# A Scope of Different Methods to Compute Wave Drift Damping in Regular Waves

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*Abstract*

The wave drift damping of bodies in regular waves is an essential data to complete low-frequency analysis in irregular seas. Using a linear potential theory, wave drift damping is not directly available and is generally estimated from the variations of wave drift forces with forward speed. The modelisation of forward speed effects, as well as drift forces may have direct consequences on the results. As the assumption of a small forward speed is generally verified, different approximations using Green's function methods have been developed at Principia and Sirehna/E.C.N (ex-E.N.S.M) during last years and are briefly compared in this paper.

## 1. Some methods to compute forward speed effects

Consider a body moving at a forward speed $C$ under the action of regular incident waves, the velocity of a point $M$ of the boby is given by :

$$\overrightarrow{V}_E(M;t) = \overrightarrow{C} + \overrightarrow{W}(t) + \overrightarrow{\Omega}(t) \times \overrightarrow{OM} \tag{1}$$

where $W(t)$ and $\Omega(t)$ denote the radiation translation and angular velocities.

The basic free surface condition of the problem written in the frame attached to the body is given by :

$$\left( \frac{\partial^2 \Phi}{\partial t^2} - 2C \frac{\partial^2 \Phi}{\partial t \partial x} + C^2 \frac{\partial^2 \Phi}{\partial x^2} + g \frac{\partial \Phi}{\partial z} \right) \Bigg|_{z=\zeta(x,y;t)} = 0 \tag{2}$$

where $\zeta$ denotes the position of the free surface.

Note that this condition has been linearized from the kinematic and dynamic conditions given by the Bernoulli-Lagrange equation, so that terms in $U^2$ and $U\overrightarrow{\text{grad}}U^2$ have been neglected.

A first approximation of this condition is to write it on the undisturbed free surface, i.e in $z = 0$ This was made by J.Bougis [1], who developed a Green function with 4 poles, and where the physical instabilities occuring at $\tau = \omega C/g = 1/4$ clearly appear. This formulation has been recently reexaminated in order to improve cpu time requirements by J. Bougis and T. Coudray. When gathering the contribution poles in another way than previously, one decomposes the Green's function in :

$$G(M, M'; t) = G_0(M, M'; t) + G_{\tilde{\omega}}(M, M'; t) + G_F(M, M'; t) \tag{3}$$

Where $G_0(M, M'; t)$ is the contribution of the opposite image source (1/r-1/r'), $G_{\tilde{\omega}}(M, M'; t)$ gathers the contributions of poles $K_1$ and $K_3$ and $G_F(M, M'; t)$ gathers the contributions of poles $K_2$ and $K_4$. (poles $K_1$, $K_2$, $K_3$, $K_4$ are presented in [1] and [2]). Function $G_{\tilde{\omega}}(M, M'; t)$ is found to be representative from the wave pattern generated by a diffraction-radiation source, while $G_F(M, M'; t)$ is representative from the wave pattern generated by a moving source. This physical re-organisation greatly simplifies the variations of the integrands so that cpu requirements can be reduced.

An interesting feature of this formulation is that for Strouhal number ($\tau$) less than 1/4, a MacLaurin development in power of $\tau$ can be made, allowing to control the accuracy of the Green function. Moreover, the expressions of the components of the development of the $G_{\tilde{\omega}}(M,M';t)$ function can be expressed with Bessel and Struve functions for which expeditive computation methods are available. On the other side, the $G_F(M,M';t)$ function at zero order is strictly equal to the Green function of the Neumann-Kelvin problem ([2],[7]). As this function is well-known to be of particular difficulties to be computed, and as one can imagine what the upper order of the development of $G_F(M,M';t)$ can be, a possible approximation is to consider the image source for the function $G_F(M,M';t)$. To preserve some consistance in the development, one finally keeps only the first order for function $G_{\tilde{\omega}}(M,M';t)$ so that the Green function expression becomes :

$$G(M,M';t) = G_0'(M,M';t) + G_{\tilde{\omega}}(M,M';t) = \mathcal{R}e\{e^{-i\omega t}\mathcal{G}(M,M')\} \tag{4}$$

with :

$$G_0'(M,M';t) = \left[\frac{1}{|MM'|} + \frac{1}{|MM'_1|}\right]\cos\omega t \tag{5}$$

$$G_{\tilde{\omega}}(M,M';t) = \left. G_{\tilde{\omega}}(M,M';t)\right|_{\tau=0} + \tau\frac{d}{d\tau}\left. G_{\tilde{\omega}}(M,M';t)\right|_{\tau=0} + O(\tau^2) \tag{6}$$

