A discrete time model of the transient hydrodynamics Green function.

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The solution of transient hydrodynamics problems in the frame of the linearized potential theory always involves the computation of convolution integrals. In the usual B.E.M approach, this task represents the major part of the whole CPU time. As an example, let us consider the generation of surface waves by the prescribed motion ($V.n$) of a body (S) around its equilibrium position in a perfect fluid. The velocity potential $\Phi(P,t)$ is known to be the solution of the following boundary integral equation:

$$\frac{\Phi(P,t)}{2} - \iint_S \Phi(M,t)\frac{\partial G_0(M,P)}{\partial n(M)}ds(M) = -\iint_S G_0(M,P)\, V.n\,(M,t)\, ds(M)$$
$$+\iint_S ds(M)\int_0^t \Phi(M,\tau)\frac{\partial F(M,P,t-\tau)}{\partial n(M)}d\tau - \iint_S ds(M)\int_0^t V.n\,(M,\tau)\,F(M,P,t-\tau)\,d\tau \qquad (1)$$

where G_0 anf F are respectively the impulsive and memory part of the time domain Green function $G(M,P,t)$ [Brard (1948), Finkelstein(1957)] which may be written as:

$$G(M,P,t) = G_0(M,P)\,\delta(t) + H(t)\,F(M,P,t) \qquad (2)$$

with :

$$G_0(M,P) = -\frac{1}{4\pi}\left(\frac{1}{r} - \frac{1}{r_1}\right)$$

$$F(M,P,t) = -\frac{(r_1)^{-3/2}}{2\pi}E(\cos\theta,\tau)$$

$$E(\cos\theta,\tau) = \int_0^\infty J_0(\lambda\sin\theta)\sin(\tau\sqrt{\lambda})\,e^{-\lambda\cos\theta}\sqrt{\lambda}\,d\lambda$$

with : $\tau = \frac{t}{\sqrt{r_1}}$, $\cos\theta = -\frac{z+z'}{r_1}$,

$R = [(x-x')^2 + (y-y')^2]^{1/2}$, $r = [R^2 + (z-z')^2]^{1/2}$

and $r_1 = [R^2 + (z+z')^2]^{1/2}$

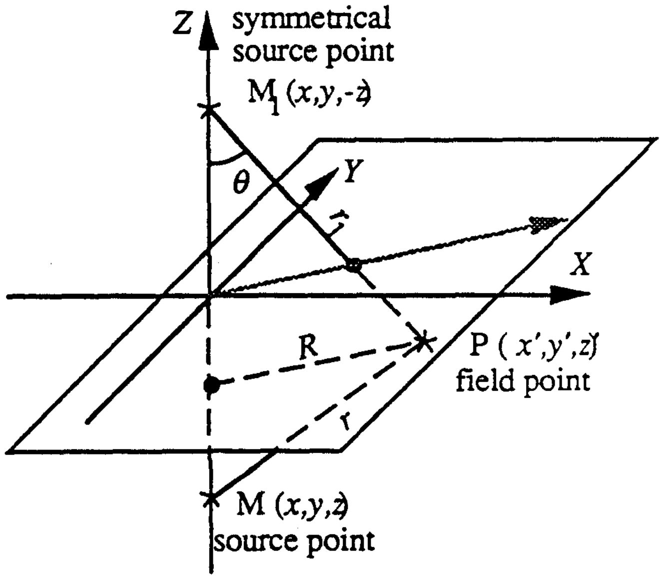

Figure 1: definition sketch

E being a function of only two variables: a space variable $\cos\theta$ and a time variable τ. As may be seen in the above formula (1), the evaluation of the right hand side of the integral equation at the current time t requires the calculation of convolution integrals of the general form:

$$I(M,P,t) = \int_0^t q(M,\tau)\,F(M,P,t-\tau)\,d\tau \qquad (3)$$

$F(M,P,t)$ being given from (2), and $q(M,t)$ being a known past value of either Φ or $V.n$ on the body surface. In the numerical implementations of (1), the computation of these right-hand-side terms represents the major part of the required total CPU time, and necessitates a very large amount of mass storage.
At every time step, the function F must be evaluated for every couple (M,P) of control points, and must also be stored in order to be reused at the next time step. From now on, let us choose a couple of fixed point M and P, respectively the source point and the field point (see fig.1). Let N be the number of control points over the mean wetted body surface, and k the time subscript varying from 0 to K.
The free surface memory terms like (3) are usually computed using of a discrete form of the convolution integral which is then written as :

$$I(M,P,t_K) = \sum_{k=0}^{k=K} q(M,t_k)\,F(M,P,t_{K-k})\,\delta t \qquad (4)$$

,or, in a more compact form : $I_K = \sum_{k=0}^{k=K} F_{K-k}\, q_k$ (5)

In the recent past, the efforts made to accelerate the numerical BEM codes based on this mode of evaluation of I_K concerned mainly two topics :

- derivation of new alternative expressions of the Green function, better suited to numerical calculation (Jami (1982), Newman (1985), Beck & Liapis (1987)),
- tabulation of the memory part of the Green function $E(\cos\theta,\tau)$ in order to replace the evaluation of the function F_k by a simple interpolation in a table (Ferrant (1988), Magee & Beck (1989)).

An ARX Model of the Green function.

We propose here an alternative method to compute the RHS terms I_K of (1) using a discrete time model of $E(\cos\theta,\tau)$. In this method, convolutive series like (5) are replaced by a simple numerical linear filtering process. This approach, which is the standard and natural way to proceed in automatic control and system theories, is not yet widespread in our research community. It has been used sometimes to modelize the linear and non-linear response of floating structures to a seaway [Jefferys & al.(1990)-(1991)]. It could be summarized as follows :
A couple of points (M,P) being given, the space parameter $\cos\theta$ is fixed and E becomes a function of the single time variable τ. Let us now consider this couple of points and the surrounding fluid as a *linear system* , with an *input* $q(M,t)$ and an *output* $I(M,P,t)$. Then, the function $E(t)$ is nothing but the impulse response function of this linear system. So, under the added assumption that it is time-invariant, the system may be identified by standard techniques to an ARMAX[1] discrete time model. In the present case, the impulse response being given by an analytic form (2), no perturbation terms need to be included in the model ; then the moving average terms were dropped out and we retained the ARX model :

$$I_K = \sum_{i=1}^{i=r} \delta_i\, I_{K-i} + \sum_{j=0}^{j=s} \omega_j\, q_{K-b-j} \qquad (6)$$

[1] AutoRegressive Moving Average with eXogenous input.

where r and s are the orders of the model, and b is an eventual pure delay between the input and the output. The *identification* of $E(\cos\theta,\tau)$ was performed for a set of equally distant discrete values θ_j of the space parameter in the range [0.1,1.0]. For any given set of orders and delay (r,s,b), an overdetermined linear system was derived from (5) and (6) and solved in a least mean square sense to determine the coefficients $\delta_i(\cos\theta_j)$, $\omega_i(\cos\theta_j)$ of the corresponding ARX model (6).

cosθ	(r,s)=(5,5)	(10,10)	(20,20)	(30,30)
1.0	2.1 10-2	1.0 10-3	2.6 10-5	4.5 10-8
0.8	2.0 10-2	8.6 10-3	3.0 10-5	3.6 10-9
0.6	4.1 10-2	1.2 10-2	4.6 10-5	8.3 10-8
0.4	5.8 10-1	1.6 10-1	4.7 10-3	2.6 10-4
0.2	8.6 10-1	6.3 10-1	2.0 10-1	1.5 10-2
0.1	9.4 10-1	8.1 10-1	5.9 10-1	2.3 10-1

Table 1: ε as a function of (r,s) and cosθ.

The numerical identification was performed within the following parameters domain : $0.1 \leq \cos\theta \leq 1.0$, $0 \leq \tau \leq 30.$, $\Delta\tau = 0.2$; furthermore, we observed that optimal results were obtained with : r=s and b=1. Then this model structure was retained in all the numerical trials.

