Impulse response functions at steady forward speed

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The problem of a steadily translating floating body, which is also forced in an unsteady transient motion, is formulated under the assumption that the flow is potential, started from rest, and with the boundary conditions linearized about the mean position of the body as well as the quiescent free-surface. The resultant initial-boundary value problem is then recast as an integral equation by way of Green's second identity. This may be done either entirely in the moving coordinates (as Liapis does) or by starting with the more general "body-exact" problem in stationary coordinates, and then shifting this result to moving coordinates while also linearizing the body boundary condition. In either case a free-surface Green function is required, and if the potential due to a source of unit strength travelling beneath the free-surface is chosen, i.e.

$$G(\vec{x};\vec{\xi},t-\tau) = G^{(0)} + G^{(f)} = \left(\frac{1}{r} - \frac{1}{r_1}\right) + 2\int_0^\infty dk\,[1-\cos(\sqrt{gk}\,(t-\tau))]\,\exp{(k\mathcal{Z})}\,J_0(kR), \quad (1)$$

then the integral equation may be written,

$$\begin{aligned} 2\pi\phi + \iint_{S_0} dS\;\phi\,G_n^{(0)} + \int_{-\infty}^{t} d\tau \iint_{S_0} dS\;\phi\,G_{tn}^{(f)} \\ +\frac{U}{g}\int_{-\infty}^{t} d\tau \int_{\Gamma_0} d\eta\,\left[G_t^{(f)}\,(2\phi_\tau - U\,\phi_\xi) + U\,\phi\,G_{t\xi}^{(f)}\right] \\ = \iint_{S_0} dS\;G^{(0)}\,\phi_n + \int_{-\infty}^{t} d\tau \iint_{S_0} dS\;\phi_n\,G_t^{(f)} \end{aligned} \quad (2)$$

where the subscripts have been used to denote differentiation. Equation (2) is discretized, and finally solved using a three-dimensional panel method. Impulse response functions for a Wigley hull, along with various other ship forms have been calculated using both an impulsive acceleration and an impulsive velocity forcing of the body. The results for the Wigley hull are in general agreement with the experiments of Gerritsma.

The fluid velocity in a moving coordinate system is usually expressed as the gradient of the potential $[\Phi_0(\vec{x})+\phi_D(\vec{x},t)+\phi_R(\vec{x},t)]$, where the first term represents the limiting form of the body's steady disturbance and the free-steam potential, while the last two combine to describe the unsteady disturbance. Due to the small amplitude of motion, the radiation potential is further considered to be a linear superposition of six canonical potentials, each corresponding to motion in one rigid-body mode.

$$\phi_R(\vec{x},t) = \sum_{k=1}^{6} \phi_k(\vec{x},t)$$

Equation (2) may be used to solve for any of the canonical radiation potentials by using the appropriate body boundary condition,

$$\frac{\partial\phi_k}{\partial n} = n_k\dot{\alpha}_k + m_k\alpha_k, \qquad k = 1,\dots,6$$

where, $\alpha_k(t)$ and $\dot{\alpha}_k(t)$ are the body's unsteady motion and velocity in mode k, respectively. Note that the steady and the unsteady problems are coupled through the presence of the 'm-terms', which are defined to be, $(m_1, m_2, m_3) = -(\vec{n}\cdot\vec{\nabla})\nabla\Phi_0$, and $(m_4, m_5, m_6) = -(\vec{n}\cdot\vec{\nabla})(\vec{r}\times\nabla\Phi_0)$. In the results presented here, however, the effects of the body's steady disturbance have been neglected and only the simplified m-terms have been used: $m_k = (0,0,0,0,U n_3, -U n_2)$.

Under the asumptions discussed in the preceeding, solving for the canonical radiation potentials becomes a matter of calculating the response of a linear system to an arbitrary, but known, forcing. Furthermore, the response of a linear system to an arbitrary excitation is completly characterized by its response to a unit impulse. In the past, the transient radiation problem has generally been formulated in terms of an impulsive velocity forcing of the body (following Cummins or Ogilvie) however, it is also possible to use an impulsive acceleration. These particular impulses correspond to imposing the following body boundary conditions:

$$\begin{aligned} \phi_n(\vec{x},t) &= n(\vec{x})\,\delta(t) + m(\vec{x})\,h(t) \qquad &\text{impulsive velocity} \\ \hat{\phi}_n(\vec{x},t) &= n(\vec{x})\,h(t) + m(\vec{x})\,r(t) \qquad &\text{impulsive acceleration} \end{aligned} \tag{3}$$