Extensive work has been made when neglecting the $C^2\frac{\partial^2\Phi}{\partial x^2}$ in the free surface condition eq (2). This was previously done by Grekas [4] who wrote equation (2) on the undisturbed free surface ($z=0$). The Green function obtained is therefore simplified compared to the one obtained with the complete free surface equation, and can be computed quite easily. Note that the formulation is valid for $\tau$ less than 1/2 and that no particular instabilities appear at $\tau = 1/4$.

Cpu requirements for theses two last formulations are about four times less than for the complete formulation.

An alternative method developed by Rong Zhao and O.M. Faltinsen [3] is to consider that the waves from diffraction and radiation interact with the local steady flow around the body. This means that the free surface condition is written on $z=\zeta(x,y;t)$ at first order, i.e each term of equation (2) is written as : ($a = x, z$ or $t$)

$$\left.\frac{\partial}{\partial a}\right|_{z=\zeta(x,y;t)} = \left.\frac{\partial}{\partial a}\right|_{z=0} + \left.\frac{\partial^2}{\partial a\partial z}\right|_{z=0} \tag{7}$$

The hydrodynamic problem is solve in separating the fluid domain in an inner and outer domain. In the outer domain the free surface condition is written on $z=0$, while it is written on $z=\zeta(x,y;t)$ in the inner domain.

A last simplification can be made on equation (2), neglecting $-2C\frac{\partial^2\Phi}{\partial t\partial x}$. Therefore, the free surface condition is identical to the one of the diffraction radiation problem without forward speed, for which extensive work has been published and expeditive Green function algorithms exist. The coupling with forward speed is therefore only represented by the use of the frequency of encounter and the "$M_j$ terms" arising from the zero-normal velocity condition. This formulation gives significant results at first order even for high speed vehicle [8]. However the limit of validity to compute drift forces is not clearly established.

The following table presents some features of the different methods.

| Formulation | Free surface condition | | | | validity for drift forces up to |
|---|---|---|---|---|---|
| | $\frac{\partial^2\Phi}{\partial t^2}$ | $g\frac{\partial\Phi}{\partial z}$ | $-2C\frac{\partial^2\Phi}{\partial t\partial x}$ | $C^2\frac{\partial^2\Phi}{\partial x^2}$ | |
| Complete formulation | • | • | • | • | no limit |
| Dev in Strouhal | • | • | • | • | $\tau < 1/4$ |
| Grekas, Zhao, Faltinsen | • | • | • | | $\tau < 1/2$ |
| frequency of encounter | • | • | | | ? |

## 2. Drift forces and wave drift damping

Drift forces are defined as the constant term arising from the second ordre theory in regular seas. They depend quadratically on the potential of the first order. Therefore the accuracy of the first order results is essential to compute successfully the drift forces. Three different formulations remain to compute these efforts, the pressure integration, the momentum conservation, and the theorem of Lagally.

The formulation on drift forces by pressure integration must be carried out up to the second order, so that some waterline integrals arise which must be precisely computed. A mesh refinement is therefore often necessary. Presentation of this formulation can be found in Pinkster [9], and has been completed by B. Molin [10]. A recent survey by Korsmeyer and al [6] for which about 12000 panels were used, has shown that the convergence of this method is slow.

An alternative method is to use the conservation of momentum equation. This was done by H. Maruo [11], J.N Newman [12], and B. Molin [10]. The convergence of this method is far better than for pressure integration.

Another alternative is to apply the Lagally's theorem, which defines the conditions of application of the conservation of momentum for a fluid domain with singularities. The Lagally's force given by

$$(8) \qquad F_{lag} = \iint_\Sigma \sigma \vec{V}(M;t) + \left(\overrightarrow{\mathrm{grad}}_\Sigma \mu(M;t) \times \vec{n}\right) \times \vec{V}(M;t)dS + \int_\Gamma \mu(M;t)\vec{l} \times \vec{V}(M;t)dl$$

is representative for the drift forces if the flow kinematic is represented by a mixed source ($\sigma$) and dipole ($\mu$) distribution. The $\vec{V}(M;t)$ represents the fluid velocity generated by the regular part of the singularities.

