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LARGE-AMPLITUDE OSCILLATION OF TWO-DIMENSIONAL BODIES IN A VISCOUS FLUID WITH A FREE SURFACE

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Nonlinear flow due to forced sinusoidal heaving of a surface-piercing cylinder is analysed. This problem is of importance in the design and in the motion response of various floating structures that experience significant nonlinear wave and vortex-induced forces.

Field equations governing this problem are the incompressible Navier-Stokes equations. Free-surface conditions, determined by kinematic and stress-continuity relations, can be found in [1]. On the solid body the no-slip condition is to be satisfied. Approximate conditions are used at the contact points (intersection of free surface and body boundary) and on the open boundary. At the contact points, the tangential component of the velocity on the body is obtained assuming the shear stress to be zero. Pressure at the open boundary is assumed to be that of the static case for finite duration of time. Initial conditions corresponding to a quiescent fluid are used.

Solutions to the fully nonlinear problem are obtained using a primitive-variables based finite-difference method. The *projection* formulation developed in [2] is used for advancing the solution in time. According to this formulation, an auxiliary velocity field, which does not satisfy the equation of continuity, is first computed. The auxiliary field is then projected onto the divergent-free velocity and curl-free pressure-gradient fields. This method is extended in [3] for solving viscous free-surface flow problems in curvilinear coordinates. A grid-generation procedure based on variational principles and the concept of reference space has been successfully developed earlier and reported in [4]. At the Øystese workshop, we demonstrated that this grid-generation method is effective in handling steep and multivalued free-surface boundaries and also in providing means to control grid properties such as coordinate spacings and cell-area distribution [5]. The applications in [5] were for inviscid-fluid problems.

The computations for a viscous fluid proceed differently. The auxiliary velocity field is computed using the momentum equations initially with the pressure-gradient term neglected. A Poisson equation is solved to obtain the pressure field subject to the appropriate free-surface and body boundary conditions. Divergence-free velocity field is then computed from the auxiliary velocity field by taking into account the correctional effects of the pressure results. For the results presented in this work, first-order upwind scheme is used to treat the nonlinear convective terms of the momentum equation. Other spatial derivatives in the flow and grid equations are discretized to second order accuracy. The pressure Poisson equation is solved directly using a vectorized LU-decomposition code. Grid equations are solved iteratively, with the values of the previous instant of time as the initial guess, using successive over-under relaxation method.

Solutions are obtained for a range of frequencies ω and amplitudes a of oscillation and beam-to-draft B/D ratios. First, we present the results corresponding to small-amplitude oscillation ($a/B = 0.095$, $B/D = 2.0$, and $\omega\sqrt{B/2g} = 1.25$). Only the contribution of the pressure term in the stress-vector equations is considered in the evaluation of the heave-force component. Time histories of the heave force and the corresponding vertical displacement are given in Fig. 1. Positive values of the displacement curve indicate upward direction of motion. It can be noticed that the response is very sinosoidal. Next, solutions for a large-amplitude oscillation case ($a/B = 0.30$, $B/D = 1.0$, and $w\sqrt{B/2g} = \sqrt{2}$) are computed. Time-history of the computed force, as shown in Fig. 2, reveal important nonlinear effects. Velocity-vector and vorticity-contour plots for this latter case are given in Figs. 3a to 3f. When the body is descending, vortices are generated at the body sides near the sharp edges (Figs. 3a, 3b). As the body ascends the flow is primarily towards the void generated by the upward movement of the body (Figs. 3c, 3d). Figs. 3e and 3f depict well-defined vortices formed in the wake as the body-keel is approaching closer to the free surface. Secondary vortices are also generated at the keel by the primary wake vortices. It can be seen in these figures that parts of the vortices generated at earlier instants of time are also shed into the fluid.

Having developed an effective method for solving viscous free-surface flow problems, we can readily consider the effects of other modes of body oscillation. Inviscid-flow results are being obtained by the same method so as to assess the importance of viscosity in all the above cases. Extension to three-dimensional problems is also being planned.

