The Time Domain Diffraction Problem

F. T. Korsmeyer
Department of Ocean Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139 USA

Introduction

In a linear analysis of the interaction of a free surface with a floating body, we have the opportunity to choose between a frequency-domain or a time-domain approach. Traditionally the frequency domain has received greater attention. In the past five years, however, time-domain investigations of this problem have been reported by numerous authors (see References). With the exception of Lin and Yue (1990), who used a quasi-nonlinear method, a noteworthy omission in these studies are simulations of the interaction of an ambient sea with a floating body. Typically, time-domain results (impulse-response functions) are Fourier transformed and compared to frequency-domain results (added-mass, damping, and exciting force functions). The work reported here is part of an effort to build on our success with the radiation problem (Korsmeyer (1988)) and develop a capability to simulate the interaction of a pseudo-random sea with a floating body which may have forward speed.

Problem Formulation

We consider the linearized problem of a fixed body in a semi-infinite fluid. The fluid is ideal and its velocity field is irrotational. A coordinate system with the z-axis opposite to the direction of the force of gravity and with the x- and y-axes in the plane of the free surface is centered in the body waterplane. A disturbance of the free surface travels to the body from spatial infinity and the solution to the problem is a description of the scattering of the incoming disturbance. We may describe the velocity field by a scalar potential function the "diffraction potential" ϕ_D, which satisfies Laplace's equation, and we decompose this potential into the incoming waves the "incident potential" Φ_I, and the scattered waves the "scattered potential" ϕ_S.

In the initial-boundary-value problem for the scattered potential we require appropriate boundary conditions at spatial infinity, and the above decomposition leads to the following boundary condition on the body:

$$\vec{\nabla}\phi_S \cdot \vec{n} = -\vec{\nabla}\Phi_I \cdot \vec{n},$$

where $\vec{n}$ is the unit normal of the body surface.

In the radiation problem we had initial conditions at $t = 0$ as there was no fluid motion until it was excited by the Heaviside body motion at $t = 0$. However this problem is not quite so straightforward. Here, since we wish Φ_I to be an impulsive wave, we need to begin its construction at spatial infinity as $t \to -\infty$. We excite short waves initially, followed by longer and longer waves, so that they coalesce in an impulse at some convenient location in space, at a time which we may set to be $t = 0$. This is a plane wave system which contains all wave lengths with unit amplitude. The description of the incident velocity field and pressure is a generalization of that derived in King (1987). The former is used to provide the body boundary condition on ϕ_S and the latter may be integrated over the body surface to provide the Froude-Krilov force.

We may recast the initial-boundary-value problem for the scattered potential as an integral equation by Green's theorem and the appropriate transient Green function as has been done in the radiation problem (for details see Korsmeyer 1988).

The equation we solve is identical to that of the radiation problem, with the exception of the lower limit on the convolution and the body boundary condition, and appears:

$$
\begin{aligned}
2\pi\phi_S(\vec{x},t) + \iint_{S_B} d\vec{\xi}\,\phi_S(\vec{x},t)G^{(0)}_{n_\xi}(\vec{x};\vec{\xi}) \\
&= -\int_{-\infty}^{t} d\tau \iint_{S_B} d\vec{\xi}\,(\Phi_I)_{n_\xi}(\vec{\xi},\tau)G^{(F)}_{t}(\vec{x};\vec{\xi},t-\tau) + \phi_S(\vec{\xi},\tau)G^{(F)}_{n_\xi t}(\vec{x};\vec{\xi},t-\tau) \\
&- \iint_{S_B} d\vec{\xi}\,(\Phi_I)_{n_\xi}(\vec{\xi},t)G^{(0)}(\vec{x};\vec{\xi}),
\end{aligned}
$$

where $G^{(0)}$ and $G^{(F)}$ are the Rankine and wave parts of the transient free-surface Green function, respectively. In this equation, the acceleration due to gravity, the fluid density, and the body length have been set equal to one.

