FURTHER USE OF THE TIMMAN-NEWMAN RELATIONS IN ASSESSMENT OF 3D HYDRODYNAMIC ANALYSES

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Introduction

This work forms part of an investigation into the behaviour of floating bodies having a low forward speed in waves. We have shown in [1] that in order to satisfy exactly the Timman-Newman reverse flow relationships, it is necessary to pose the problem using a form of the free surface boundary condition which includes the coupling between the steady and unsteady potentials. The extra coupling terms are relevant when the body is not slender or deeply submerged. When this formulation is employed as the basis of a numerical analysis for arbitrary bodies, as for example by Nossen, Grue and Palm [2], the reverse flow relations may be used as one of the checks on the reliability of the numerical algorithms and the adequacy of a discretisation.

In the following we compare results from three methods for calculating the hydrodynamic coefficients on an oscillating body moving with a small forward speed. Two of the methods result from the development of general procedures for the computation of wave drift damping on arbitrary bodies. The third is a special formulation for the hydrodynamic coefficients, which inherently satisfies the reverse flow relations provided that the zero speed cross coupling coefficients are symmetric.

Overview of Formulation

Details of the boundary value problem are given in [1] and [2]. We use a system of coordinates (x, y, z) moving along the horizontal x axis at the mean speed U of the body, with the z axis measured vertically upwards from the undisturbed free surface. The velocity potential is separated here into the steady component

$$\phi_s = U\chi_s = U(\chi - x) \quad ; \tag{1}$$

and the unsteady component associated with motions ξ_j of the body at encounter frequency σ

$$\Phi_u = Re\,[i\sigma \sum_{j=1}^{6} \xi_j \phi_j e^{i\sigma t}] \quad . \tag{2}$$

Under the assumption of low forward speed, on $z = 0$ the steady potential satisfies the rigid wall condition, and the unsteady potentials satisfy

$$-\nu\phi_j + 2i\tau\nabla_2\phi_j \cdot \nabla_2\chi_s + i\tau\phi\nabla_2^2\chi + \frac{\partial\phi_j}{\partial z} = 0 \tag{3}$$

where $\nu = \sigma^2/g$, $\tau = U\sigma/g$. We note the coupling in the free surface boundary condition (3) between the steady disturbance χ_s and the unsteady potential ϕ_j; and the absence of terms in τ^2.

As the basis of each of our three numerical methods we use integral equations, discretised by quadratic isoparametric elements. First we solve for χ, using as Green function a Rankine source and its image about the still free surface. Then in the equations for ϕ_j we use a reversed flow Green function, satisfying the far field form of (3) corresponding to there being no steady disturbance. This is employed in a modified form of integral equation, which makes use of an integral on the inner waterplane $S_{F'}$, to reduce the singularity on the body surface, in the manner described in [3]. This results in

$$[1-\iint_{S_{F'}}(\nu G-2i\tau G_x)dxdy]\phi_j(\xi)+\iint_{S_B}[\phi_j(x)-\phi_j(\xi)]\frac{\partial G}{\partial n}dS =$$
$$-2i\tau\iint_{S_F}\phi_j(\nabla_2 G\cdot\nabla_2\chi+\frac{1}{2}G\nabla_2^2\chi)dxdy$$
$$+\iint_{S_B}(G+\frac{i\tau}{\nu}\nabla G\cdot\nabla\chi_s)n_j dS \quad (j=1,\ldots,6) \tag{4}$$

The first term on the right hand side is integrated over the undisturbed surface S_F outside the body.

Method 1

In this, equation (4) is solved directly for the radiation potentials, using the reversed flow Green function described above. Calculation of this function is difficult and, in our present algorithms, quite slow compared with calculation of the zero speed Green function. Furthermore, the potential ϕ_j on the free surface S_F is an unknown in the integral equation. It is, however, possible to truncate S_F close to the body because of the decay of the steady disturbance χ.

Method 2

This employs a perturbation expansion in the forward speed parameter τ, and is similar to that first developed by Nossen et al [2]. Differences in the numerical implementation are our use of higher order elements and a modified integral equation. The procedure is first to solve for the zero speed potential ϕ_j^o, and then to use this in an integral equation for the forward speed correction of order τ. In each of the integral equations the unknowns are only required on the body surface. As also the zero speed and order τ Green functions are much easier to calculate than in method 1, this approach appears to be much more efficient.

Method 3

This too is based on a perturbation expansion of ϕ_j to order τ. The hydrodynamic coefficents are also expressed as a perturbation series in τ, and the correction at order τ is derived in terms of ϕ_j^o and its derivatives. The procedure has been described in [1]. In the numerical disretisation it is necessary to extend the integral on S_F further than in the other methods, but the solution is fast since only the zero speed potential is required.