References

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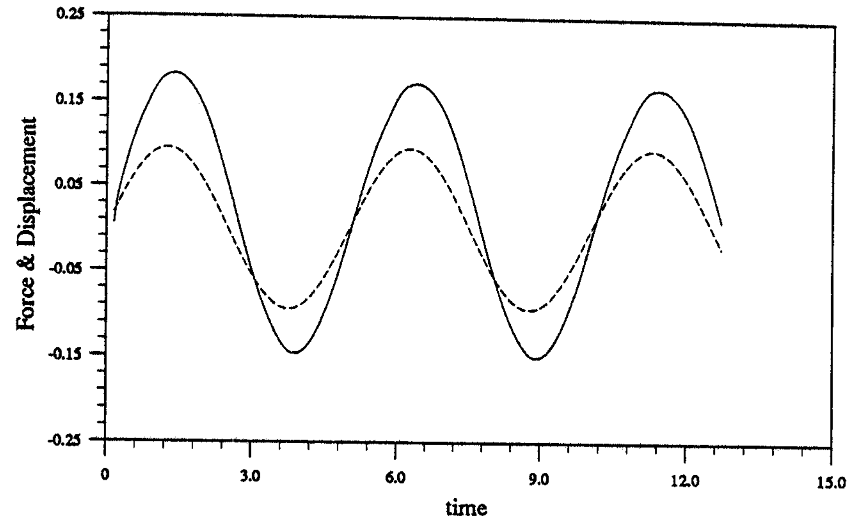

Figure 1: Time history of heave force (solid line) and displacement (dashed line) for the case $a/B = 0.095$, $B/D = 2.0$, and $w\sqrt{B/2g} = 1.25$.

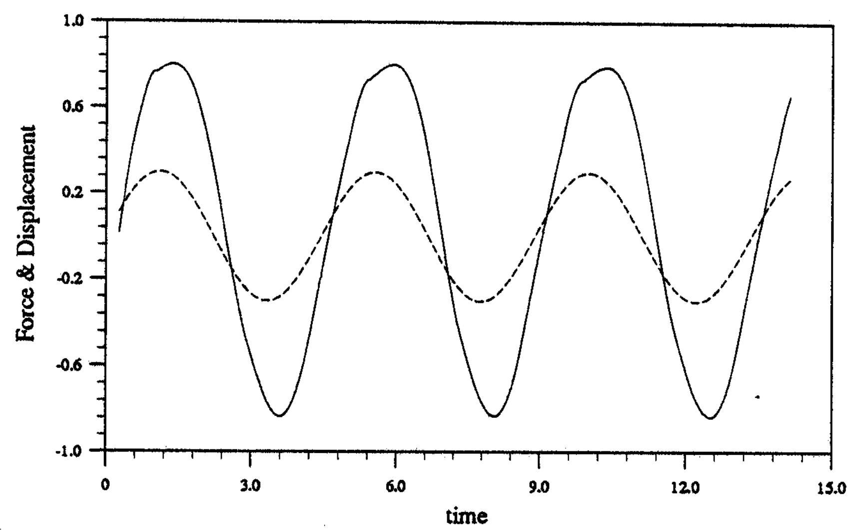

Figure 2: Time history of heave force (solid line) and displacement (dashed line) for the case $a/B = 0.30$, $B/D = 1.0$, and $w\sqrt{B/2g} = 1.414$.