Table 1 above shows the mean relative error ε between the classical method (5) and the present ARX model method (6). One can notice that the order must be increased as cosθ tends to zero, which is a consequence of the oscillatory behaviour of the function E in this limit. This is also the reason why the identification was not performed in the range $0 \leq \cos\theta \leq 0.1$.

After this preliminary step of identification, which has to be done once for all, the method was tested on a simple case. A point source of sinusoïdal strength appears at t=0 at the location M(0,0,-1). The potential on the free surface was computed by both methods (5) and (6) on a radial cut up to Radius=20 and Tmax=40.

On the figure 2 we have plotted the results obtained by using increasing order models (r=s = 5 , 10 ,20, 30) as dotted lines together with the reference results as a solid line. In that case, despite the choice of the "worst" value cosθ =0.1 for the space parameter, the (30,30,1) model gave an average relative error of only $3.8\ 10^{-2}$.

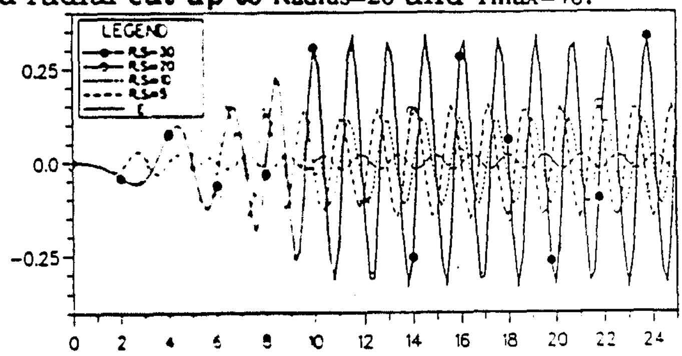

Fig2: potential at the free surface cosθ =0.1

In fact, the discrepancy between the two methods remains undistinguishable on this figure (black dots signal some results obtained by the ARX method, and the solid line for the reference solution computed by the convolutive form)

<u>Expected benefits of the dicrete time model method.</u>

Let us now evaluate the amount of CPU time and storage volume which can be saved by using this method in a BEM algorithm based on (1). Let N be the number of control points over the hull, K the number of time steps, and Q the number of discrete values of $\cos\theta$ for which the identification has been performed and stored. Thus, the calculation of $I(M,P,t_K)$ for the whole body requires:

discrete convolution method	ARX model method
➪ $N(N+1)K$ memory locations for the storage of the previous values of q and of the influence coefficients F(M,P),	➪ $N(Nr+s+1)+Q(r+s+1)$ memory locations for the storage of: N^2r previous values of I_K $N(s+1)$ previous values of q_K $Q(r+s+1)$ model coefficients,
➪ N^2K evaluations of the function F	➪ 0 (!) evaluation of the memory part F of the Green function.
➪ $N^2K(K-1)/2$ multiplications and access to the coefficients file.	➪ $N^2K(r+s+1)$ multiplications and access to the files,

Thus, the CPU-time reduction factor is roughly K/r for the storage volume and $K/4r$ for the number of multiplications and file accesses. If the entire simulation duration is considered, K may be of order 500; so, keeping r=s=30, these ratio are respectively equal to 16.7 and 4.2 . (or 6.7 and 1.7 if K is the truncation order of the convolutive series, say K=200). Furthermore, the most important gain arises from the total suppression of the N^2K evaluation of F, either by direct calculation or by tabulation. The gain associated with this improvement is difficult to evaluate a priori. It depends on the performance of the Green function routine. In the simple evaluation test reported herein, even with a fast evaluation algorithm for the Green function (Ferrant 1988), the maximum overall reduction gain in CPU time reached 80 ! The price to pay for this drastic cut in CPU-time requirement and storage volume is the preliminary identification of $E(\cos\theta,\tau)$; but one has to keep in mind that this has to be done once for all.

At the moment, these improvements remain potential. A lot of obstacles remains to be tided over before implementing the method in a transient seakeeping software. First of all, the ARX model must be extended up to $\cos\theta=0$ in order to cover the whole possible range of this parameter in realistic calculations. The second point is the lowering of the model order. We are now devising a new method of identification in order to address these to points.

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