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THE APPROXIMATE EVALUATION OF SECOND-HARMONIC VERTICAL FORCES ON A TENSION-LEG PLATFORM

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The computation of second-order sum-frequency wave loads on a tension-leg platform (TLP) is extremely time consuming and the relevant code cannot be routinely run on serial computers. Recently, Newman (1990) obtained an asymptotic form of the second-harmonic potential at large depths. The final expression is so simple that the second-order potential can be easily calculated by hand-on calculators as long as the Kochin function at the weather side is given. In this paper, it is shown that this asymptotic solution can be used in the approximate estimation of second-harmonic vertical wave loads on a TLP. Since the asymptotic solutions are valid at large depths, they are applied to the computation of vertical forces only.

For analysis, Cartesian coordinate system (x, y, z) as well as polar coordinate system (r, θ, z) with the origin at $z = 0$, and the z axis positive upward is used. The leading asymptotic term of the second-order double-frequency potential $\phi^{(2)}$ at large depth z is given by Newman (1990) in equation (16):

$$\phi^{(2)}(r,\theta,z) \sim \frac{z}{8\pi K}\int_0^{2\pi} d\alpha \int_0^{\infty} \rho d\rho \left(R^2+\rho^2-2r\rho\cos(\theta-\alpha)\right)^{-3/2} q(\rho,\alpha), \qquad (1)$$

where K is the infinite-depth wavenumber and $R = \sqrt{r^2+z^2}$ the spherical radius. The function q represents the free-surface inhomogeneity and is given approximately by

$$q(r,\theta) \sim \frac{F(\theta)}{\sqrt{Kr}} e^{-iKr(1+\cos\theta)}, \qquad (2)$$

where $F(\theta)$ is a function of far-field wave amplitudes. Upon substituting (2) into (1), we obtain

$$\phi^{(2)} \sim \frac{z}{8\pi K^{\frac{3}{2}}}\int_0^{\infty} d\rho \int_0^{2\pi} d\alpha F(\alpha) e^{-iK\rho(1+\cos\alpha)} \sqrt{\rho}\left(R^2+\rho^2-2r\rho\cos(\theta-\alpha)\right)^{-\frac{3}{2}}. \qquad (3)$$

In evaluating the above integral, Newman (1990) first used the stationary phase method for the θ integral assuming $K\rho$ is large. The subsequent ρ integral yields

$$\phi^{(2)} \sim \frac{F(\pi)e^{i\frac{\pi}{4}}}{4\sqrt{2\pi}K^2} \frac{z}{R(R+x)}. \tag{4}$$

For field points close to the vertical axis, (4) can be reduced to

$$\phi^{(2)} \sim \frac{F(\pi)e^{i\frac{\pi}{4}}}{4\sqrt{2\pi}K^2}\frac{1}{z}. \tag{5}$$

We can see in (5) that the leading term of $\phi^{(2)}$ decays only at $1/z$. In (4) and (5), the second-harmonic potential at large depths is completely described by the first-order far-field wave amplitude function F at $\theta = \pi$. It appears that the second-order pressure field given by (4) may be the same for a number of different geometries as long as they generate the same far-field wave amplitudes at $\theta = \pi$.

In the above analysis, the second-order Stokes waves and free waves due to a body forcing term are not considered, since they will be exponentially small at large depths. From the same reasoning, the second-harmonic vertical force on the deep portion of a body due to the quadratic products of linear quantities can be neglected.

We next show how we can use (4) and (5) to estimate second-harmonic vertical wave loads on a TLP. For general three-dimensional bodies, The function F can be related to the classical Kochin function $\mathcal{H}$ as follows:

$$F(\theta) = \frac{-2iK^3A}{\sqrt{2\pi}}e^{-i\frac{\pi}{4}}(1-\cos\theta)\mathcal{H}(\theta+\pi). \tag{6}$$

The Kochin function $\mathcal{H}$ represents the far-field amplitude of the first-order diffraction potential $\phi_D^{(1)}$ through the relation

$$\phi_D^{(1)} \sim \sqrt{\frac{K}{2\pi r}}e^{Kz-iKr-i\frac{\pi}{4}}\ \mathcal{H}(\pi+\theta) \quad \text{for } r >> 1, \tag{7}$$

and $\mathcal{H}$ is given by

$$\mathcal{H}(\theta) = \int\int_{S_B}\left(\frac{\partial\phi_D^{(1)}}{\partial n} - \phi_D^{(1)}\frac{\partial}{\partial n}\right)e^{Kz'}e^{-iK(x'\cos\theta+y'\sin\theta)}\,dS, \tag{8}$$

where S_B denotes a mean body surface. After computing Kochin functions for a given geometry, we can easily evaluate $\phi^{(2)}$ using (4). Particularly, (4) and (5) lead to the following *closed-form* expressions for second-harmonic vertical forces $F_z^{(2)}$ on a TLP. For illustration, we select the ISSC TLP which consists of four cylindrical columns (radius $a = 8.435$m and draft $D = 35$m) and four rectangular pontoons forming a square. More detailed geometric characteristics are given in Eatock Taylor & Jefferys (1985).