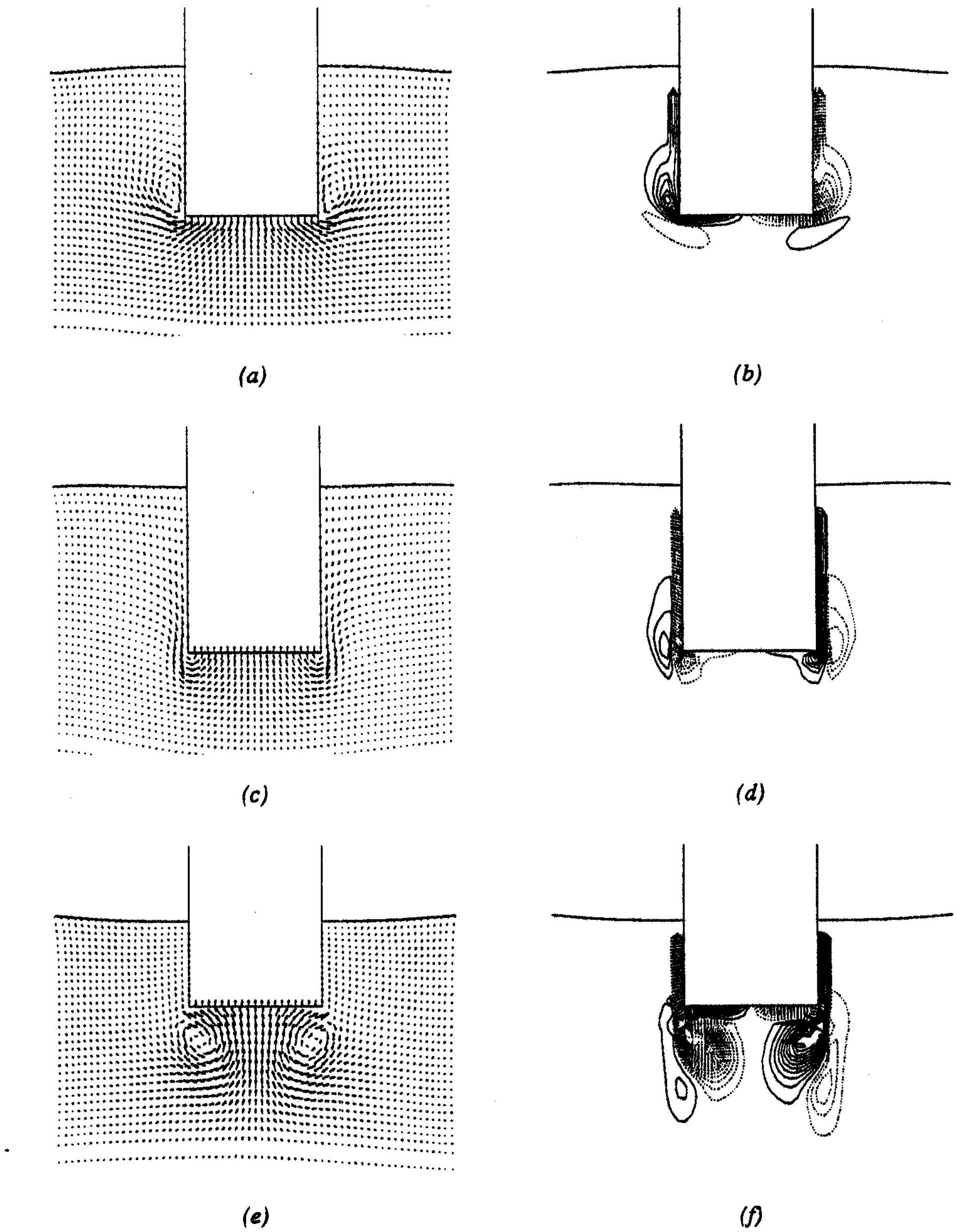

Figure 3: Velocity-vector and vorticity-contour plots for the case $a/B = 0.30$, $B/D = 1.0$, and $w\sqrt{B/2g} = 1.414$ at $t = 2.55T$ (a & b), $t = 2.87T$ (c & d), and $t = 3.18T$ (e & f) where T is the period of oscillation. Solid lines in the vorticity-contour plots represent negative vorticity and dotted lines positive vorticity.

Ferrant: Are you able to quantify individually the viscous and the wave damping components of the total damping? Wave radiation is a non-monotonic function of frequency. Did you vary the frequency in order to observe its influence? The frequency you use in your computations seems to be above the significant frequency range for wave radiation. I believe that the choice of a lower frequency would lead to more significant viscous – free surface interaction effects. Can you comment on that?

