot be estimated in this way.

An alternative to the convolution model of a linear system is the continuous (or discrete) time differential (or difference) equation. System identification methods (ref.2) evaluate the best parameters of a (usually discrete time) 'auto regressive moving average' model.

$$y(t) = a_1\, y(t-1) + a_2\, y(t-2) \ldots + b_1\, u(t-1) + b_2\, u(t-2) \ldots$$

which best transforms the input time series u(k) into the output series y(k). Between 2 and (say) 10 a and b parameters may be required, depending on the complexity of the impulse response function g(t). However, this is far fewer than the 100 or so which may be required in the pure convolution (moving average) model. An analogy with decimal expansion vs. rational fraction representations of a number is helpful; an infinite number of terms can be needed to represent a number which is compactly expressed as the ratio of two primes.

In principle, a similar nonlinear difference equation might be developed for the quadratic (or perhaps higher order) transfer functions of hydrodynamics. But the potential number of terms in y(t-l)y(t-m), u(t-l)u(t-m) and y(t-l)u(t-m) is daunting and the evaluation of the best fitting parameters is dependent on the assumed model order. Conventional methods for problems containing a simple nonlinearity such as quadratic damping use formulations where the problem is linear in the parameters and effectively assume that the nonlinear term is an additional input. This would not work well in this case; given the best 100 term model, all 100 possible 99 term models would have to be identified to assess the necessity of the 100 terms!

Recent work at Sheffield University (ref.3) has produced a technique for 'orthogonalising' the data which permits each term to be evaluated individually and its contribution to the final output determined. It also ensures that third and higher order terms, if present, do not interfere with the estimation of lower order terms, a problem which affects the traditional frequency domain approach. In the surge drift problem, viscous Morison forcing on columns with varying immersion and wave drift damping are likely to be third order.

To assess the applicability of these techniques, data from a TLP test in a towing tank was analysed to derive a difference equation model. To minimise the number of terms required, it is important to choose an appropriate sampling rate; our first attempt threw away a little of the power in the input spectrum but did not account for the impressive ability of the structure to turn wave elevation into drift at higher frequencies. The results were not particularly good, especially at wave frequencies and the model could not reproduce the mean offset without the addition of a constant term. Investigation in the frequency domain indicated that the low frequency portion of the spectrum of sampled u^2 was different from that evaluated by squaring after resampling.

The first model was derived by consideration of all possible products of terms up to a lag of 30 steps and up to an order of 4. One small term in yu^2 was identified but no other third or fourth order terms were found necessary; this should be a relief to those of us who find second order hydrodynamics quite difficult enough. It seems plausible that this term is related to wave encounter damping, although some caution is needed in deriving contniuous time models from the discrete time representation. In particular, all the y*u terms may disappear in a continuous time model, which is currently under development in parallel with the theory. The model shown in table 1 is the first 20 terms of the initial model, including only quadratic terms; the nonlinear coherence tests indicate that this is borderline and we should probably include third order terms. The error reduction ratio denotes the contributiuon of each term to decreasing the one step ahead prediction in the orthogonalised domain. Most of the error reduction is due to the previous value of y; however, the real input to the low frequency linear dynamics comes from the nonlinear terms, so the absolute value of ERR may be a misleading measure of the importance of a term. Its relative value is a good guide to the optimal model size. Note that the large lags on the input u are due to a time shift to intruduced to ensure causality.

Figure 1 diplays arbitrarily selected portions of the response time history of the model, driven by the measured input and compared against the original measurements for a section of the data outside the interval used to derive the model. A reasonable fit is apparent. Fig.2a,b shows the transfer function between waves and model response compared with theory at wave frequencies (0.06 - 0.11 Hz); here the fit is very good. Figure 3a,b displays the transfer function between observed and fitted behaviour. At wave frequencies, the fit is excellent, leaving something to be desired in the low frequency range. Finally, fig.4 compares the mean drift forces predicted by SESAM:WADAM with those derived by exciting the NARMAX model with a sinusoid at various frequencies. The fit is not wonderful but there are mitigating circumstances, discussed below.

The noise on the motion measurements is very small but this is not the case with the wave data. The identified model incorporates a linear noise model; this means that the value of y(t) depended linearly on terms proportional to the error between the observed and predicted behaviour at earlier time steps. It can be shown that this is needed to obtain unbiased estimates of the parameters in the presence of correlated noise. The wave probe measurement is corrupted by radiated and diffracted waves, the former due to the previous motion of the platform. Additionally, wall effects can generate significant cross tank standing wave systems which interact with the incoming waves to generate additional drift forces. Precalibration of the tank without the model might help resolve some of these problems but the incident waves due to diffracted/radiated waves re-reflected by the wavemaker would not be included, nor would the effects of the cross-tank wave system. This is a strong argument for absorbing wavemakers and wide tanks.

To predict the surge motion, the NARMAX model must be run without noise inputs since there is no 'ideal' response with which to generate an error term. In this case, the errors are a significant input to the system since the quadratic interaction amplifies the effects of small relatively high frequency terms so strongly; this phenomenon was also displayed in the sampling rate problem. Note that these problems are not unique to NARMAX modelling; conventional frequency domain methods which try to average away errors, instead of modelling them, will be even more strongly affected.

