Near Field Internal Waves Generated by a ship in a Shallow Thermocline

Marshall P. Tulin, University of California, Santa Barbara
Touvia Miloh, Tel-Aviv University
Pei Wang Yao, University of California, Santa Barbara
Yitao Yao, University of California, Santa Barbara

The internal wave fields generated by a ship traveling over or through a shallow thermocline are detectable by radar because of their interaction with naturally occurring wind waves. It has been a challenge for theory accurately to predict the form and amplitude of these wave fields.

Here two near field theories (A and B) are discussed and results presented, together with some experimental internal wave results based on model tests in our laboratory.

In both of these theories, weak coupling between waves on the sea surface and internal waves is assumed, so that for purposes of computing the latter, the sea surface may be replaced by a rigid lid. It is also assumed that the effects of stratification are sufficiently weak in comparison to the effects pertaining in the absence of stratification, that the internal wave field near the ship may be assumed a perturbation of the homogeneous flow field there; therefore the internal wave near-field appears as if forced by the invicid, homogeneous near-field, which can be calculated exactly. This forcing is largely due to the displacement of isopycnic surfaces due to the passage of the ship. The response of these surfaces near the ship, resulting in the generation of internal waves, is then calculated numerically based on approximate field equations (A and B). In each of these theories, different assumptions have been made as described below. They do share in common that each provides that the boundary conditions on the hull are satisfied approximately, even in the presence of the internal wave perturbation.

The earliest theory, A, is restricted from the beginning to the case of a sharp interface (two-layer), and is otherwise asymptotic to the long wave limit, i.e., the internal wave perturbation near the ship is assumed to have a spectral content concentrated at small values of kh, where k is the wavenumber and h the thermocline depth. The field equation relates the wave deflection of the sharp thermocline in terms of the horizontal co-ordinates (planar). The effects of dispersion are given by a 2-d integral of Cauchy type, and may be neglected in the near field when the thermocline is very small compared with the ship length. The other effects, including convection of the wave by the flow field, are represented by a second order PDE, which in the case of sufficiently large ship speed (in comparison to the internal wave speed) is of hyperbolic type. This may be solved numerically by the method of characteristics. In the resulting solutions, the disturbance is seen to propagate transverse to the direction of motion. A feature of this theory is that the ship is not assumed thin, and the influence of the largely transverse internal wave on the hull is corrected by a suitable distribution of planar quadrupoles placed on the hull centerline. Non-dispersive calculations for spheroidal double-models have been made and show a wave of depression originating on either side of the hull over its forward section and propagating outwards transversely, followed by a wave of elevation originating over the aft portion of the hull. The net effect is the creation of a triple-lobed pattern at a short distance aft of the ship.

The limitations of theory A motivated the development of a theory capable of dealing with arbitrary density stratifications and without asymptotic assumptions concerning kh. It makes use of the essentially transverse nature of the internal wave field and of the slenderness of the hull; it may be considered a slender body theory. In B, the field equations describe the behavior of the internal wave stream-function in the transverse plane. The upstream influence is entirely represented by the homogeneous flow about the hull. The 2-d equation may therefore be solved successively for planes marching aft, and the development of the wave observed. Non-linear wave effects may also be included. The calculation is based upon finite element techniques. Comparisons are made between theory A and experimental results.

The resulting deflection pattern may be normalized through appropriate scale factors for the downstream (x) and transverse (y) directions & for a given ship & thermocline show considerable similarity for a large range of Froude numbers (F = ship speed / limiting interfacial wave speed). This is shown as Figure 1, where the vertical deflections have been non-dimensionalized by the thermocline depth.

For a Froude number of about 3, a comparison of calculated & experimental kinematical wave patterns is shown as Figure 2a & of wave amplitudes at various longitudinal cuts as Figure 2b. The calculated wave patterns are based on far field theory using an amplitude spectrum determined from a transverse cut through the triple lobe near field pattern at $x/(LF^{1/2}) = 0.3$, where x is measured from the ship mid-section and L is the ship length. In the transverse cut at this special location, the near field pattern on the ship centerline reaches an apex & begins to spread thereafter, see Figure 3 where the entire calculated near field pattern is shown. A comparison of the calculated amplitude spectrum, $A^*(kh)$, in this case, and the values derived from the experimental wave measurements is shown as Figure 4.