The second-harmonic vertical force on four columms of the ISSC TLP can be obtained from (4):

$$F_z^{(2)} = \rho_0 \omega a A K \ \mathcal{H}(2\pi) \ \frac{a}{D} \sum_{k=1}^{4} \frac{1}{\sqrt{1+\mu_k^2}(\sqrt{1+\mu_k^2} + \mu_k \cos\theta_k)}, \tag{9}$$

where ρ_0 is a fluid density and the ratio $\mu_k = r_k/D$, with r_k being the radial distance to the center of each column. For the ISSC TLP, $\mu_k = \mu = 1.74$ and $\theta_k = (2k-1)\pi/4$. Thus we obtain

$$F_z^{(2)} = \rho_0 \omega a A K \ \mathcal{H}(2\pi) \ \frac{a}{D} \left(\frac{4}{1+\mu^2/2} \right). \tag{10}$$

It is of interest to see that the factor $4/(1+\mu^2/2)$ approaches to 4 for the limit $\mu << 1$. From this we can see that the vertical force acting on four closely-spaced columns can be magnified as large as four times in addition to the corresponding increase of the Kochin function at $\theta = 2\pi$. Analogous N^2 increase of the mean drift forces for long waves and N closely-spaced bodies can be found in Eatock Taylor & Hung (1985) and McIver (1987).

The vertical force on two pontoons that are parallel to the wave direction ($z = -D$ or $-D_u$, $y = \pm L$) can be obtained from the integral:

$$B \int_{-L'}^{L'} \frac{z}{\sqrt{x^2+L^2+z^2}(\sqrt{x^2+L^2+z^2}+x)} dx = 2B \frac{L'z}{L^2+z^2}, \tag{11}$$

where B is the pontoon width, D_u the draft of pontoon upper surface, and $L' = L - a$. Similarly, the vertical force on the right and left side pontoons ($x = \pm L$) that are perpendicular to the wave direction can be obtained from the integral:

$$B \int_{-L'}^{L'} \frac{z}{\sqrt{y^2+L^2+z^2}(\sqrt{y^2+L^2+z^2} \pm L)} dy$$

$$= 2B\{\arcsin\left(\frac{\frac{-z^2}{\sqrt{L^2+z^2}\pm L} \mp L}{\sqrt{L^2+z^2}} \right) - \arcsin\left(\frac{\frac{-z^2}{\sqrt{L^2+L'^2+z^2}\pm L} \mp L}{\sqrt{L^2+z^2}} \right)\}. \tag{12}$$

From (10) $\sim$ (12), we can easily estimate the order of magnitude of each force on individual members of the ISSC TLP. For example, the vertical force on the pontoons is approximately 38 % of that on the four columns, and has the opposite sign. Among the pontoon members, the contribution from the upwave pontoon, which is perpendicular to the wave direction, is the most significant and is as large as 57 % of the force on four columns with opposite sign.

Finally this result is compared with a more complete second-order computation (Kim, 1991). Approximate solutions show qualitative agreements with the numerical results, especially for the maximum values, while there exists a shift of the frequency for maximum values. We do not have this kind of frequency shift when the approximate solutions for four columns are compared with the corresponding numerical results.

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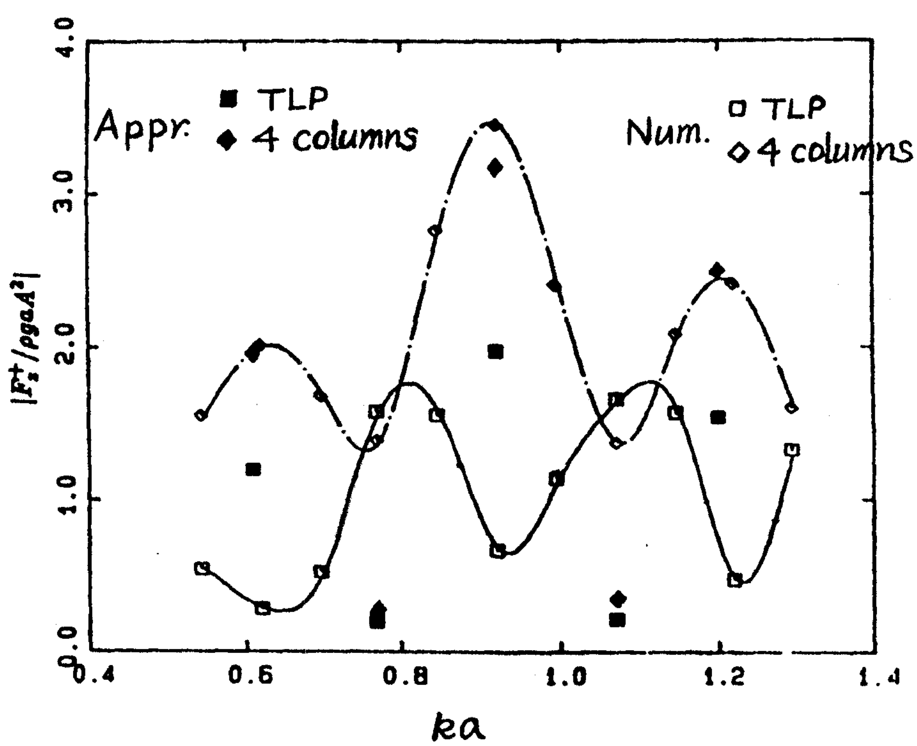

Yue: You show the effect of decreasing relative column spacing on the second-order sum-frequency wave load. If one is interested in the tendon tension load, then the pitch moment is important and the decreasing spacing will have the (opposite) effect of decreasing the moment arm and hence the tension load. It appears that there should be an optional leg spacing for given leg geometry and resonant frequencies.

Kim: The present method may be effectively used to find the optimum geometric characteristics of a TLP for a given natural frequency.

McIver: Can you clarify when Newman's approximation is valid? How is it influenced by the geometry under consideration?

Kim: The details are given in Newman (1990, JFM). In general, the asymptotic solution is valid at large depths and near the origin. For example, the ratio of the horizontal distance to the square of the depth must be small. In addition, this asymptotic solution cannot be applied at very low frequencies. The present results do not violate these assumptions.