Finally, it is worth noting that the low frequency response is due to a force with extremes much larger than a Gaussian process - the time series of $u^2(t)$ appears almost like a series of pulses striking the platform and exciting its resonant response. Thus, it is rather impressive that an identification scheme which depends solely on the one step ahead prediction error can characterise the response as well as it has done.

References

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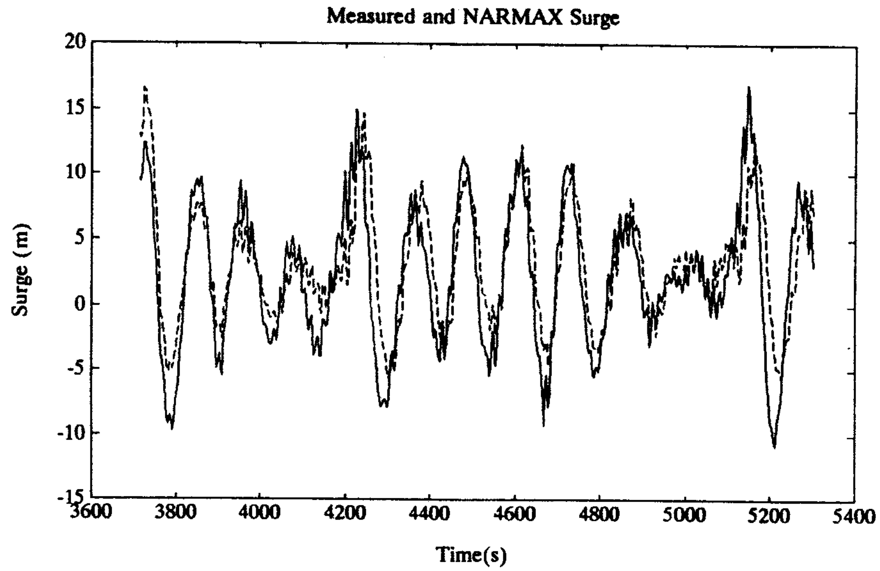

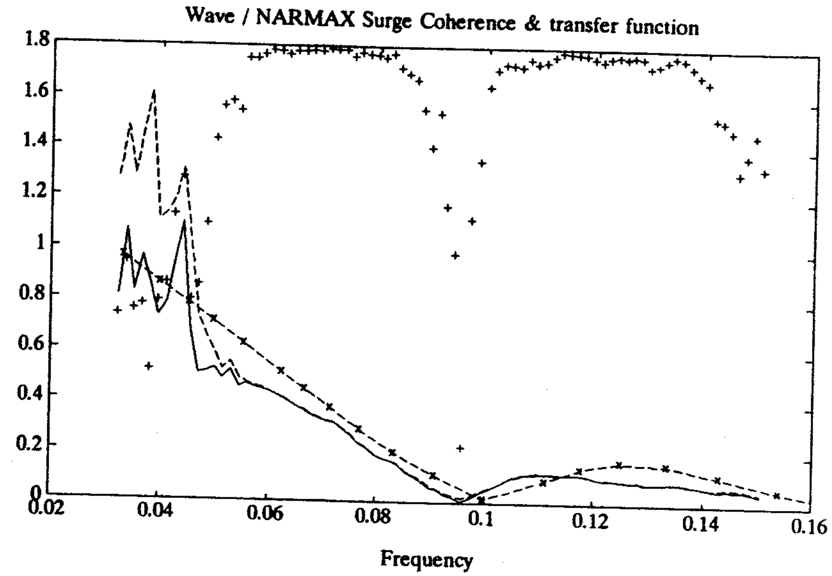

Term	Parameter estimates	Error reduction ratio
y(t-1)	0.14912E-01	0.94685E+00
u(t-11)	-0.84935E-01	0.23920E-01
u(t-5)	-0.10184E+00	0.82419E-02
y(t-2)	-0.54393E+00	0.12356E-01
u(t-7)	0.98068E-01	0.26400E-02
u(t-13)	0.28650E-01	0.10493E-02
y(t-3)u(t-8)	0.55212E-02	0.35341E-03
u(t-12)u(t-4)	-0.92537E-02	0.31819E-03
y(t-8)	-0.10555E+00	0.43762E-03
y(t-1)u(t-12)	0.74157E-02	0.26981E-03
u(t-9)	0.57665E-01	0.22829E-03
u(t-8)	0.33859E-01	0.27084E-03
u(t-17)	0.32683E-01	0.19731E-03
u(t-11)u(t-13)	-0.68410E-02	0.15737E-03
y(t-3)	0.11041E+00	0.12667E-03
u(t-18)u(t-19)	-0.54622E-02	0.94952E-04
y(t-19)y(t-20)	0.11721E-02	0.50568E-04
y(t-11)u(t-18)	0.31183E-02	0.43486E-04
y(t-4)u(t-2)	-0.28828E-02	0.56274E-04
u(t-10)	0.56095E-02	0.29783E-04
e(t-6)	0.26915E+00	0.14173E-03
e(t-2)	-0.16744E+00	0.40147E-04
e(t-4)	0.12433E+00	0.29699E-04
e(t-1)	-0.13731E+00	0.20706E-04
e(t-5)	0.95441E-01	0.17499E-04
e(t-3)	0.13617E-04	0.34790E-12