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TRANSVERSE WAVE CUT ANALYSIS BY A RANKINE PANEL METHOD

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During the 1960's significant interest was expressed in developing algorithms for the prediction of ships' wave resistance given the corresponding steady wave field. Originally wave pattern analysis was built upon experimental measurements of the wave elevation and/or its gradients behind the ship and earned its living as alternative to the à la Froude prediction of the wave resistance. Different techniques proposed for the wave pattern analysis are reviewed comprehensively by Eggers, Sharma and Ward (1967).

Recently, similar interest has been reanimated; this time, however, in order to serve slightly different goals. The free wave spectrum, before treated mainly as an intermediate result, is the ship's most characteristic footprint and may be used for identification purposes. Moreover, wave spectrum calculations may be employed in the investigation of the accuracy and robustness of numerical solutions to the ship wave problem, very much like the Karman-Trefftz plane calculations in lifting surface theory.

The scope of this study is to adopt Egger's *transverse wave cut analysis* inside the frame of numerical solvers for the wave resistance problem based on Rankine panel methods. Besides the self-contained interest in the numerical prediction of the free wave spectrum, attention is also focused on the evaluation of the wave resistance by conservation of the fluid's momentum. Comparison of the so derived resistance to the result of pressure integration over the hull establishes a solid criterion for the evaluation of the robustness of the numerical solution.

The free wave spectrum

Consider a cut of the wave field perpendicular to the track of the ship at $x = \text{const.}$ and $-\infty < y < \infty$, along which the Fourier Transform of the wave elevation $\zeta(x,y)$ and its longitudinal slope are defined as

$$C(x,v) = \int_{-\infty}^{\infty} dy \zeta(x,y)\, e^{ivy} \qquad \text{and} \qquad C_x(x,v) = \int_{-\infty}^{\infty} dy \frac{\partial \zeta(x,y)}{\partial x}\, e^{ivy} \quad . \tag{1}$$

The free wave spectrum is give by the limit at $x \to \infty$ of

$$H(v;x) = \frac{1}{8\pi} \frac{2u^2-1}{u^2} \left[C(x,v) + \mathrm{i}\frac{C_x(x,v)}{u} \right] e^{iux} \quad , \quad v \in [-\infty, +\infty] \quad , \tag{2}$$

where u and v are normalized by g/U^2 and satisfy the dispersion relation $u^2 = \sqrt{u^2+v^2}$.

The wave elevation and its slope are numerically predicted, rather than experimentally measured, by means of a Rankine panel method which is based on Green's theorem and a quadratic spline representation of the unknown potential over the free surface and the hull (Nakos and Sclavounos (1990)). The solution scheme is free of numerical damping and consequently it allows accurate prediction of the wave field at large distances downstream of the hull.

The transverse cut method is preferred over its longitudinal counterpart mainly due to the attractive property of the transverse wave elevation records to be of finite extent. On the other hand the dependency of H (eqn. (2)) on the longitudinal position of the cut is completely eliminated only at infinite distances from the hull and under the assumption of no discretization errors in the enforcement of the free surface boundary condition.

Numerical experience suggests that near-field effects are, for all practical purposes, negligible at distances of about half a ship length downstream of the stern. Nevertheless, the discretization of the free surface induces numerical dispersion (see Nakos and Sclavounos (1990)) and therefore prevents the complete elimination of the dependence of H on the longitudinal position of the wave cut.