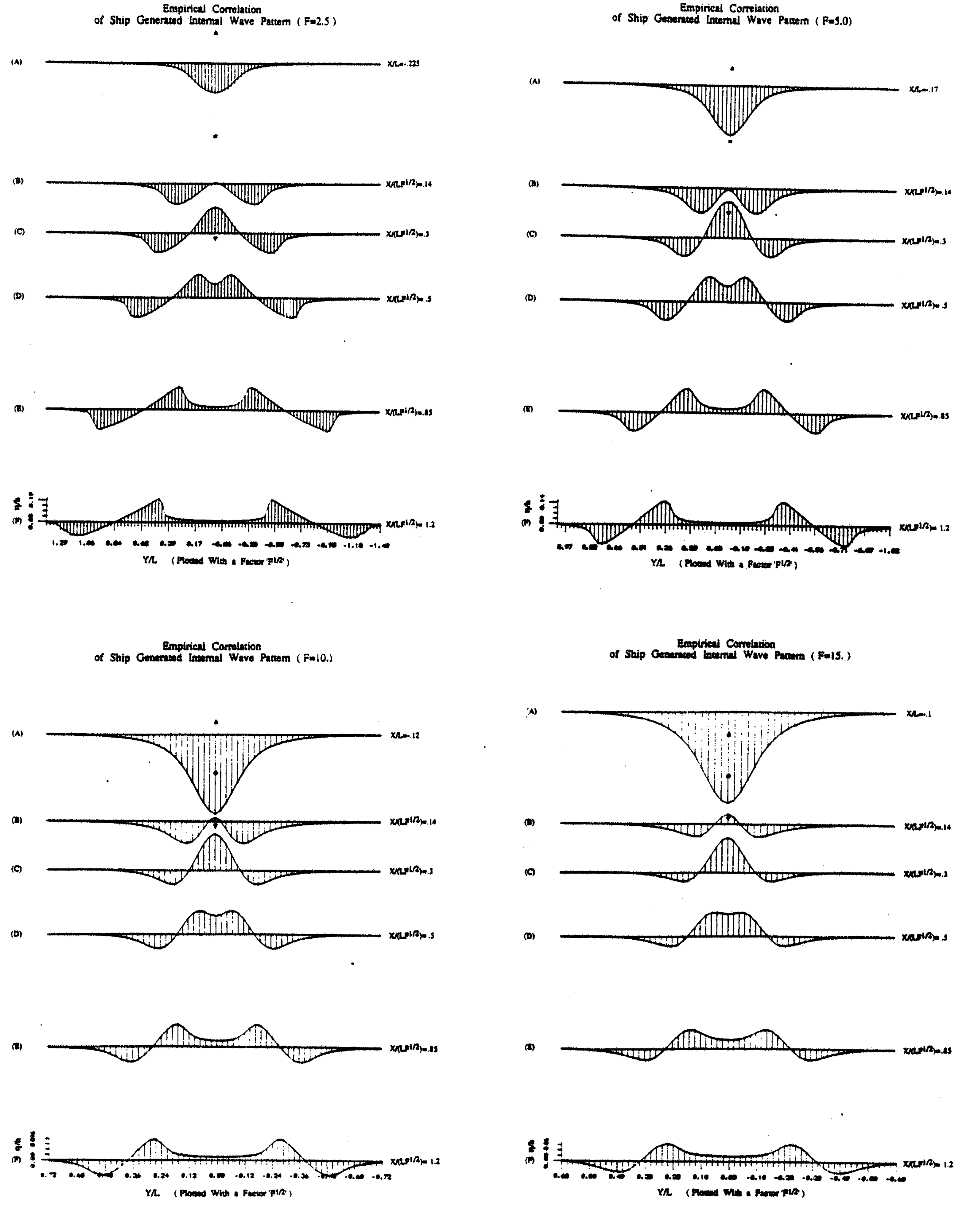

Figure 1 : Empirical Correlation of Ship Generated Internal Wave Pattern (Δ - Bow; * - Midship; ∇ - Stern)

