COMPUTATION OF NONLINEAR WAVE AND WAVE RESISTANCE

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1. Introduction :

With the perfect fluid hypothesis, the calculation of wave resistance can be achieved successfully by Rankine Source Method. The linearized free surface conditions are fullfilled on the plane of free surface at rest. The main limitation of these models lies in the prediction of the wave elevation along the hull for thin ships and also in the wave resistance for more full ships. Many authors, like L. Larsson [1] ,V. Bertram [2] and H. Raven, have attempted to overcome these limitations by using nonlinear models. In the present work, we propose an approach of the fully nonlinear problem of the wave resistance.

2. Formulation :

2.1. Exact free surface conditions :

Following Dawson, we write here in streamlines coordinates a practical approximation of the exact kinematic free surface condition, which affect only slightly the results [3], and the exact dynamic free surface condition :

$$\Phi_{ll}\Phi^2_l + g\,\Phi_z = 0 \quad \text{on } z = \zeta(x,y)$$

$$\zeta(x,y) = [\,U^2 - \Phi^2_l\,]/2g \quad \text{on } z = \zeta(x,y)$$

l being the streamlines of the flow on the free surface, the subscripts designing the partial derivatives.

- This condition, after linearisation around the double model, is used in the Dawson method. The radiation condition is satisfied by an upward finite difference scheme with four points, the second and fourth order derivatives being cancelled.

In Cartesian coordinates, the same conditions are written :

$$\Phi_{xx}\Phi^2_x + 2\Phi_x\Phi_y\Phi_{xy} + \Phi_{yy}\Phi^2_y + g\,\Phi_z = 0 \text{ on } z = \zeta(x,y)$$

$$\zeta(x,y) = [\,U^2 - (\Phi^2_x + \Phi^2_y + \Phi^2_z)\,]/2g \text{ on } z = \zeta(x,y)$$

- This condition, after linearisation, is used by Nakatake [4]. The partial derivatives of first and second order are calculated by analytical formulas obtained by derivation of the influence coefficients of the Hess-Smith formulas. The radiation condition is satisfied by backward shifting of the control point of the facet, like in Bertram's method [2] where the panels are above the free surface. These two methods have been compared for the linear Neumann-Kelvin problem.

2.2. Iterative method:

To solve the nonlinear problem we divide the potential in two parts :

$$\Phi = \Phi_0 + \Phi_1 = Ux + \varphi \quad \text{where } \varphi \text{ is the unknown potential.}$$

The kinematic free surface condition becomes :

$$\varphi_{xx}\Phi^2_{0x} + 2\,\Phi_{0x}\Phi_{0y}\varphi_{xy} + \varphi_{yy}\Phi^2_{0y} + 2\ [\varphi_x(\Phi_{0xx}\Phi_{0x} + \Phi_{0xy}\Phi_{0y}) + \varphi_y(\Phi_{0xy}\Phi_{0x} + \Phi_{0yy}\Phi_{0y})] + g\,\Phi_z = 2\,[\Phi_{0xx}\Phi^2_{0x} + 2\,\Phi_{0x}\Phi_{0y}\Phi_{0xy} + \Phi_{0yy}\Phi^2_{0y} - U(\Phi_{0xx}\Phi_{0x} + \Phi_{0xy}\Phi_{0y})] - NLT$$

$$NLT = \Phi_{0xx}\Phi^2_{1x} + 2\,\Phi_{0x}\Phi_{1x}\Phi_{1xx} + \Phi_{1xx}\Phi^2_{1x} + \Phi_{0yy}\Phi^2_{1y} + 2\,\Phi_{0y}\Phi_{1y}\Phi_{1yy} + \Phi_{1yy}\Phi^2_{1y} + 2\,\Phi_{0x}\Phi_{1y}\Phi_{1xy} + 2\,\Phi_{0y}\Phi_{1x}\Phi_{1xy} + 2\,\Phi_{1x}\Phi_{1y}\Phi_{0xy} + 2\,\Phi_{1x}\Phi_{1y}\Phi_{1xy}$$

where all terms in NLT are calculated at the previous step at the location of the free surface.

The problem is solved by a panel method with sources of constant strength located on the real form of the hull, up to the displaced free surface, and on the displaced free surface.