where $\delta(t)$ is the Dirac delta function, $h(t)$ is the Heavyside step function, and $r(t)$ is the ramp function. Given the form of the forcing, it is useful to decompose the solutions in a similar fashion,

$$\begin{aligned} \phi(\vec{x},t) &= \psi^{(0)}(\vec{x})\,\delta(t) + \chi(\vec{x},t)\,h(t) \\ \hat{\phi}(\vec{x},t) &= \psi^{(\infty)}(\vec{x})\,r(t) + \hat{\chi}(\vec{x},t)\,h(t) \end{aligned} \tag{4}$$

Substituting these decompositions into the original initial-boundary value problem and applying Green's theorem, together with the appropriate Green function in each case, will produce a set of integral equations from which all of the functions in Eq. (4) may be calculated. The impulse response function is, in either case, simply the force on the body due to that impulse, and may be calculated by integrating the consequent pressure over the body surface. Since the steady disturbance has been assumed large compared to any of the unsteady potentials, a consistantly linearized Bernoulli equation will take the form,

$$p_k = -\rho\left(\frac{\partial\Phi_k}{\partial t} + \vec{W}\cdot\vec{\nabla}\Phi_k\right) \tag{5}$$

and the force on the body in mode j, due to an arbitrary motion in mode k is,

$$\begin{aligned} F_{jk} = \; & -\rho\int\!\!\int_{S_0} dS\,\frac{\partial\Phi_k}{\partial t} n_j + \rho\int\!\!\int_{S_0} dS\,\Phi_k m_j \\ & +\rho\int_{\Gamma} dl\,\Phi_k n_j(\vec{l}\times\vec{n}_{2D})\cdot\vec{W}. \end{aligned} \tag{6}$$

The potential in Eq. (6) is that due to an arbitrary forced motion of the body, and it may be calculated from either of the canonical radiation potentials,

$$\Phi_k(\vec{x},t) = \begin{cases} \displaystyle\int_0^t d\tau\,\phi_k(\vec{x},t-\tau)\,\dot{\alpha}_k(\tau) \\ \displaystyle\int_0^t d\tau\,\hat{\phi}_k(\vec{x},t-\tau)\,\ddot{\alpha}_k(\tau) \end{cases} \tag{7}$$

Substituting (4) through (7) and into (6) will allow the force on the body to be written in terms of the decomposed potentials. The result is,

$$F_{jk}(t) = \begin{cases} -\mu_{jk}(\vec{x})\ddot{\alpha}_k(t) - b_{jk}(\vec{x})\dot{\alpha}(t) - c_{jk}(\vec{x})\alpha(t) - \displaystyle\int_0^t d\tau\,K_{jk}(\vec{x},t-\tau)\,\dot{\alpha}_k(\tau) \\ -\mu_{jk(\vec{x})}\ddot{\alpha}_k(t) - \hat{b}_{jk}(\vec{x})\dot{\alpha}(t) - c_{jk}(\vec{x})\alpha(t) - \displaystyle\int_0^t d\tau\,\hat{K}_{jk}(\vec{x},t-\tau)\,\ddot{\alpha}_k(\tau) \end{cases} \tag{8}$$

where,

$$\mu_{jk}(\vec{x}) = \rho \int\int_{S_0} dS\, \psi_k^{(0)} n_j$$

$$b_{jk}(\vec{x}) = \rho \left(\int\int_{S_0} dS\, \chi_k^{(0)} n_j - \int\int_{S_0} dS\, \psi_k^{(0)} m_j - \int_\Gamma dl\, \psi_k^{(0)} n_j (\vec{l} \times \vec{n}_{2D}) \cdot \vec{W} \right)$$

$$\hat{b}_{jk}(\vec{x}) = \rho \left(\int\int_{S_0} dS\, \psi_k^{(\infty)} n_j - \int\int_{S_0} dS\, \hat{\chi}_k^{(\infty)} m_j - \int_\Gamma dl\, \hat{\chi}_k^{(\infty)} n_j (\vec{l} \times \vec{n}_{2D}) \cdot \vec{W} \right)$$

$$c_{jk}(\vec{x}) = -\rho \left(\int\int_{S_0} dS\, \psi_k^{(\infty)} m_j + \int_\Gamma dl\, \psi_k^{(\infty)} n_j (\vec{l} \times \vec{n}_{2D}) \cdot \vec{W} \right)$$