The wave drift damping is defined to take into account the variations of drift forces with forward velocity for low-frequency simulations. The linear dependance of drift forces with speed is the basic assumption.

$$(9) \qquad \mathcal{F}_{WDD} = \frac{\partial F_{drift}}{\partial V} V(t)$$

Note that a more accurate method to provide the variation of drift forces in a low-frequency time domain simulation is to compute the low-frequency forces at each time step of the simulation. This approach is available in an option of software Diodore for which pressure integration up to the second order is computed at each time step. Corresponding results will be presented in a next paper.

## 3. Results

Numerical results are presented on a cylinder free to surge (radius=a; draft=3a) as in [13]. The effect of the different free surface conditions has been investigated on drift forces on figure 1,2. For the drift force computations, the pressure integration method and the near field method are almost insensitive to the different free surface conditons, while Lagally's theorem gives results slightly different for each free surface conditon. The differencies are highlighted for the wave drift damping where the results for the Grekas formulation or the complete formulation are higher than for the two other methods (figure 3). A comparison with the results of [13] (figure 3) shows the influence of the stationnary potential and the discrepancy between the frequency of encounter method and the complete or Grekas method.

Results for a group of 4 cylinders using the frequency of encounter method and the Strouhal development plotted on figure 4 are in a correct agreement with [13] where the stationnary potential is taken into account.

Last tests were performed on the Turret moored Production Ship (TPS2000) [5]. Small velocities were considered from -0.5 m/s to 1.5 m/s with a step of 0.5 m/s for small headings (0°, 10°, 20°). Two different sets of velocities were considered to investigate the linear dependance of the wave drift coefficient (figure 5). The first set is for ($C = \pm 0.5$ m/s), and the second ($C = +0.5$ and $C = +1.5$ m/s). Important differencies between the wave drift coefficients are observed. Note also that negative wave drift coefficients can be obtained. Last plot (figure 6) shows the differencies between the wave drift damping computed either with the frequency of encounter method and the pressure integration or with the complete formulation and the Lagally's theorem. Differencies between the results are concentrated on the extreme values and for the large frequencies.

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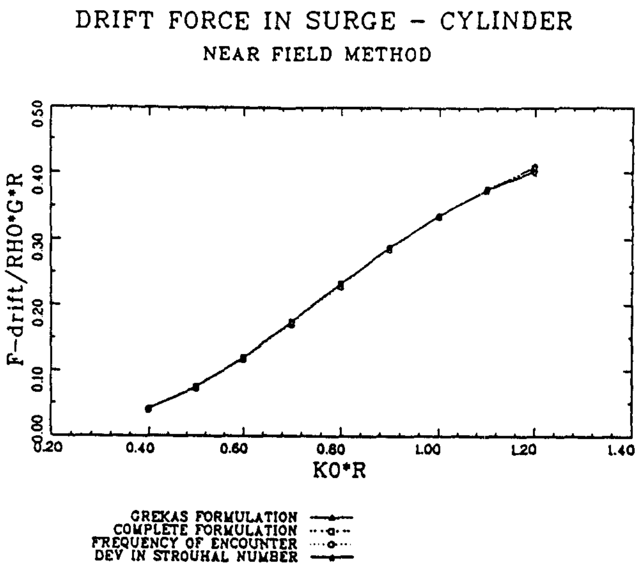