We solve this problem computationally by a panel method which approximates the geometry by a set of plane quadrilaterals on which the unknown potential is assumed constant. The convolutions are computed by the trapezoid rule. We call this program TIMIT for: TIme domain, MIT. Our choice of a value of time to begin the computation is based on experience, and depends (as will be shown below) on where in space we select for the coalescence of the impulse. While the computation may begin at $t < 0$, the duration of the computation is of the same order as in the radiation problem.

Results

In our past work on the radiation problem we were thinking of applications to offshore structures primarily; however, our interest has broadened to include ships as well. We find that in performing computations for ship-like bodies the challenges are similar to those we have experienced with compact bodies.

Our first aim is to establish the validity of the diffraction computations. In Figure 1, we present results for the Wigley hull at zero speed in head seas. In these computations, $\Phi_I(\vec{x},t)$ is an impulse which coalesces at midships at $t = 0$. Note that the force on the body has non-zero value before $t = 0$ because at the bow of the ship, before the waves coalesce at midships, Φ_I appears as a very compact, "reversed," Cauchy-Poisson disturbance. King termed this phenomenon "non-causal," because the response is apparent before the impulse; however, it is *not* apparent before the *excitation*. Physically, this is indeed a causal system.

Note that the impulse-response functions in Figure 1 contain oscillations with a period of approximately 0.7 which persist to the end of the record. These are due to the manifestation, in the time domain, of irregular frequencies as discussed in Korsmeyer (1988) for the radiation problem. This period of 0.7 corresponds to the first transverse mode of the homogeneous, interior, Dirichlet problem. That is, the wave length is a large fraction of twice the body beam. Just as in the radiation problem, the amplitude of these oscillations may be reduced by a finer spatial discretization of the body, but given any such discretization, cannot be reduced beyond a certain value by a refinement of the temporal discretization.

In Figure 2, we present a comparison in the frequency domain with results from a frequency domain radiation/diffraction panel-method code: WAMIT. The body was represented by the same set of panels in both codes, and the time-domain results shown in Figure 1 were Fourier transformed by Filon quadrature without any special attention to the ends of the record such as tapering or asymptotic extension. This latter technique was shown to be useful for removing irregular frequency effects in the radiation problem in Korsmeyer (1988). Here, however, the first irregular frequency is very high compared to the range of frequencies of interest in ship-motion analysis. Whether irregular frequencies will be a concern in cases with forward speed we do not yet know.

There is no special significance to exciting this problem with an impulse located at $\vec{x} = 0$. The impulse may be anywhere on the free surface and if some location is more convenient, then we might choose it instead. Consider, for example, a location ahead (up-wave) of the body. In this case, all of the body will experience a Cauchy-Poisson incident potential. Figure 3 presents results for such a case. Here, the impulse coalesces at $x = 1.5$ (which we call the "offset") at $t = 0$. The results for a midships location of the impulse are plotted as well for comparison. We can see that in the offset case, the force on the body is as we expect from an incoming surface profile which has a leading wave-front followed by successively shorter waves. In this case, the duration of the record which would be required by a simulation has lengthened a bit due to dispersion. On the other hand, the record has diminished to negligible response before the wavelength has arrived which will excite the irregular frequency effects. This may turn out to be a convenient feature in a simulation.

Naturally, all of the information in the record with zero offset is also present in any non-zero offset record provided that the initial and final time values allow excitation by the same range of wave frequencies. In Figure 4, the Fourier transforms of the records in Figure 3 are presented. In the plot of the exciting force amplitude we can see that the records do indeed have the same frequency content. In the plot of the exciting force phase, we can see that the offset case has different phase information, as we expect, but this may be adjusted to the midships result by multiplication by a factor of $\exp(ikl)$ where l is the offset. This result is plotted as well.

It is also the case that we need not excite the body with an impulse either. As shown in King (1987) other excitations may be used and the impulse-response function obtained through a double Fourier transform technique involving the transform of the particular excitation used. It was suggested that this is a convenient method for controlling the frequency content of the solution in an effort to improve numerical behavior. In our investigations, so far, we have not perceived any advantage to this technique.