$$K_{jk} = \rho \left(\int\int_{S_0} dS\, \dot{\chi}_k n_j - \int\int_{S_0} dS\, \left(\chi_k - \chi_k^{(\infty)}\right) m_j - \int_\Gamma dl\, \left(\chi_k - \chi_k^{(\infty)}\right) n_j (\vec{l} \times \vec{n}_{2D}) \cdot \vec{W} \right)$$

$$\hat{K}_{jk}(\vec{x}) = \rho \left(\int\int_{S_0} dS\, \dot{\hat{\chi}}_k n_j - \int\int_{S_0} dS\, \left(\hat{\chi}_k - \hat{\chi}_k^{(\infty)}\right) m_j - \int_\Gamma dl\, \left(\hat{\chi}_k - \hat{\chi}_k^{(\infty)}\right) n_j (\vec{l} \times \vec{n}_{2D}) \cdot \vec{W} \right).$$

Note that $\chi(\vec{x}, \infty) \equiv \chi^{(\infty)} = \psi^{(\infty)}$ and $\hat{\chi}(\vec{x}, 0) \equiv \hat{\chi}^{(0)} = \psi^{(0)}$, and that simple integral equations for all four limiting values of, χ and $\hat{\chi}$ may be derived along the lines outlined above. Also, by applying Green's theorem to $\psi^{(0)}$ and $\chi^{(0)}$, or to $\psi^{(\infty)}$ and $\hat{\chi}^{(\infty)}$, and using the boundary conditions which they satisfy, it may be shown that,

$$\begin{aligned} b_{jk} = &\quad \hat{b}_{jk} \quad &= 0 \qquad &\text{for } j = k \\ (b_{jk} + b_{kj}) = &\quad (\hat{b}_{jk} + \hat{b}_{kj}) \quad &= 0 \qquad &\text{for } j \neq k. \end{aligned} \tag{9}$$

If the body's unsteady motion, $\alpha(t)$, is taken to be a time harmonic function at constant frequency ω, the impulse response functions may be related to the more commonly used frequency dependent added mass and damping coefficients as follows,

$$A_{jk}(\omega) = \begin{cases} \mu_{jk} - \dfrac{1}{\omega}\left(\displaystyle\int_0^\infty dt\, K_{jk}(t) sin(\omega t) \right) \\ \mu_{jk} + \displaystyle\int_0^\infty dt\, \hat{K}_{jk}(t) cos(\omega t) \end{cases}$$

$$\frac{B_{jk}(\omega)}{\omega} = \begin{cases} \dfrac{1}{\omega}\left(b_{jk} + \displaystyle\int_0^\infty dt\, K_{jk}(t) cos(\omega t) \right) \\ \dfrac{\hat{b}_{jk}}{\omega} + \displaystyle\int_0^\infty dt\, \hat{K}_{jk}(t) sin(\omega t) \end{cases}$$

$$C_{jk} = c_{jk} \tag{10}$$

The coefficients μ_{jk} may be identified as the infinite frequency limits of the added mass coefficients, while $\hat{b}_{jk} = (b_{jk} + \int_0^\infty K_{jk} dt)$ are the zero frequency limits of the damping coefficients. The C_{jk} are simply the restoring force coefficients.

The heave-heave responses of a Wigley hull, at Froude number of 0.3, to the two impulses considered above are shown in figure 1. In figure 2 the two responses have been Fourier transformed and are compared to the experimental results of Gerritsma for the added mass coefficients.

The singularity at $\tau = U\omega/g = 1/4$ (as has been noted by many) manifests itself in the time domain as a slowly decaying oscillatory tail in the impulse response function and may be clearly seen in figure 3. This tail is troublesome because, in practice, it is desirable to have a response which has essentially decayed to zero. Several *ad hoc* methods have been suggested for artificially removing the singularity, however, it is possible to capture its behavior asymptotically. Consider the Green function, Eq. (1). As $t \to \infty$ the pressure due to this travelling source is, to leading order,

$$G_t \sim \mathcal{R}e \frac{-\sqrt{2}\, i\, t}{r_1^2 \sqrt{R r_1}} (|Z| + iR) \exp\left(\frac{Z t^2}{4 r_1^2}\right) \exp i \left[\frac{t^2 R}{4 r_1^2} + \frac{1}{2} \tan^{-1}\left(\frac{R}{|Z|}\right) + \frac{\pi}{4}\right]$$

