Ship Oscillations and Wake Solitons

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Abstract

Stimulated by satellite photographs of narrow V-wakes behind a ship, Brown et al (1989) have made direct field measurements behind the coast guard cutter of narrow V-wakes behind a ship *Point Brower*. They observed a distinctly nonlinear feature of a pair of nearly steady wave envelopes, one on each side of the ship path. Beginning approximately from the bow, the axis of an envelope is a straight line inclined at about 11° from the ship path and therefore inside the Kelvin wedge of half angle $\theta = 19.5^\circ (\tan\theta = 1/\sqrt{8})$. These two envelopes persist beyond 1 km behind the ship, with some periodic modulations along their axes. Brown et al. suggest that these groups of finite amplitude waves may be relevant to fairly wide V-wakes under light to moderate wind. At present their role in the radar image of the narrow V-wakes has not been directly confirmed, since the complete physics must involve a myriad of factors whose importance varies from case to case. Nevertheless the existence of these nonlinear envelopes appears to be hard to dispute. It is therefore interesting to examine the origin of their formation.

Brown et al (1989) suggested that the observed wave groups resemble the oblique soliton envelopes which are nonlinear solutions to the two dimensional cubic Schrodinger equation, found by Zakharov & Rubenchik (1974). Accepting this suggestion, one remaining question here is how the envelopes are initiated at the ship.

A likely cause of the soliton initiation, yet unexplored in the literature, is the oscillation of an elongated ship, induced by ambient sea waves under a moderate wind. Although ship oscillations were not recorded by Brown et al, the existence of ambient waves was

noted as a possible explanation for the wavy modulation of the observed wave envelope from the theoretical soliton. Because of Doppler's effect, an incident wave of frequency ω_o and inclined at the angle α with respect to the ship path appears to a ship-bound observer as oscillations at the encounter frequency

$$\omega = \omega_o \left(1 - \frac{V\omega_o}{g} \cos\alpha\right) \tag{1.1}$$

where V is the ship speed. According to the linearized theory the ship is induced to oscillate at the same encounter frequency ω. For a head sea, $\alpha = \pi$, ω is greater than ω_o of the incident waves. The advancing ship therefore acts as a plane piston, traveling in its own center plane and radiating relatively short, nearly plane waves away from its course, resulting in wave packets trailing in the wake. This heuristic picture near the ship is supported by the linearized slender-body theory of Sclavounos & Nakos (1990). In their studies forced heaving of an advancing ship is shown to produce straight wave crests parallel to the ship path, provided that the ratio $V/C = Uk/\omega$ is large enough where k, and C are respectively the wave number and the phase velocity of the radiated waves. In the observations of Brown, et al. the wave crests are theoretically estimated to be inclined with respect to the ship axis at 22.8°. Their estimate is based on the measured frequency and the assumptions that waves are stationary with respect to the ship. This inclination is likely due to the usual hull form where the bow is more pointed than the stern, hence the equal-phase lines should be slightly inclined with respect to the ship axis. If one adopts the reasoning of Gu (1989), the sharp interface bounding the turbulent breaking bow wave whose position oscillates with the induced heave or pitch, may also act as an oscillating piston inclined with the ship axis, hence enhances the inclination of the radiated wave crests. Once leaving the ship, the wave packets are then expected to evolve under the influence of nonlinearity, as stipulated by Brown et al.

The edges of Kelvin waves in the photograph (Fig 2) of Brown et al appear quite far ouside the envelope solitons. Since Kelvin waves are caused by the finite volume displacement of a steadily advancing ship, it is a reasonable first step to exclude them by considering only a ship of negligible volume, so as to focus attention on the nonlinear evolution of the radiated waves in the far wake. Diffraction should also be unimportant if the ship is slender and advances into head or following seas. Specifically we choose a ship-bound coordinate system with the y axis coinciding with the ship axis and the positive x axis pointing to the port side of the ship. A uniform ambient current V along the y axis is then present. If attention is focussed on the far wake, the amplitude of the radiated waves, which can be obtained by a separate calculation based on existing linearized theories can be considered as given along the centerline of the ship. This boundary condition is different in physical origin from that used by Akylas et al(1989), and Hall & Buchsbaum (1990) in that the waves comprising the envelopes are not stationary with respect to the ship, and may also represent the effect of other oscillating and traveling line disturbances such as a strip of surface pressure. In short, the near wake (linearized) is just the superposition of oscillating point sources (Lighthill, 1967) along a line segment. At the outer limit of the near wake, one sees essentially plane waves, with long but finite crest length, propagating away from the ship axis. In the far wake the envelopes of these laterally radiated waves are shown to be governed by a one dimensional cubic Schrodinger equation in a moving and distorted coordinate system. After some manipulations, the boundary value problem is reduced to a standard initial value problem. Known results from earlier mathematical theories are used to infer physical consequences in the present problem. Numerical illustrations are also included for either forced heave or pitch. Several predicted features deserve experimental confirmation in the laboratory.

Although motivated by the field experiments of Brown et al, their data were taken only at one fixed station and no records of ship motion or ambient sea waves are available. This precludes a decisive check of the present theory. Neverlethess some gross estimates show qualitative relevance, including the inclination of the envelope axis.

Cao, Y.: It seems to me that in the photograph of the wake of the ship "Point Brower", the waves along the line of half-angle $\theta \approx 11°$ are individual waves. Also, it seems to me that you are comparing the solutions of the cubic Schrödinger equations, which is the wave envelope, to those individual waves. If what we see in the photograph were the wave envelope, there should be carrying waves of very short length compared to the wave length of the envelope, so that the Schrödinger equation can be used. However, I do not see very short waves along the line of $\theta \approx 11°$. Could you explain?

Mei & Naciri: The quality of the transparencies is probably not good enough for you to see the short waves. Please refer to the paper of Brown, *et al* in JFM for clearer photos. They will be apparent.

Tulin: How does your theory predict the inclination of the wave crests on the ray? How does this compare to observation?

Mei & Naciri: The observed inclination begins very near the ship. I speculate that the rather blunt bow with its breaking waves acts as an oscillating piston inclined with respect to the ship axis. This creates wave crests which are inclined near the ship. In our theory, this aspect is treated in the boundary condition. Numerical experiments show that, with inclinations, the qualitative features of the far field solitons are unchanged.* Only the axes of the solitons are changed slightly.
*Ref.: Mei & Naciri: Proc. Roy. Soc. London, March, 1991.

Tuck: Due to the randomness in the sea-wave excitation, ships oscillate at a very broad spectrum of frequencies. Each component frequency presumably creates its own soliton, at its own angle. The net effect must be a very diffuse pattern, over a large range of angles. But this is not what the photograph shows. Instead, the photograph shows an <u>amazingly</u> narrow trailing interior wake pattern!

Mei & Naciri: It is unlikely that the wake solitons are observed all the time. The theory simply suggests that if the oscillations induced by the incident sea are narrow-banded, wake solitons should occur as predicted. Experiments by Brown, *et al*, being incomplete (partially recorded), can only be regarded as a motivation for theoretical works. Whether or not this or any theory can fully explain their observations is uncertain. Indeed without onboard measurement of ship motions, one cannot be sure that the ship oscillation is not narrow-banded. More than looking at a few photographs is needed here.