Figure 1

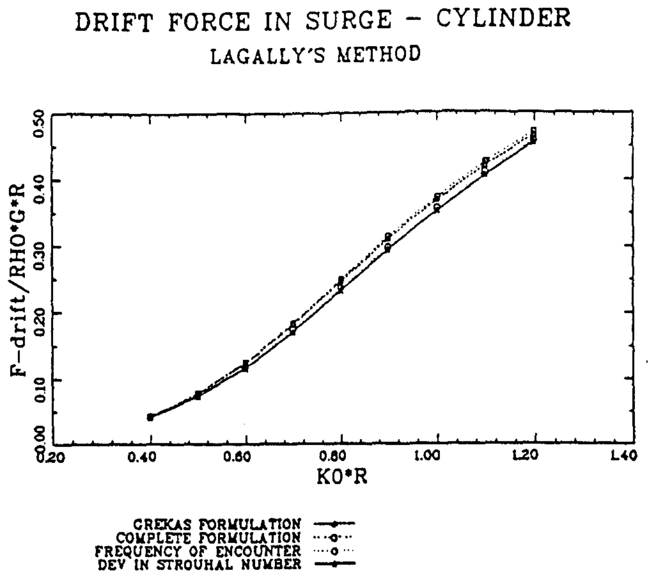


Figure 2

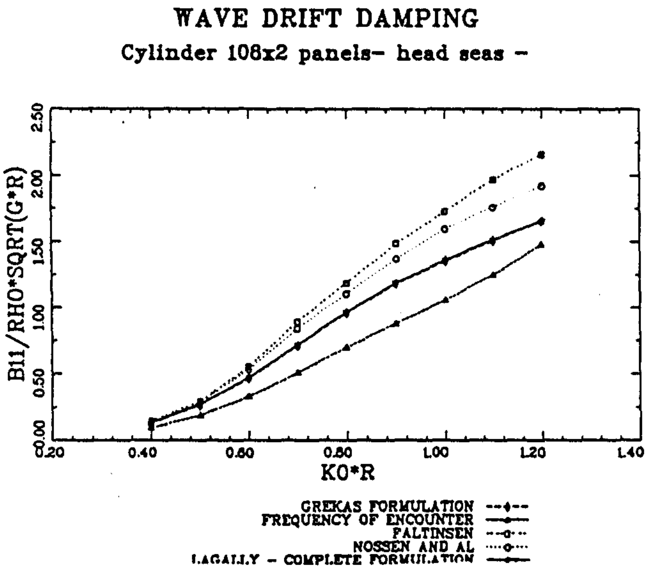


Figure 3

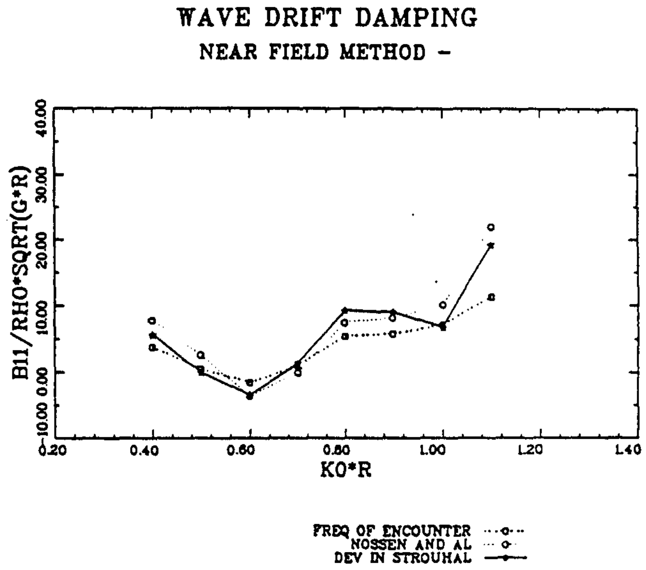


Figure 4

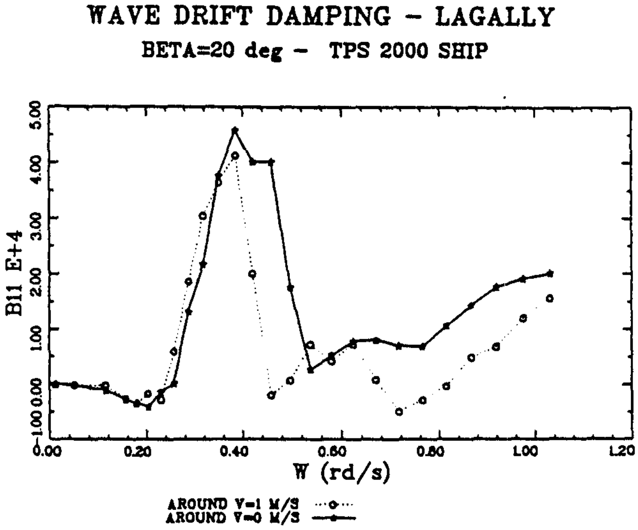


Figure 5

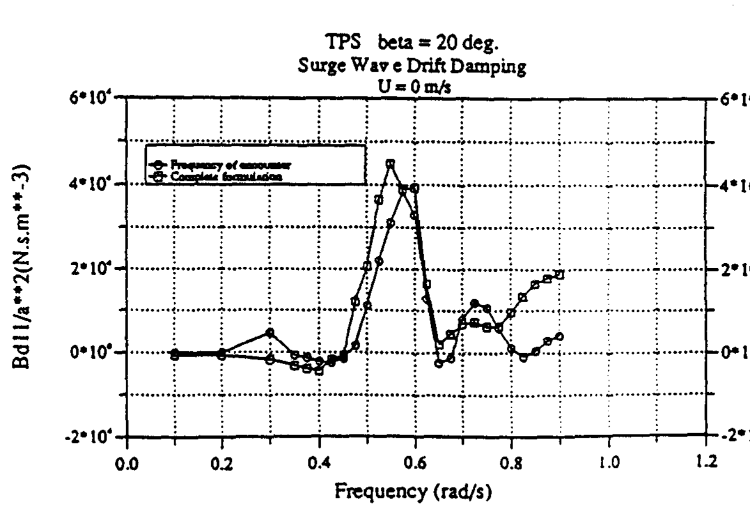


Figure 6

## DISCUSSION

MILOH: I am happy to see that you find the Lagally theorem superior for numerical computation. However, looking at eq. (8) in your abstract it was difficult for me to show the equivalence of your Lagally expression for a doublet distribution with the more familiar expression $\int_S (\vec{\mu} \cdot \nabla) \vec{\vartheta} \, dS$.

BERHAULT et al.: A derivative of the Stokes formulae can be written:

$$\int_S \vec{n} \, \vec{\nabla} \cdot \vec{V} - \vec{n} \wedge (\vec{\nabla} \wedge \vec{V}) - (\vec{n} \cdot \vec{\nabla}) \vec{V} \, ds = \int_C \vec{V} \wedge \vec{dl}, \tag{1}$$

where C is the contour line of an open surface S. Applying formulae (1) to the vector $\mu\vec{\vartheta}$, we get the identity:

$$\int_S (\mu \, \vec{n} \cdot \nabla) \cdot \vec{\vartheta} \, dS = \int_S (\vec{\nabla}\mu \wedge \vec{u}) \wedge \vec{\vartheta} \, ds + \int_C \mu \vec{l} \wedge \vec{\vartheta} \, dl,$$

GRUE: The effect of the forward speed square-term on the diffracted/radiated waves and the wave drift forces was studied for example by Zhao & Faltinsen (1988), *Applied Ocean Research*. Their conclusion was that this term is insignificant for $\tau = U\sigma/g \leq 0.15$, and small for $\tau < 0.20$. This means for example that the ship waves (the short time dependent waves) are insignificant for $\tau \leq 0.15 - 0.2$ . What is your experience with this?