(Newman 1990). The decay is like $t\, e^{-t^2}$ and the oscillation frequency is increasing like t^2. Now, consider what happens when the body has a steady forward speed, in which case $R, r_1 \to Ut$, $\tan^{-1}\left(\frac{R}{|Z|}\right) \to \frac{\pi}{2}$, and

$$G_t \sim \frac{\sqrt{2}}{U^2 t} \exp\left(\frac{Z}{4U^2}\right) \sin(\omega_c\, t)$$

where, $\omega_c = \frac{g}{4U}$. An exponential decay of the response with time has been replaced by a $\frac{1}{t}$ decay, and osscillations are now at a constant frequency, which is of course what is observed. This analysis shows that the asymptotics of the Green function, to leading order, contain information only at the frequency corresponding to $\tau = 1/4$, and therefore justifies the notion that numerical damping of the portion of the Green function which is calculated asymptotically effects the results only at the critical frequency. However, it also suggests that the linear response of a ship at forward speed can be entirely captured by making calculations out to a certain point, and then attaching an asymptotic tail of the form $\mathcal{A}_0 \sin(\omega_c t + \delta_0)$, which, given the physical nature of the singularity, is the more appropriate thing to do. Such a matching is shown in figure 4.

Fig. 1 Wigley hull at Fn=0.3, heave-heave memory function

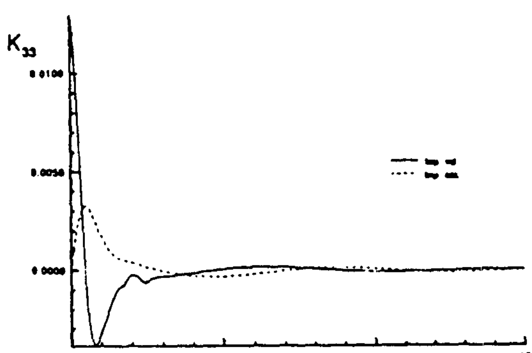


Fig. 3 Heave-pitch memory function for a Wigley hull Fn=0.3

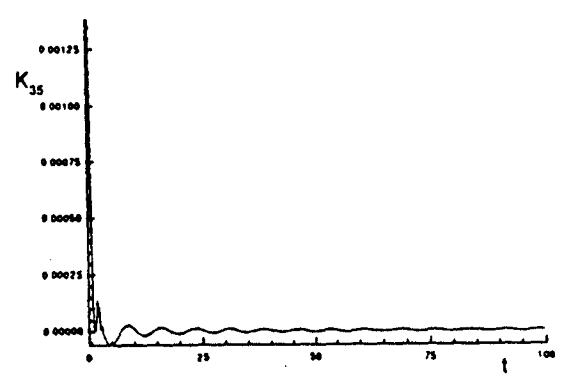


Fig. 2 Added mass coefficients

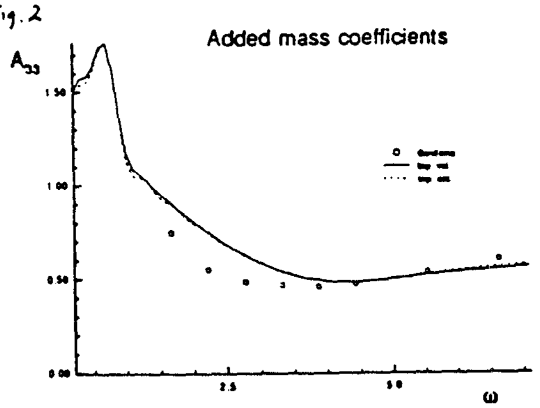
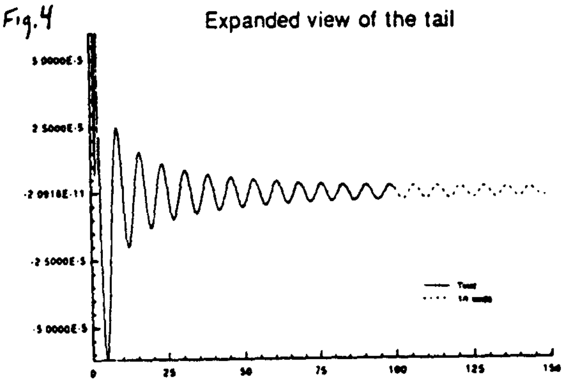


Fig. 4 Expanded view of the tail



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DISCUSSION

R.W. Yeung: It is perhaps worthwhile to mention that the $\tau = 1/4$ singularity is actually observable experimentally. In the "Ringing Phenomenon" that was described in the first Weinblum Lecture by J.V. Wehausen (Schifftechnik, 1978), a slowly decaying but oscillatory transient was observed after a ship moves over a bump. This was explained by the apparent frequency occuring at $\tau = 1/4$.