Frequency	Damping coefficient		
νa	Method 1	Method 2	Method 3
0.2000	2.310 2.318	2.322 2.315	2.315
0.5000	2.533 2.541	2.545 2.540	2.545
0.8000	2.224 2.253	2.249 2.250	2.246
1.0000	1.902 1.910	1.929 1.913	1.931
1.2000	1.641 1.647	1.661 1.663	1.643
1.4000		1.505 1.520	1.523
1.6000		1.455 1.441	1.442
1.8000		1.447 1.421	1.429
2.0000		1.448 1.445	1.458

Table 1 Cross-coupling damping coefficients. For each dimensionless frequency the upper line gives $-b_{13}(U)/\rho a^2 U$, and the lower line gives $-b_{31}(-U)/\rho a^2 U$

Results

We consider the floating hemisphere, for which graphical results have been given by Nossen et al [2]. Table 1 lists dimensionless values of the damping coefficents coupling surge (1) with heave (3), namely $-b_{13}(U)$ and $-b_{31}(-U)$ for different dimensionless frequencies. These have been calculated at a Froude number $(U/\sqrt{ga}) = 0.04$, where a is the radius. We note that at zero forward speed $b_{13} = b_{31} = 0$, and so the results give a useful indication of reliability in predicting forward speed effects. The Timman-

Newman relation requires that $b_{13}(U) = b_{31}(-U)$, and similary for the added mass coefficients. Method 3 satisfies this identically, so only one set of results is provided in the table (and these are independent of the value of τ at which the calulation is performed in method 3). The characteristics of the boundary element meshes are listed in table 2.

Method used	Planes of symmetry	Elements on discretised S_B	Elements on discretised S_F	Outer radius of S_F
1	1	18	30	4.4a
2	2	64	96	4.9a
3	2	64	320	25.8a

Table 2 Characteristics of boundary element meshes

References

1. Wu, G.X., and Eatock Taylor, R., 1990, The Hydrodynamic Force on an Oscillating Ship with low Forward Speed, J. Fluid Mech., Vol. 211, pp.333-353.

2. Nossen, J., Grue, J. and Palm E., 1991, Wave Forces on Three-Dimensional Floating Bodies with Small Forward Speed, J. Fluid Mech., Vol. 227, pp.135-160.

3. Eatock Taylor, R. and Chau F.P., 1991, Wave Diffraction - Some Developments in Linear and Non-Linear Theroy, Proc. of Offshore Mech. and Arctic Eng., Vol.1-A, pp.19-27.

DISCUSSION

MARTIN: Referring to your table 1, do you know ***which*** results are closest to the exact answer, and *why* the Timman-Newman relations are not satisfied for Methods 1 and 2?

EATOCK TAYLOR & TENG: Discretization and truncation errors lead to imperfect satisfaction of the T-N relations with Methods 1 and 2. I am not keen to speculate which of the results in table 1 are closest to the exact answer, although I would point out that the variability in all cases shown is less than 1 %. In most cases the results from Method 2 satisfy the T-N relations more closely than those from Method 1.

GRUE: What is the role of the m_j-tems which appears in Method 3 for the practical evaluation of f_{13}^2. What is the role of using high-order methods in comparison with low-order methods in the evaluation of the drift forces at zero forward speed and small forward speed (and then the wave drift damping coefficient)?

EATOCK TAYLOR & TENG: The m_j terms appear in the formulation of Method 3 as published in [1]. For the calculation of the results in table 1, however, the integral involving these terms on the body surface was transformed again by application of Tuck's result. This leads to an integral involving the product of the steady and unsteady tangential components of velocity. We believe that one of the advantages of using higher order elements is in the calculation of these velocities as gradients of the velocity potentials.

CLARK: Apparently method 3 of your paper obtains the added mass and damping in radiation problems with forward speed by direct calculation using zero-speed potentials. In a paper submitted for publication Prof. Sclavounos and Dr. Emmerhoff have achieved something similar for the wave drift damping coefficient in the corresponding diffraction problem. In the wave drift damping procedures that you mention in the paper have you considered calculating the wave drift damping coefficient directly? (i.e. without numerical differentiation of the drift force).

TAYLOR & TENG: Method 3 (our ref. [1]) only requires the steady disturbance potential and the zero-speed oscillatory potential for the calculation of added mass and damping (ϕ_{01} and ϕ_{10} respectively in the notation of Emmerhoff and Sclavounos). Their formulation of wave drift damping, however, requires an additional interaction potential ϕ_{11}. We have not so far sought to calculate this interaction term, so the answer to your question is 'no'.