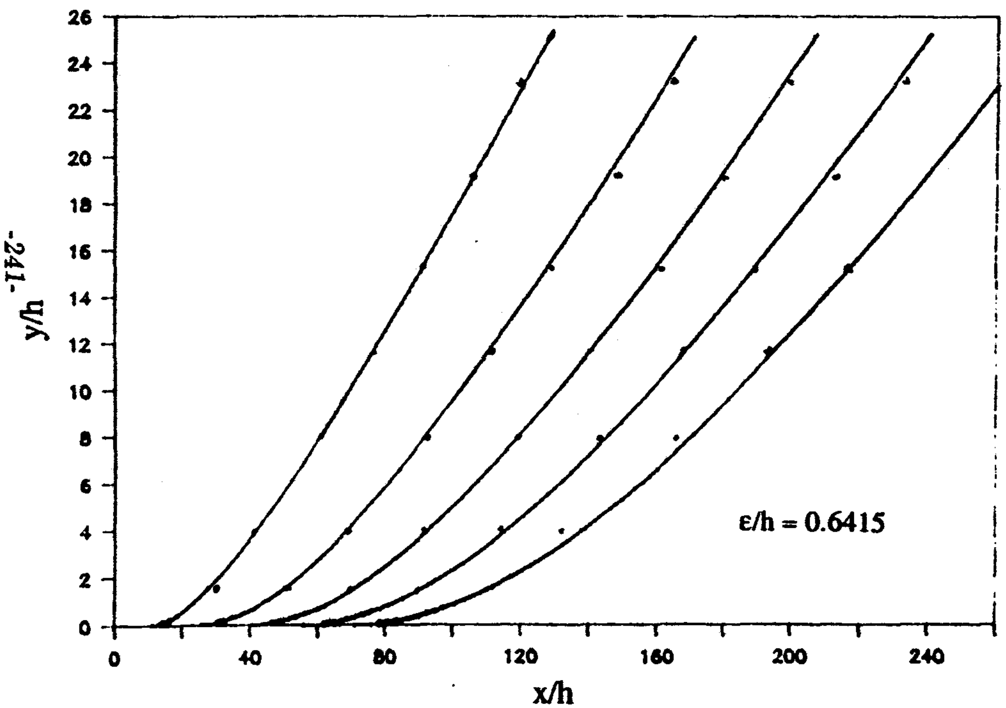

Figure 2a : Kinematical Wave Pattern From Far Field Theory (solid line) and Comparison with Experimental Results (dot)

