

## Ship Motions with Nonlinear High Speed Effects

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### Introduction

This describes ongoing efforts to improve predictions of ship motions at high forward speed in waves of moderate amplitude by accounting for the nonlinear effects of steady forward motion flow on the motions in waves. Conventional ship motion theories (strip theory) often do not result in good predictions for ships with forward speed, even for slender ships in moderate waves. While proper three-dimensional theories exist for the zero speed case, three dimensional theories with forward speed are still in the development stage. Forward speed enters the ship motion problem in three ways in addition to the frequency of encounter effect: the body boundary condition, the free surface boundary condition, and the change of hull form due to sinkage and trim.

Forward speed effects enter the body boundary condition through the so-called  $m$ -terms (presented below), which involve second derivatives of the steady forward motion potential. A limited amount of experience with the Ogilvie-Tuck<sup>1</sup> rational strip theory, which includes forward speed effects in the free surface boundary condition more completely than do strip theories, as well as more complete expressions for the body boundary conditions, resulted in considerably improved prediction of added mass, damping, and motions for the few hull forms examined. In strip theory, the flow field is taken as just the uniform stream, with the disturbance due to the body neglected. Recently Sclavounos and Nakos<sup>2</sup> have shown greatly improved results for two hull forms by taking the double-body flow as the steady forward motion potential and linearizing the unsteady problem about this.

It is reasonable to expect that change in the mean wetted hull surface due to sinkage and trim at forward speed will also result in changes to the motions as well, particularly at high speeds.

In the past several years there have been developed improved solutions of the steady forward motion problem. More recently, numerical solutions of the full nonlinear steady motion problem are becoming available. This together with the continued improvement of available computers holds forth the possibility of fully accounting for the effects of forward speed on ship motions.

The present work outlines an approach being taken currently to utilize the full nonlinear steady forward motion potential as the reference solution in developing a full three-dimensional linear ship motions theory. Bertram<sup>3</sup> has recently undertaken a similar theory, based on the steady forward motion solution of Jensen<sup>4</sup>.

### Exact Boundary Value Problem

The exact boundary value problem in fixed coordinates  $x_0$  for the arbitrary motion of a three-dimensional body in an inviscid fluid with a free surface is given by

$$\nabla^2 \Phi = 0 \quad \text{in the fluid}$$

$$(\mathbf{V}_S - \mathbf{V}) \cdot \mathbf{n} = 0 \quad \text{on the ship surface}$$

the kinematic free surface condition

$$D/Dt(\zeta - z_0) = 0 \quad \text{on } z_0 = \zeta$$

the dynamic free surface condition

$$\Phi_t + (1/2)V^2 + gz_0 = 0 \quad \text{on } z_0 = \zeta$$

which can be combined as

$$\Phi_{tt} + 2\nabla\Phi \cdot \nabla\Phi_t + (1/2)\nabla\Phi \cdot \nabla(\nabla\Phi \cdot \nabla\Phi) + g\Phi_{z_0} = 0 \quad \text{on } z_0 = 0$$

and suitable conditions on the behaviour of  $\Phi$  at large distances from the body.

### Nonlinear Steady Forward Motion Problem

If we consider only the steady forward motion of the body, then in coordinates  $x$  moving with the body and writing the potential as

$$\Phi(\mathbf{x}_0, t) = U\bar{\phi}(\mathbf{x})$$

and the resulting flow field as

$$\mathbf{W} = U\nabla(\bar{\phi} - x)$$

the above problem becomes

$$\mathbf{W} \cdot \mathbf{n} = 0 \quad \text{on } \bar{S}$$

$$(1/2)\mathbf{W} \cdot \nabla(W^2) + g\bar{\phi}_z = 0 \quad \text{on } z = \bar{\zeta}$$

where the steady free surface is given by

$$\bar{\zeta} = -(1/2g)(W^2 - U^2)_{z=\bar{\zeta}}$$

### Unsteady Free Surface Boundary Condition

If we next linearize the original problem about the above steady solution for small motion, writing the total potential as

$$\Phi(\mathbf{x}_0, t) = \phi(\mathbf{x}, t) = U\bar{\phi}(\mathbf{x}) + \varphi(\mathbf{x}, t)$$

and decomposing the unsteady linear potential  $\varphi$  in the usual way

$$\varphi = [A(\varphi_0 + \varphi_\tau) + \sum_{j=1}^6 \xi_j \varphi_j] e^{i\omega t}$$

the resulting boundary value problem is

$$\frac{\partial}{\partial n}(\varphi_0 + \varphi_\tau) = 0 \quad \text{on } \bar{S}$$

$$\varphi_{jn} = i\omega n_j + U m_j \quad \text{on } \bar{S}$$

where

$$(n_1, n_2, n_3) = \mathbf{n}$$

$$(n_4, n_5, n_6) = \mathbf{x} \times \mathbf{n}$$

$$(m_1, m_2, m_3) = -(\mathbf{n} \cdot \nabla)(\mathbf{W})$$

$$(m_4, m_5, m_6) = -(\mathbf{n} \cdot \nabla)(\mathbf{x} \times \mathbf{W})$$

$$\begin{aligned} & -(\varphi_t + \mathbf{W} \cdot \nabla \varphi) \left[ (1/2) \frac{\partial}{\partial z} (\mathbf{W} \cdot \nabla W^2) + g \bar{\phi}_{zz} \right] / [g + \mathbf{W} \cdot \mathbf{W}_z] + \varphi_{tt} + 2\mathbf{W} \cdot \nabla \varphi_t + \\ & \mathbf{W} \cdot \nabla (\mathbf{W} \cdot \nabla \varphi) + \frac{1}{2} \nabla \varphi \cdot \nabla (W^2) + g \varphi_z = 0 \quad \text{on } z = \bar{\zeta} \end{aligned}$$