Further Investigation

Our intention is to be able to perform simulations in the time domain for stationary or moving bodies. All of the computation will be carried out in the time domain. First, the radiation and diffraction problems will be solved for all modes of motion of interest to generate the impulse-response functions. This is the computationally intensive part of the problem. Then with this data, any number of different simulations using a quasi-random Φ_I could be run. The advantage over a frequency domain approach is the straightforward inclusion of forward speed, and the real-time data available in the simulation, like relative motion, propellor inflow velocity time-histories, *etc.*

Acknowledgment

This work is supported by the Office of Naval Research.

References

Beck, R. F. and Liapis, S. 1987 Transient motions of floating bodies at zero speed. *J. Ship Research* **31**, 3, 164-176.

King, B. 1987 Time-domain analysis of wave exciting forces on ships and bodies. The Department of Naval Architecture and Marine Engineering Report No. 306, The University of Michigan, Ann Arbor, Michigan.

Korsmeyer, F. T. 1988 The first- and second-order transient free-surface wave radiation problems. PhD Thesis, Massachusetts Institute of Technology, Cambridge Massachusetts.

Lin, W. M. and Yue, D. K. P. 1990 Numerical solutions for large-amplitude ship motions in the time domain. *Eighteenth Symp. on Nav. Hydro.*,Ann Arbor, Michigan.

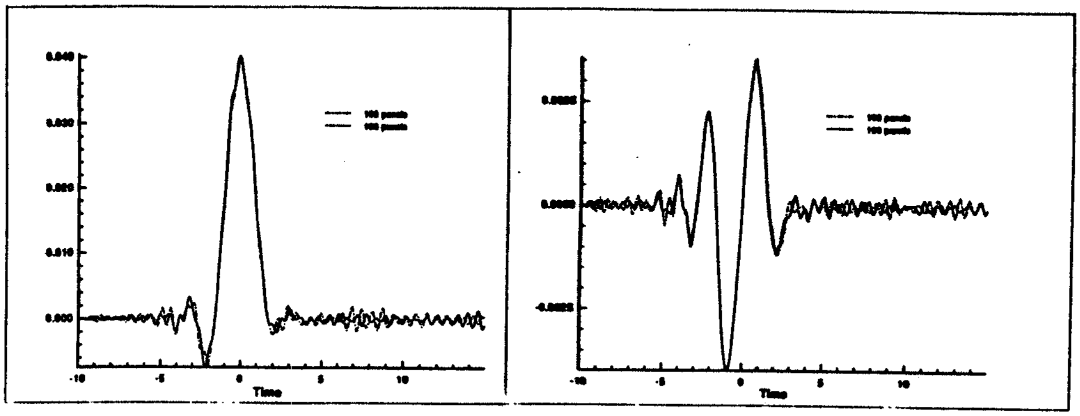

Figure 1. Heave and pitch exciting force (moment) i.-r.f. for a Wigley hull

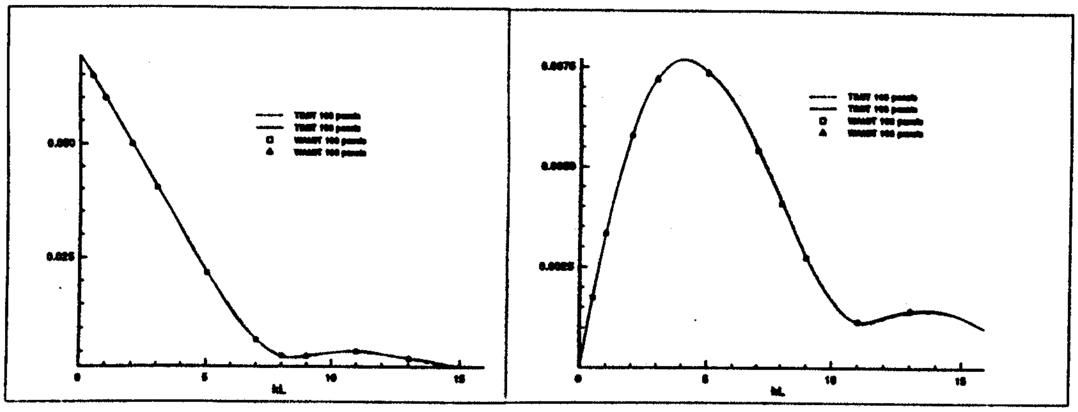

Figure 2. Heave and pitch exciting force (moment) amplitudes for a Wigley hull.