The iterative scheme is then :

- First iteration : solution of the linear problem on the fixed hull [5] [6]:

Free surface equation is solved with Φ_0 as the double model potential and Φ_1 equal zero. The first order derivatives in x and y and second derivatives in xy and yy of the potential φ are calculated analytically by modified Hess-Smith formulas. The second derivative in xx is calculated by the upward finite difference scheme used by Dawson. After resolution, Φ_1 is computed by $\Phi_1 = Ux + \varphi - \Phi_0$. Forces are obtained by dynamic pressure integration on the hull and displacements of the hull (trim and sinkage) are calculated by using linear hydrostatic restoring coefficients.

- Other iterations: displacement of the mesh, nonlinear free surface condition :

The free surface is displaced vertically following the wave height obtained at the first iteration, without the term in Φ^2_z .

The hull is meshed up to the displaced free surface. This method is today only in use for mathematical hulls, but the generalisation is easy to perform.

Φ_0 can no more be calculated by a double model method. We compute Φ_0 by simulation of a double model. The influence coefficients are obtained by symetrisation of the hull above the free surface.

The potential Φ_1 is taken equal to its value at the previous iteration. After solution of the linear system in φ , Φ_1 and all other quantities are updated. At each step, we calculate the wave resistance and the displacements by integration of the total pressure on the hull, after substracting the gravity forces. The value of the residual part of the exact kinematic free surface condition is then calculated using analytical influence coefficients, except for the term in xx which is computed by a finite difference scheme.

The process generally converges rapidly in few iterations.

3. First numerical results :

This method was first tested on the Wigley hull (80x8x5) with Froude numbers between 0.266 and 0.482 and the computed wave profiles are compared with Japanese experiments [7].

Meshes of free surface at a Froude number Fn = 0.397, and wave elevations for linear and nonlinear models are shown in figure 1.

Figure 2 shows the main difference between linear and nonlinear calculations at Fn = 0.310. With the linear model, the wave profile at the waterline is damped, especially at the first crest. The nonlinear model is in better agreement with experiments.

This behaviour is observed for all Froude numbers. In our modelisation, it seems that the accuracy improvement is mainly due to the presence of nonlinear terms in the free surface equation, rather than the displacement of the free surface, which gives a significant contribution to the wave height only for higher Froude numbers.

Figure 3 shows the wave profile at each iteration for a Froude number of 0.482. Even at this high Froude number, we can see that convergence is nearly reached at the second nonlinear iteration.

The comparison of wave resistance for linear Dawson model and our nonlinear model shows little difference (< 3 %), with slightly better results for nonlinear modeling, especially for trim.

This method has been tested on other mathematical hulls and compared with experiments. The conclusions are the same.

4. Conclusion :

The nonlinear method allows the computation of the wave at waterline with a better accuracy than linear methods. The influence of the method on wave resistance of full ships will be tested.

Some problems remain to be addressed :

- For hulls with an half-angle of entrance around 20 degrees, the method does not converge for all speeds. This lack of convergence increases with the angle of entrance and with the speed.

- To apply the method to general shape of hulls, the problem of the intersection of a non mathematical hull with a displaced free surface must be studied carefully. The problem of the sensitivity of the results at the mesh near the free surface must be taken into account, especially for hulls varying rapidly at waterline, like bow bulbs or flat bottoms.

References:

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[6]J.J. Maisonneuve , 1989, "Résolution du problème de la résistance de vagues des navires par une méthode de singularités de Rankine" - Thèse de Doctorat, E.N.S.M. Nantes , 1989

[7]"Summary of the cooperative experiment on the Wigley Parabolic model in Japan" - Proceedings of the Second DTNSRDC Workshop on Ship Wave-Resistance Computations, Washington, 1983, pp 5-35