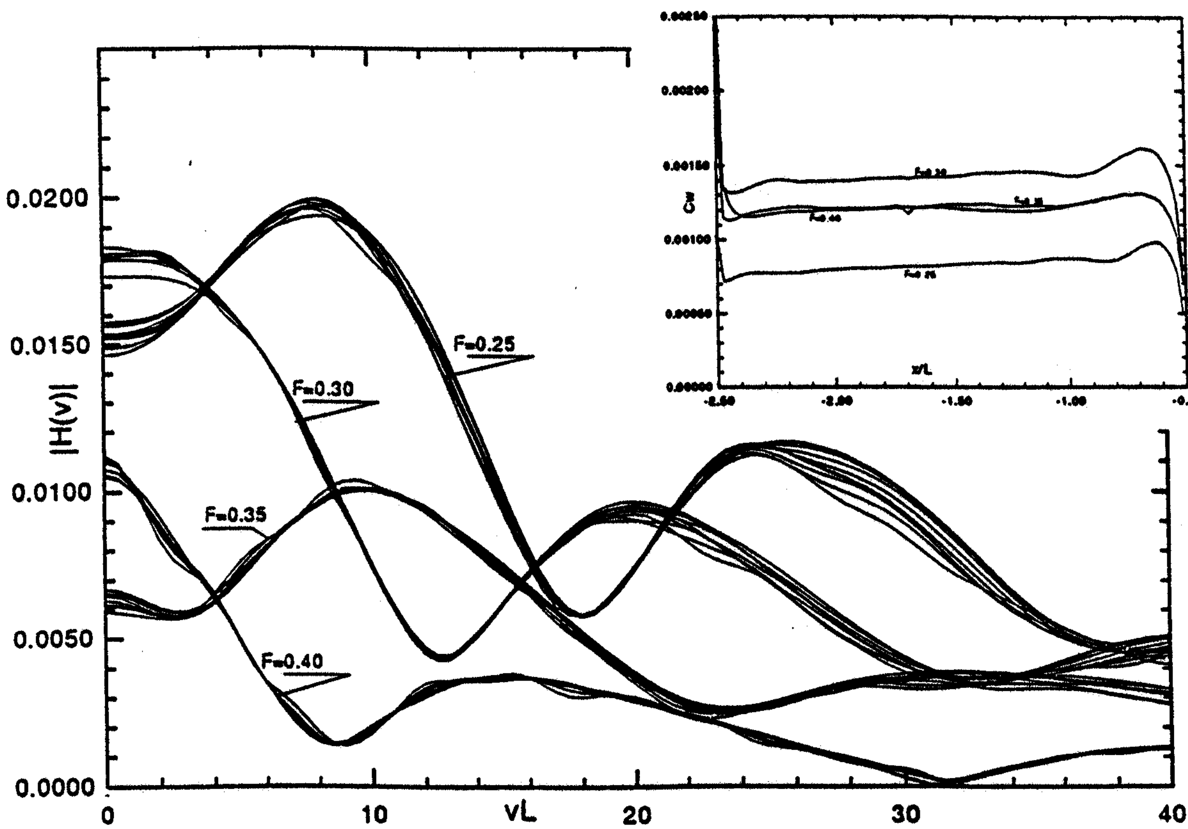

Figure 1 : Free wave spectrum and wave resistance for the Wigley model

Figure 1 illustrates the free wave spectrum for the Wigley model advancing at Froude numbers varying over the range $[0.25, 0.40]$. The sets of lines corresponding to each Froude number are obtained by considering different transverse wave cuts uniformly distributed over the interval $x \in [-L, -2L]$. The ship's bow and stern are at $\pm L/2$, while the computational domain extends longitudinally to $-2.5L$ and it is discretized by 4092 panels over half the free surface and the hull. The wave flow is linearized about the uniform incoming stream (Neumann-Kelvin problem). Evidently, the effect of finite distance from the wavemaking source and of approximation errors induce a small oscillatory

'noise' which may be eliminated by some extrapolation scheme of even low degree of sophistication. Convergence of the spectrum with respect to the position of the wave cut is typically lost close to the downstream end of the computational domain due to the abrupt truncation of the free surface.
Similar discussion applies to calculations of the free wave spectrum due to the Series-60-$c_B = 0.6$ hull, illustrated in Figure 2. In this case the wave flow is linearized about the corresponding double-body flow and the computational domain consists of 5820 panels extending downstream to $-2.6L$.