Bingham: Thank you for the reference. Precisely because this is a physical phenomenon, I would argue that it is appropriate to capture it as best one can and not to suppress it as one would an irregular frequency, for example.

J.Grue: In 2-D we were able to show, by use of Fourier transforms, that the fluid velocities and the linear oscillatory and second order steady wave forces remain bounded as $\tau \rightarrow 1/4$. The strict proof was given for a submerged circle. Numerical experience with a submerged ellipse is the same. The references are Grue & Palm J.F.M. 1985, Mo & Palm J.S.R. 1987. Is there any fundamental difference between a 3-D surface piercing body and a 2-D submerged body regarding the behavior of the fluid flow properties as $\tau \rightarrow 1/4$?

Bingham: Professor Miloh has shown (Dagan & Miloh J.F.M. 1982) that for a general oscillating singularity traveling beneath the free-surface the wave amplitude becomes unbounded like $(\omega_c - \omega)^{-1/2}$ in 2-D and like $\ln(\omega_c - \omega)$ in 3-D as the oscillation frequency approaches the critical frequency. I must read these references which you suggest, but my feeling is that whether a body is surface piercing or not, it may be described by a distribution of singularities, and so should exhibit the same sort of resonance as $\tau \rightarrow 1/4$.

E. Tuck: You showed your two results (close to each other) for the added mass, compared to Gerritsma's experimental results (moderate agreement). I would have expected you to also show a comparison with frequency-domain computations. I know that Korsmeyer has done this comparison successfully at zero speed. Have you made such a comparison at non-zero speed?

Bingham: Korsmeyer's comparison is between a time domain and a frequency domain solution of the same linearization of the problem, and hence the agreement is essentially to graphical accuracy. I have compared my results with a Rankine panel method, (SWAN, Nakos & Sclavounos) which is linearized about the double-body flow, and their results differ from both mine and Gerritsma's by essentially the same amount as mine differ from Gerritsma's for the diagonal terms; however, for the cross-coupling coefficients, SWAN and Gerritsma are in very close agreement while my results differ by an amount similar to what was shown for the diagonal terms. SWAN can also use a Neumann-Kelvin linearization and that would be the analogous comparison to Korsmeyer's. I will be doing this very soon.

F. Noblesse: What do you think is the cause of the discrepancies between your time-domain predictions and the frequency-domain predictions of Nakos and Sclavounos?

Bingham: In SWAN the m-terms are calculated from the double-body flow, while my results are only with the simplified m-terms. I believe that this accounts for most of the discrepancy, although the different free-surface conditions may also contribute.

A. Magee:

- **Q1:** What is the advantage, numerical or otherwise, in using the impulsive acceleration instead of the usual impulsive velocity?
- **Q2:** Can you show the improvements in your results using the fix-up for the long-time tail. It seems that if it works so well, your curves obtained by the two inputs should collapse into one. I used exactly the same method in a postprocessor for the time-domain program at Michigan. So did Brad King in his thesis. We were not convinced that it was so super, but merely a necessary evil to obtain the Fourier transforms of responses due to non-impulsive inputs. Hence we didn't write anything about it. (A mistake perhaps!) I think that I obtained much better resolution near $\tau = 1/4$ for the results in my thesis using this method.

Bingham:

- **Q1:** Actually, in my experience using an impulsive velocity is computationally more forgiving. This is because at non-zero forward speed the impulsive acceleration response tends to a linear function of time, while the response to an impulsive velocity tends to a constant. This means that errors will grow in time for the impulsive acceleration forcing. (This may be avoided my some clever manipulation of the integral equation for $\hat{\chi}_k$, but the storage requirements are then doubled.)
- **Q2:** Yes, the results are improved. I have not shown this because there are other issues involved, such as the time step size of the impulse response function vs. the frequency step size of the Fourier transform, especially near $\tau = 1/4$. I expect however, that the two curves will become graphically identical when all of these issues are resolved.