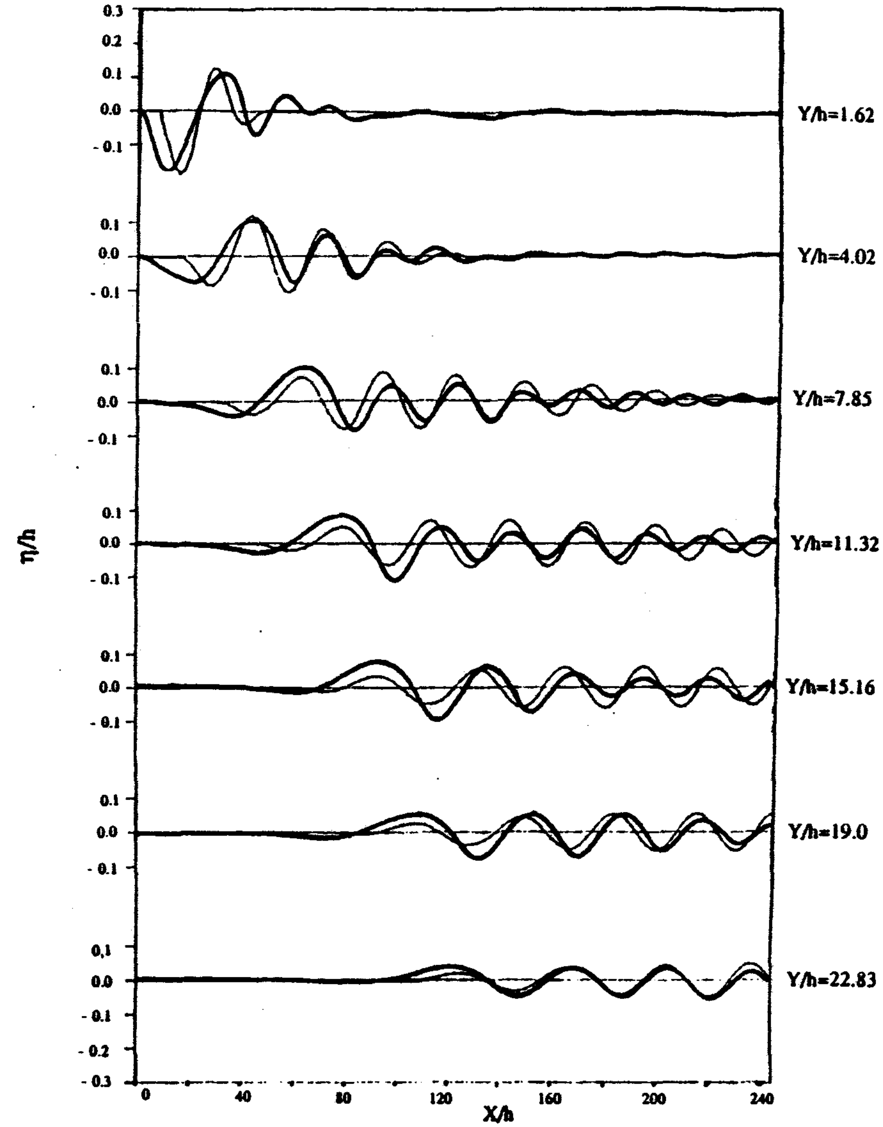

Figure 2b : Comparison of wave amplitude of theory (thin curve) with that of experiment (dark curve)

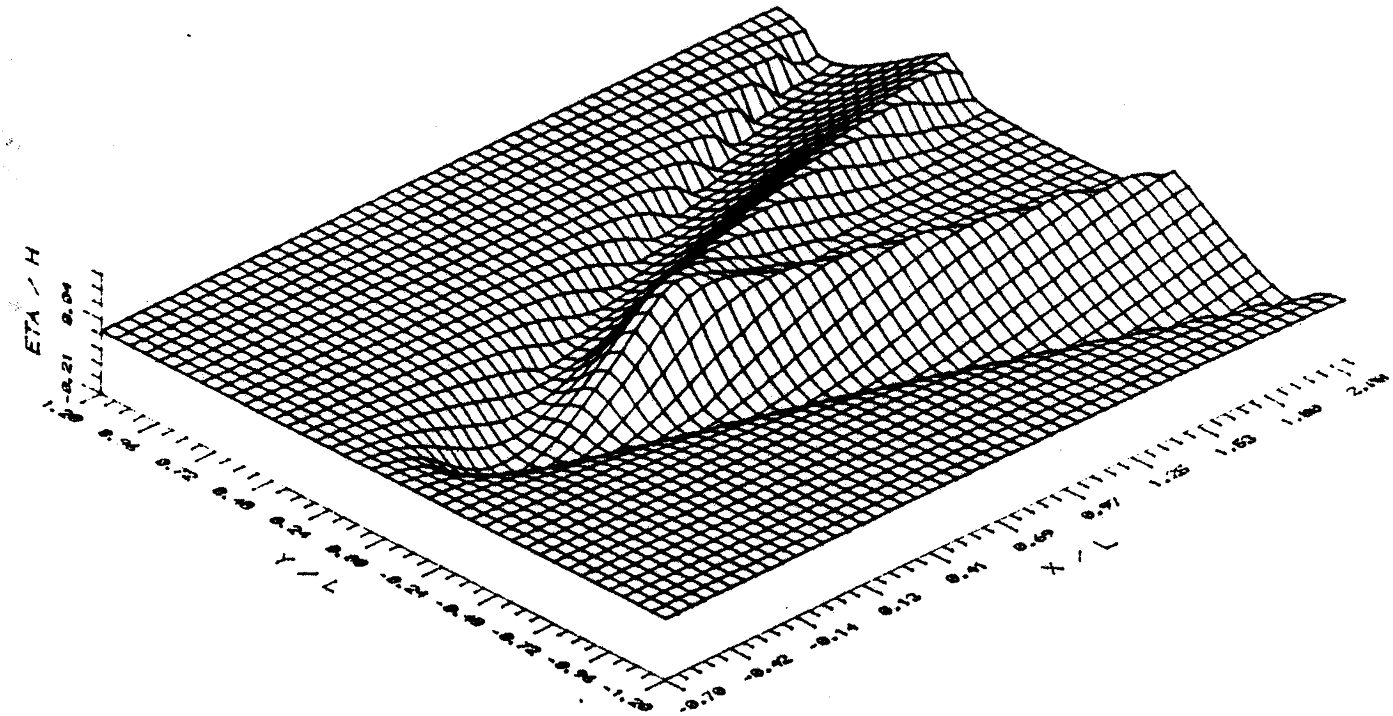

Figure 3 : Near Field Wave Pattern of Ship Generated Internal Wave
(F = 3.22 , h/L = .1)