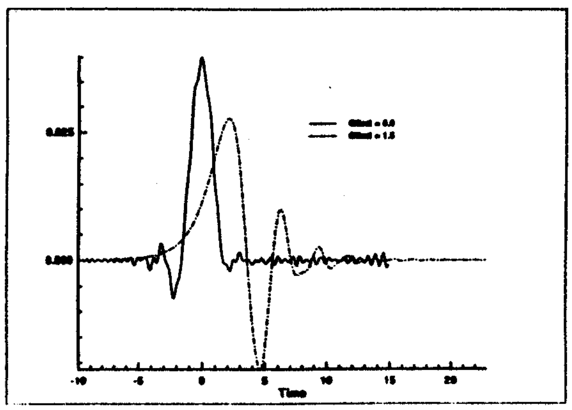

Figure 3. Heave exciting force i.-r.f. for a Wigley hull w&w/o offset.

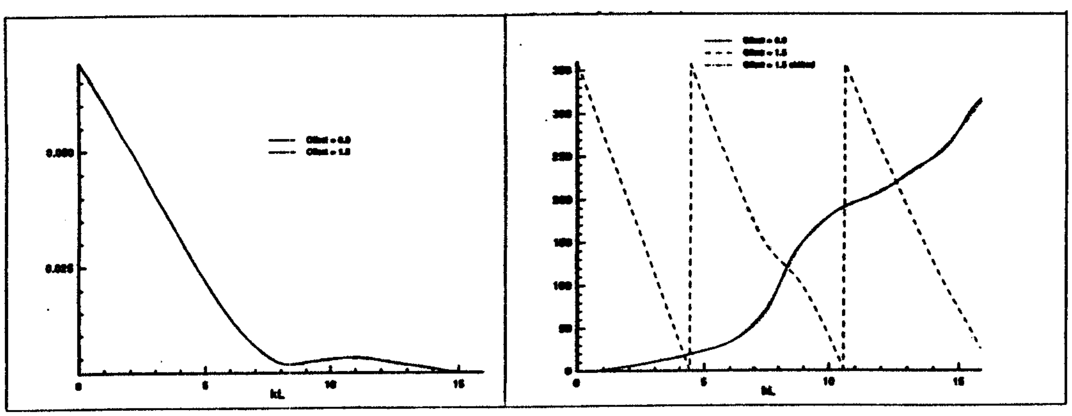

Figure 4. Heave exciting force amps. and phases, Wigley hull w&w/o offset.

Kim: In your integral equation for a scattering potential, you have two terms on the right-hand side. Which one is computationally more expensive? What is the typical ratio of CPU time between your original formulation and the modified one?

Korsmeyer: In our time domain codes we compute the wave part of the Green function and its derivatives at each time step and store these quantities on the disk for use in the convolution term at subsequent times. The computation of this convolution term requires most of the effort (primarily I/0) once the LHS matrix has been factored. In the diffraction potential formulation the convolution term requires half as much I/0, and therefore the code is twice as fast as the scattered potential formulation.

Martin: I have a comment and a question. Comment: In the frequency domain, the integral equation for $\phi_D = \phi_I + \phi_S$ is well known; all one needs is that ϕ_I satisfies Laplace's equation everywhere inside the body *i.e.*, it must not have any singularities there. Question: Why does your method excite the first irregular frequency and not, say, the seventeenth?

Korsmeyer: As shown in results from the transient radiation problem, all (infinitely many) irregular frequencies will affect the solution. It is the case, however, that their presence is felt less strongly with increasing frequency. In practical terms, we tend to see the effect of the first, and perhaps the second, irregular frequencies.

Faltinsen: You said that the irregular frequencies were not a problem for a ship. I don't think that this is correct. They have importance in the calculation of loads (bending moments, shear forces, etc.).

Korsmeyer: You are right.