BERHAULT et al.: In the numerical tests presented here, we see almost no differences between the Grekas formulation and the complete formulation. This shows that for the small Strouhal parameters (i.e $\tau = U\sigma/g < 0.03$ in our tests) the square term has effectively no influence.

NEWMAN:

1. In using Lagally's theorem it may be necessary to consider the velocity at $\underline{x}$ due to the Rankine (1/R) sources at $\underline{\xi} \neq \underline{x}$.
2. The reference [6] only considered the drift forces from momentum, not from pressure as is implied. Our recent experiences (cf Newman & Lee, Boss 92) are that both methods converge at approximately the same rate, although momentum seems preferable in general.

BERHAULT et al.:

1. If we split the total velocity in a regular part and a singular part as: $\vec{V} = \vec{\vartheta} + \vec{v}$; $\vec{v}$ is the velocity induced by the Rankine part of the singularities. If we compute the Lagally's force by

$$\vec{F} = \int_S \sigma \vec{V} \, ds$$

we can split the force in

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$$\vec{F} = \int_S \sigma \vec{\vartheta}\, ds + \int_S \sigma \vec{v}\, ds$$

where the second term is zero because of the d'Alembert paradox. The demonstration of the same behavior of $\vec{F}$ for piercing bodies is much more complicated, and may be published soon.