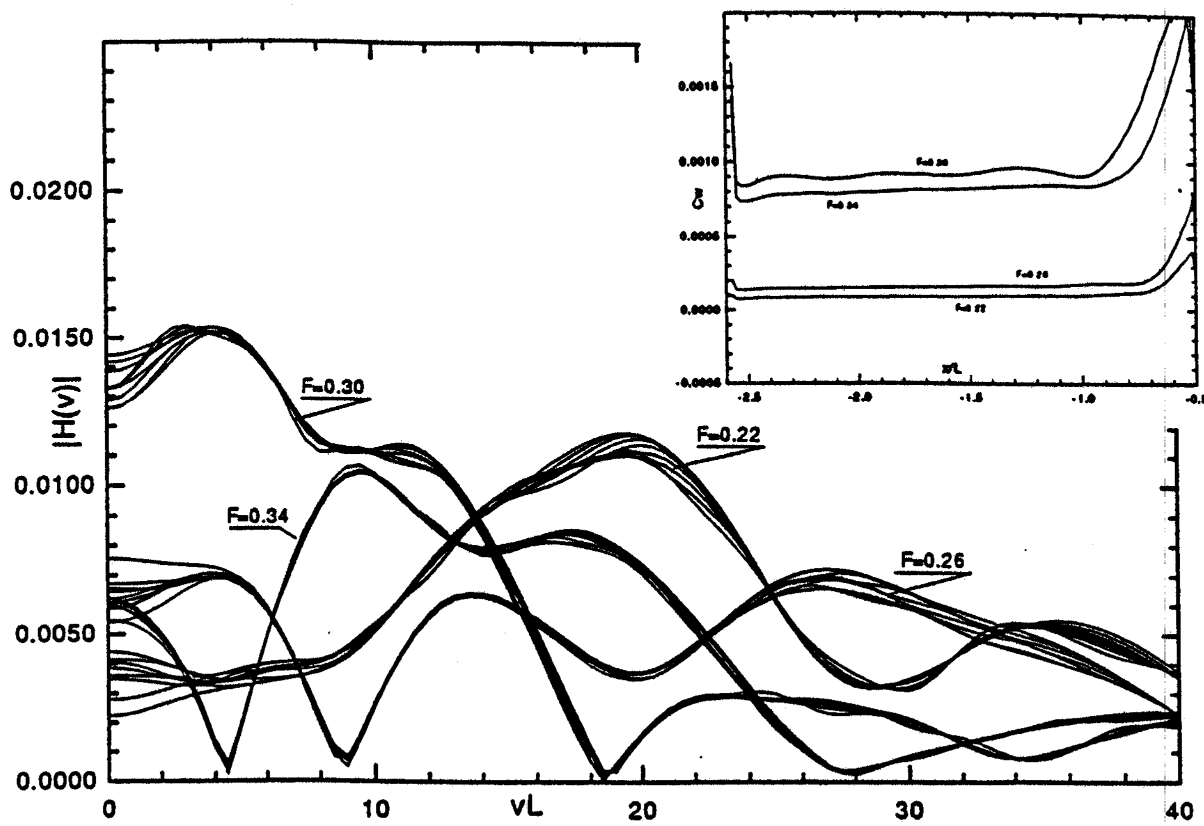

Figure 2 : Free wave spectrum and wave resistance for the Series-60-$c_B = 0.6$

The wave resistance

Consider the fluid region bounded by the hull, the free surface and a vertical control surface. Momentum is supplied to this control volume by the ship's forward motion and escapes downstream in the form of free waves. The wave resistance may be derived by momentum conservation and it is shown (see eg. Eggers, Sharma and Ward (1967)) to be

$$c_w = \frac{R_w}{\frac{1}{2}\rho U^2 S_w} = 4\pi F^4 \frac{L^2}{S_w} \int_0^\infty dv \, |H(v)|^2 \, \frac{1+\sqrt{1+4v^2}}{\sqrt{1+4v^2}} \quad , \tag{3}$$

where S_w is the wetted surface area and $H(v)$ the free wave spectrum as given by (2).
The convergence of c_w with respect to the position of the wave cut is illustrated in Figures 1 and 2 (small inserts) for the respective hull shapes and forward speeds.