with the free surface given by

$$\zeta = \bar{\zeta} - [(\varphi_t + \mathbf{W} \cdot \nabla \varphi) / (g + \mathbf{W} \cdot \mathbf{W}_z)]_{z=\bar{\zeta}}$$

### Numerical Solution

We seek a numerical solution in the form of a distribution of Rankine sources over the wetted surface of the body at rest and a suitable portion of the mean free surface  $\bar{\zeta}$  near the body. The potential solver used is that of Y. H. Kim and Lucas<sup>5</sup>. This allows a choice of flat panels with constant source strength and quadratic panels with linear source strength.

The numerical solution of the steady forward motion problem used as a reference flow is due to Y. H. Kim and Lucas<sup>5</sup>. This solution is based on an iterative method to find the exact steady nonlinear free surface position  $\bar{\zeta}$  and the corresponding steady forward motion potential  $\bar{\phi}$ .

### Unsteady Solution

Solution of the unsteady problem is based on the nonlinear steady motion potential described in the previous section, setting up boundary conditions utilizing influence coefficients from the potential solver described above, implement suitable approximations for the radiation condition, solve for the unsteady potential using a modified version of the solver from NLSWIFT, and evaluate the resulting forces and motions.

Several potential numerical problems exist. Second derivatives of the potential obtained from a panel solution are often very sensitive to exact details of panelization, etc., and consequently may be unreliable. Slavounos and Nakos<sup>2</sup> evaded this difficulty on the body by use of Stokes' theorem. In the present effort, not only are second derivatives of the steady potential  $\bar{\phi}$  required on the body, but also *third* derivatives of this potential are required on the free surface. In the present case, use of quadratic panels should greatly ease this problem, at the expense of some computer time. Suitable numerical radiation conditions also need to be applied.

### Status

At the time of writing, implementation is in progress.

## Acknowledgments

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## REFERENCES

1. Ogilvie, T. F. and E. O. Tuck, "A Rational Strip Theory for Ship Motions, Part 1," Department of Naval Architecture and Marine Engineering, University of Michigan, Technical Report Rep. 013, Ann Arbor, MI (1969).
2. Sclavounos, P. D. and D. Nakos, "Ship Motions by a Three-Dimensional Rankine Panel Method," In: *Eighteenth Symposium on Naval Hydrodynamics*, Berkeley, California (Aug 1990).
3. Bertram, Volker, "A Rankine Source Method for the Forward-Speed Diffraction Problem," Universität Hamburg, Technical Report IfS Report 508 (1990).
4. Jensen, G., "Berechnung der statären Potentialströmung um ein Schiff unter Berücksichtigung der nichtlinearen Randbedingung an der Wasseroberfläche," Universität Hamburg, Technical Report IfS-Report 484 (1988).
5. Kim, Yoon-Ho and Thomas R. Lucas, "Nonlinear Ship Waves," In: *Eighteenth Symposium on Naval Hydrodynamics*, Berkeley, California (Aug 1990).

**Kring:** How are radiation conditions enforced for the unsteady and steady flows?

**McCreight:** The steady flow solution of Kim & Lucas relies on the damping properties of the numerical scheme for handling the free surface boundary condition to enforce the radiation condition of no waves upstream. For the unsteady problem, the state of the art is to do calculations for  $\tau > \frac{1}{4}$  and rely on the scheme used for the steady solution to again suppress upstream waves. The  $\tau < \frac{1}{4}$  case is still a research problem.

**Yue:** When the nonlinear steady potential is used, it seems clear that by merely going from constant to linear variation of the solution on the panel, you will not be able to avoid the difficulty associated with the presence of up to third derivatives on the surface. (For the nonlinear problem  $\frac{\partial \phi}{\partial z}$  will have nontrivial tangential components).

**McCreight:** One higher order term in the free surface boundary condition includes a third derivative. It remains to be seen how serious a difficulty this is in practice.

**Zhao:** In your numerical calculation of the nonlinear steady potential, do you have any numerical problems at the intersection points between the body surface and the free surface?

**McCreight:** I have not noticed any such difficulty with the steady potential, even though handling of the body-free surface intersection leaves something to be desired. In the iteration to satisfy the nonlinear free surface condition, nodes are moved only vertically, so that for a flared hull there can be a gap between adjacent panels, or an overlap. This is briefly described in Ref. 5.

**Söding:** You apply higher-order panel methods involving also the curvature of the body surface. What do you do when the body has a sharp edge, for example, at the bow? Is there an additional line integral applied?

**McCreight:** No special measures are taken. Details of the theory for the potential solver are given in F. T. Johnson, "A General Panel Method for the Analysis and Design of Arbitrary Configurations in Incompressible Flows", NASA CR-3079, 1980.