Yeung & Ananthakrishnan: The case for which the results are given in the abstract is primarily to demonstrate that our solution method can handle a situation where the amplitude of oscillation is large, and at the same time capture the viscous-effects such as vortex-formation and -shedding. Solutions corresponding to a range of frequency and amplitudes of oscillation and proper quantification of viscous- and wave-damping components are being carried out. These results will be presented in a later report. Using a boundary-integral formulation based on linearized equations, Yeung and Wu have shown that viscous effects on added mass and damping can be quite significant in the low-frequency regime (*Proc.* 10^{th} OMAE, Stavanger, Norway, 1991).

Faltinsen: Stansby has attempted to solve for the viscous flow around a blunt object by the vortex-in-cell method. He found numerical difficulties in the special case of a rectangular cross-section due to the sharp corners. Did you encounter any similar difficulties?

Yeung & Ananthakrishnan: No. The numerical difficulties one faces in treating sharp corners in *certain* vorticity-stream-function type formulations are not encountered in primitive-variables based approaches such as the one used in our work.

Tuck: (1) I was under the impression that the moving contact line paradox occurred only with surface tension. If that is not so, to what extent is the slip requirement at the junction of free surface and body essential in your computations? If it is eliminated, or extended to further grid elements, do the results change much? (2) Every researcher who solves the Navier-Stokes equations seems to use a different method. I am sure that every such author believes that his method is the best. Why do you believe that your method is the best? Is there a general (perhaps impartial!) review article which ranks all such methods, so that a beginner to solution of Navier-Stokes equations can make a rational decision about which method to use, or perhaps an even more rational decision not to do it all?

Yeung & Ananthakrishnan: (1) Modelling the moving contact-line problem with the conventional no-slip condition leads to a singularity in the stresses. This is basically because of the incompatibility in the velocity condition. It has been shown by Marcovich (*Fluid Dynamics*, vol. 23, No. 2, 1988) that the no-slip condition is not satisfied over a region at the contact line and the size of this region depends also on surface tension. As a remedy, we find it plausible to assume that the shear-stress is zero at the contact region so as to determine the tangential-component of the velocity on the body. This is closely related to "local-slippage" model proposed by Huh and Masin (*J. Fluid Mech.*, vol. 81, part 3, 1977), in which it is assumed that the fluid slips freely over a small distance near the contact line. We have not experimented with changing this model and so cannot comment on the "sensitivity" of our results to its variation. However, as observed by Dussan (*J. Fluid Mech.*, vol. 77, part 4, 1976) in a related context, the precise form of a boundary condition that allows slip may not be critical to the overall flow behaviour. (2) Not every researcher uses a different method to solve the Navier-Stokes equations. We have not made any claim that our method is the best. But we have invested a lot of effort to devise a viscous-flow method that can accommodate the effects of a free surface. Generally, there are two formulations available with which to tackle problems governed by the incompressible Navier-Stokes equations, namely (a) vorticity – stream function, and (b) velocity – pressure (primitive variables) formulations. The primitive-variables based formulation is used here since the treatment of stress conditions and later extension to three dimensions are straight-forward. the numerical method is a finite-difference scheme based on curvilinear coordinates and is described in Ref.[4]. Our coordinate-generation method can handle steep wave surfaces, as well as provide us with an effective means to control certain grid properties, *e.g.* cell-like distribution. This can be adapted for implementing other field-discretization procedures such as the finite-element method. For a review on Navier-Stokes equation solvers, you may like to refer to, for example, Orzsag and Israeli, *Ann. Ref. Fluid Mech.*, 1974. The subject is an extensive one and we undertook a comprehensive evaluation before embarking on the numerical work. If there were any choice, we would prefer not to consider the Navier-Stokes equation at all! But there are numerous practical situations in which viscosity effects are important. It would be an irrational decision to bypass such a viscous-flow investigation simply for the sake of expedience.