Alternatively , the wave resistance force may be obtained by pressure integration over the wetted surface of the hull. In linear or quasilinear wave resistance theories the total flow is decomposed into a basis flow Φ and the wave flow ϕ. The free surface condition in subsequently linearized under the assumption of $\nabla\phi << \nabla\Phi$ and (in most cases) transfered onto the plane $z=0$. In consistency with the linearization assumptions, the wave resistance is given by

$$\begin{aligned} R_w &= -\rho \iint_{S_0} \left[\nabla\Phi\cdot\nabla\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi\right] n_x dS \;-\; \rho\iint_{\Delta S} [\,\nabla\Phi\cdot\nabla\phi + gz]\, n_x dS \\ &\simeq -\rho \iint_{S_0} \left[\nabla\Phi\cdot\nabla\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi\right] n_x dS \;+\; \frac{1}{2}\rho g \oint_{WL} \zeta^2 \frac{n_x}{\sin\alpha} d\ell \qquad , \end{aligned} \tag{4}$$

where S_0 is the portion of the hull below $z=0$, ΔS its difference from the actual wetted surface (runup surface) and α the flare angle.

Within the frame of Neumann-Kelvin theory it may be shown that the values for the wave resistance obtained from equations (3) and (4) are identical only for submerged bodies, a fact that has also been confirmed numerically. In the case of surface piercing bodies, even under the assumption that there exist no singularities along the waterline, relations (3) and (4) are not equivalent, an inconsistency often referred to as the 'Gadd's paradoxon' (see eg. Eggers (1979)).

Numerical evidence suggests that the steady wave flow around realistic ship forms develops singularities, especially near the stern, unless special care is taken. Work is currently in progress for the design and implementation of appropriate conditions to be applied along the waterline (or portion of it) aiming at the regularization of the flow. As a matter of fact, the agreement between the near-field and far-field evaluations of the wave resistance may be considered as a measure of the flow's smoothness. Then, and only then, robust solution of the linearized wave resistance problem may be claimed.

Acknowledgements

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References

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G.X. Wu: There are two more important reasons to use a longitudinal cut; one is that a transverse cut crosses the wake of the ship, and the other is that the width of the tank is limited.

Nakos: That is correct from the viewpoint of an experimentalist. In numerical investigations, however, the finite extent of the transverse wave records which eliminates any need for analytical "extrapolation," is the main advantage of transverse cut methods and probably over-shadows any other arguments.

Söding: One of my students made a similar evaluation of wave resistance using both pressure integration and transverse cuts of the wave field, but for a flow which satisfied the non-linear free-surface boundary conditions. The two wave resistance curves, when plotted against Froude number, were nearly parallel except in the low Froude number region where only the wave cut curve tended to zero. We attributed the speed-independent difference to pressure integration errors due to the finite number of body surface panels. Would you agree with this suspicion?

Nakos: In the case where the fully nonlinear flow is solved, the evaluation of the wave resistance by pressure integration and by far-field momentum flux should give identical results. Of course in practice these two results will differ due to discretization errors. I don't think, however that there exists any reason for this "numerical-error-effect" to be independent of the Froude number. Moreover, such an effect should disappear as the discretization becomes finer.

Kashiwagi: Regarding the two ways of evaluating the wave resistance, I have a comment. I found a similar problem in relation to the energy-conservation principle and discussed it in the Third Workshop. One of my conclusions was that, if we take into account the additional term coming from the intersection between the body and free surfaces, the agreement between the pressure integration and the far-field evaluation is perfect, (within numerical error of less than 0.1%). When not included however, the error was incredibly large.

Nakos: The conclusion drawn by Professor Kashiwagi absolutely agrees with my own experience. His study addressed the conservation of momentum in the linearized seakeeping problem, in which case I have also found that the "momentum flux" through the waterline is of major importance; unlike the case of wave resistance addressed in the present study.