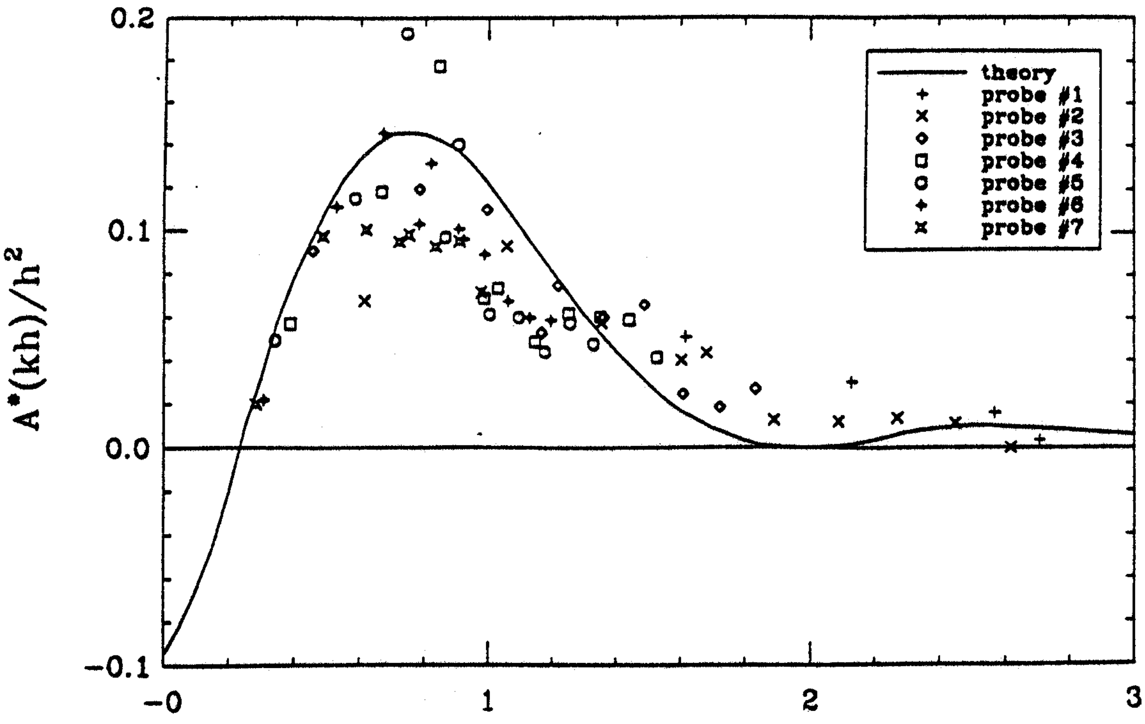

Figure 4 : Amplitude Spectrum from Near Field Theory (Solid curve) & Experiment (Symbols)

Grue: How will a large Froude number influence the effect of nonlinearity in your theoretical model? What kind of nonlinear effects, besides the generation of solitons, can appear for internal waves?

Tulin, Miloh, Yao, Yao: In method A, the equations contain nonlinear terms arising both from convection and from variations in wave speed due to changes in the interface depth. Therefore these equations can give rise to solitons, wave steepening and wave drift. In method B, which is restricted to high Froude numbers, but otherwise much less restrictive than A, the leading order nonlinear terms are retained, so that wave steepening, solitons and wave drift can also occur.

Beck: How was the thermocline generated in the experiment?

Tulin, Miloh, Yao, Yao: The tank was first filled to an appropriate level with salt water, and fresh water was gently put on top to form the upper level. Some inevitable mixing during this process results in a pycnocline with a finite thickness, just as in the ocean.