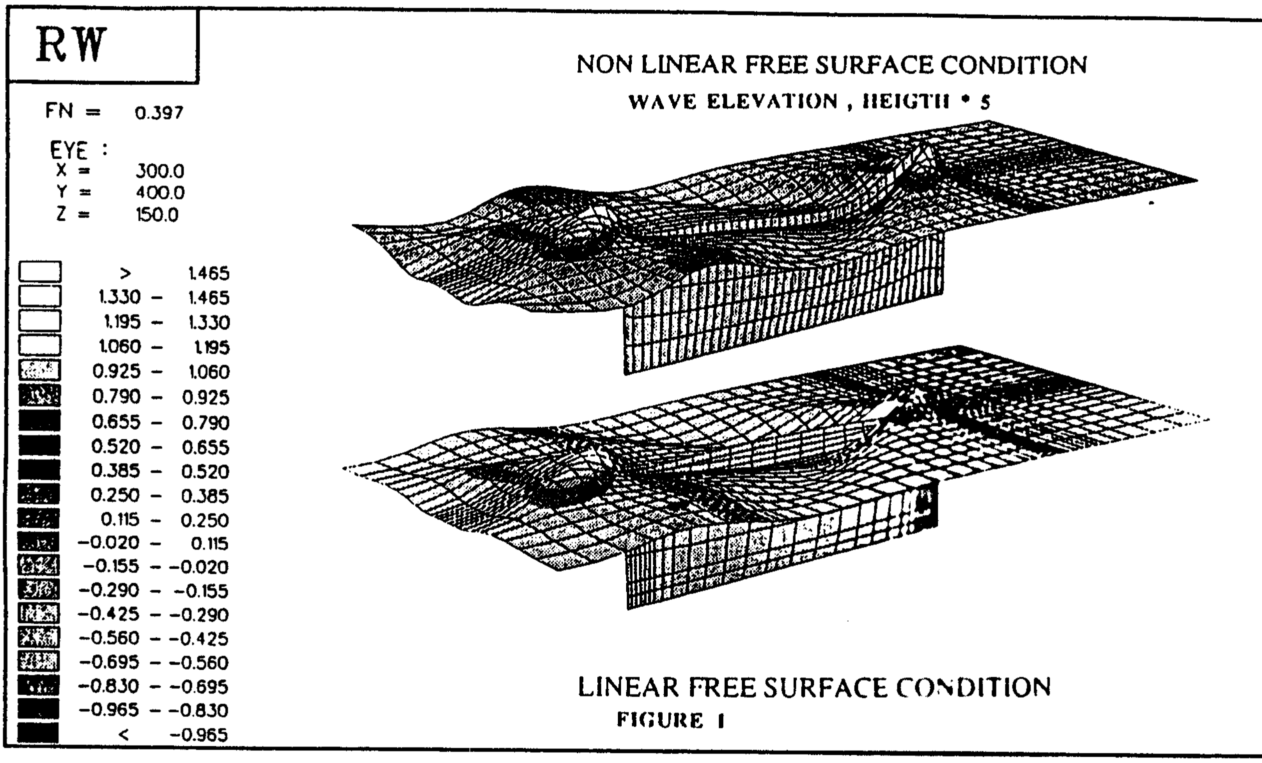

FIGURE 1

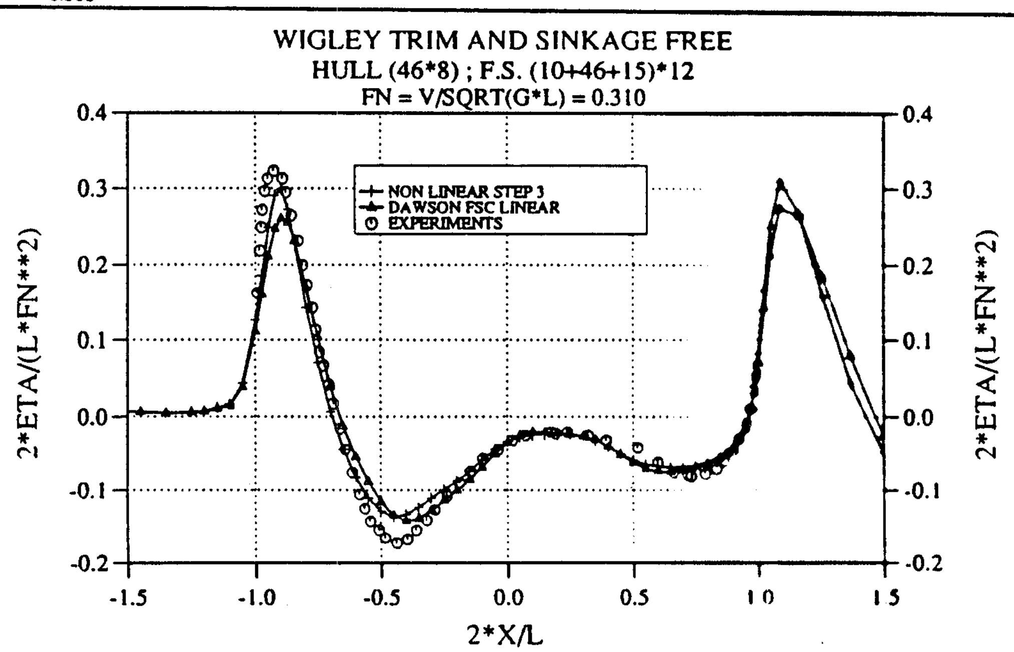

FIGURE 2

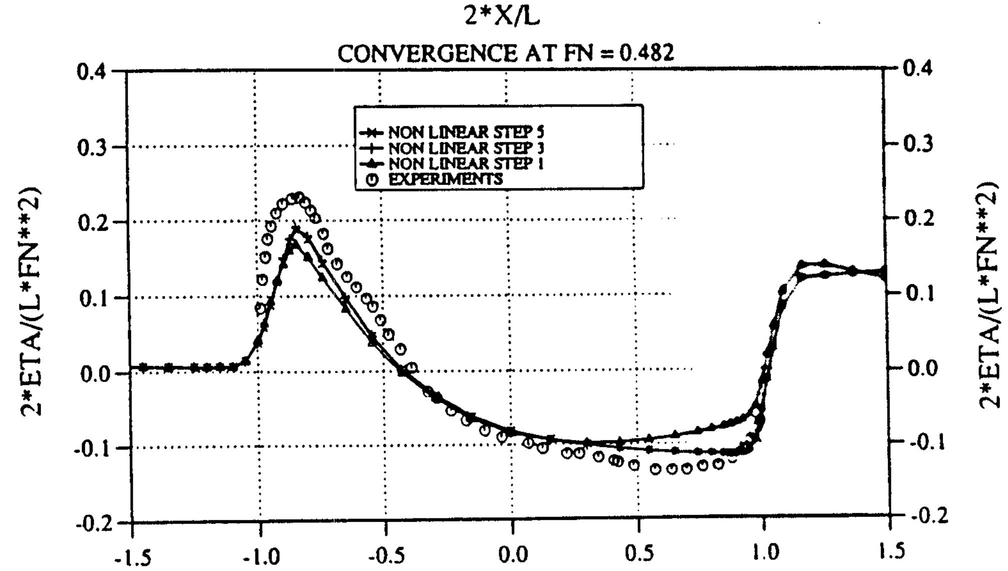

FIGURE 3

DISCUSSION

JENSEN:

1. I think you miss some Φ_z terms in the cartesian formulation of the dynamic free surface condition.
2. What kind of problems did you have with the Wigley hull at higher Froude numbers?
3. The picture comparing linear and nonlinear waves is confusing, as it shows apparently no waves for the linear solution.

DELHOMMEAU:

1. In the computation of the wave heights, I neglect the terms in Φ_z. When I took these terms into account, I sometimes observed a strong depression before the hull. I think it is better not to use them, but the reason for this must be found.
2. For the Wigley hull with your method, the wave elevation around the hull are quite realistic, but the wave resistance is too low for Froude numbers greater than 0.35.
3. The picture comparing the nonlinear and linear waves shows the meshes of the hull and free surface at the location where they are computed, the heights are given by the color of the free surface, which is not sufficiently contrasted with a grey scale. It has been modified in the final version of the paper, where the waves are shown on the displaced free surface, even for the linear model. It can be seen that the mesh of the hull is fixed for the linear waves and is fitted to the free surface for the nonlinear waves.

BERTRAM: If you use 4 point sources of equal strength instead of 1 point source for close collocation points, you should get fine results for $\Delta z = \Delta x$ even for high Froude numbers.

DELHOMMEAU: When we use your method, we have always used 4 points panels. The discrepancy of the results at great Froude numbers may be due to the non regular mesh on the free surface. We will try in a near future a more regular mesh and we hope that the results of the wave resistance will be improved.

RAVEN: It is not quite clear to me how the potentials Φ_0 and Φ_1 are updated in the course of the iteration process. According to your abstract, Φ_0 is recalculated in each iteration, but Φ_1 is taken equal to its value at the first iteration. Could you clarify this?

DELHOMMEAU: The two potentials Φ_0 and Φ_1 are updated at each iteration. The text of the first abstract is not clear on this point and is corrected in the final version.