Abstract for the 7th International Workshop on Water Waves and Floating Bodies
Val de Reuil, France 24-27 May 1992

THE EFFECT OF SECOND ORDER VELOCITIES ON DRIFT FORCES AND DRIFT MOMENTS

by John Grue and Enok Palm

Department of Mathematics, Mechanics Division, University of Oslo, Norway

1 Introduction

Knowledge of steady horizontal drift forces and yaw moment due to the combined effect of regular waves and a steady forward speed is of importance in the ship and offshore industry. Examples are the added resistance on conventional ships and high speed vessels, and the steady lateral force and yaw moment on sailing boats. Another example, important for moored ships and platforms, is the slow drift damping mecanism, which is due to the usually positive change in the mean wave drift force on a body introduced by a small forward speed. The steady drift forces and moment are of second order in the wave slope if this can be assumed small or moderate.

For zero forward speed these second order quantities can be obtained by time averaged squares of first order, linear quantities which are simple to compute.

When there is a forward speed, U, however, new terms being products of U and a steady second order velocity emerge in the expressions for the steady second order forces and moments. Assuming that the fluid motion is irrotational, the second order velocity may be derived from a velocity potential, $\psi^{(2)}$, say. Thus, we need to know the solution for $\psi^{(2)}$, which may be a rather complicated task.

For many practical problems, however, U is small, and higher order terms in U may be neglected. We shall here neglect terms of $O(U^2)$, which simplifies the problem considerably. In fact, we shall show that by this simplification we are able to express the terms containing $\psi^{(2)}$ by terms being products of two first order quantities, without really solving the $\psi^{(2)}$-problem.

2 The near field method

We consider a floating body moving horizontally with constant speed U and responding to long-crested incoming regular waves with small amplitude. Let us introduce a frame of reference (x, y, z) moving with forward speed U, in the same direction as the body, with the (x, y)-plane in the undisturbed free surface, the x-axis in the direction of the forward motion, and the z-axis positive upwards. Unit vectors (**i**,**j**,**k**) are introduced respectively along the x, y, z-directions. It is assumed that the motion is irrotational and the fluid incompressible. The total fluid velocity may then be written

$$\mathbf{v} = \nabla\Phi + U\nabla\chi_s \tag{1}$$

Here χ_s is the steady velocity potential generated by the moving body, independent of the incoming waves and Φ is the velocity potential due to incoming, scattered and radiated waves. χ_s and Φ both satisfy the Laplace equation. The corresponding fluid pressure is given by the Bernoulli equation

$$p = -\rho(\frac{\partial\Phi}{\partial t} + \frac{1}{2}U^2(|\nabla\chi_s|^2 - 1) + U\nabla\chi_s \cdot \nabla\Phi + \frac{1}{2}|\nabla\Phi|^2 + gz) + C(t) \tag{2}$$

where $C(t)$ is an arbitrary function of time.

In this section we assume that the forces and the moments are obtained by pressure integration over the body (the near field method). In the next section the same quantities will be examined by using the far field method. Φ may be written

$$\Phi = \phi^{(1)} + \phi^{(2)} + \psi^{(2)} \tag{3}$$

where $\phi^{(1)}$ is the linear oscillatory potential proportional to the wave amplitude, and $\phi^{(2)}$ and $\psi^{(2)}$ are the oscillatory and steady second order potentials proportional to the wave amplitude squared, respectively. The terms $\frac{1}{2}U^2|\nabla\chi_s|^2, gz, \frac{1}{2}|\nabla\Phi|^2|, \partial\Phi/\partial t$ and $U\nabla\chi_s \cdot \nabla\Phi$ in (2) give rise to steady second order contributions being products of two $\phi^{(1)}$-terms. The two latter terms also give contributions through products of $\phi^{(1)}$-terms and terms describing the first order body motion. All these contributions are known when the complete first order motions have been determined. The term $-\rho U\nabla\chi_s \cdot \nabla\Phi$ gives, however, also rise to a term

$$-\rho U\int_{S_B} \nabla\chi_s \cdot \nabla\psi^{(2)} n_i dS, \quad i = 1, 2, ...6 \tag{4}$$

which needs to be evaluated to consistently obtain the pressure integrated drift forces and drift moments. These terms seems to have been disregarded in the literature. In (4) we have introduced $(n_1, n_2, n_3) = \mathbf{n}$ and $(n_4, n_5, n_6) = (\mathbf{x} \times \mathbf{n})$, with $\mathbf{n}$ being the normal vector, positive out of the fluid and $\mathbf{x} = (x, y, z)$. S_B denotes integration over the wetted part of the body.

The contributions from (4) to the horizontal force components and the yaw moment may, however, for small values of U by an integral transform be written in a proper form without solving for $\psi^{(2)}$. For the horizontal forces it may be shown that

$$-\rho U\int_{S_B} \nabla\chi_s \cdot \nabla\psi^{(2)} n_i dS = -\rho U\int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}\frac{\partial\chi}{\partial x_i} dS, \quad i = 1, 2 \tag{5}$$

and for the yaw-moment we obtain

$$-\rho U\int_{S_B} \nabla\chi_s \cdot \nabla\psi^{(2)} n_6 dS = -\rho U\int_{S_F+S_B} \Psi\frac{\partial\psi^{(2)}}{\partial n} dS - \rho U\int_{S_F+S_B} \frac{\partial\psi^{(2)}}{\partial n}(x\frac{\partial\chi}{\partial y} - y\frac{\partial\chi}{\partial x}) dS \tag{6}$$

where we have introduced χ by

$$\chi_s = -x + \chi \tag{7}$$

Ψ denotes the steady velocity potential if the body is moving along the positive y-axis (corresponding to χ when the body is moving along the positive x-axis). Thus, for $i = 1, 2, 6$, (4) may be replaced by integrals over S_F and S_B, where $\psi^{(2)}$ enters in the form $\partial\psi^{(2)}/\partial n$.

3 The far field method

An often used method to obtain the steady second order forces and moments is to apply the principle of conservation of linear and angular momentum. Thereby the integration of the momentum may be replaced from the body surface to the restrained vertical cylinder at infinity.

This procedure has the merit that the geometry is very simple and that the velocity potentials for the steady flow and the wave motion may be replaced by their asymptotic values. These simplifications make it possible to use analytical methods in the evaluation of the forces and moments. Let S_∞ denote the surface of the vertical cylinder at infinity. The mean horizontal force may then be written

$$\mathbf{F} = -\overline{\int_{S_\infty} (p\mathbf{n} + \rho \mathbf{v} v_n) dS} \tag{8}$$

where a bar denotes the time average and $v_n = \mathbf{v} \cdot \mathbf{n}$. Correspondingly, using the principle of conservation of angular momentum, the yaw moment on the body is given by

$$M_z = -\mathbf{k} \cdot \overline{\int_{S_\infty} (p\mathbf{x} \times \mathbf{n} + \rho \mathbf{x} \times \mathbf{v} v_n) dS} \tag{9}$$

In the Bernoulli equation (2), $\overline{\Phi_t} = 0$, and for the second order pressure $C(t) = O(A^2)$, where A is the amplitude of the incoming waves. Furthermore, the coupling term $\rho U \nabla \chi_s \cdot \nabla \psi^{(2)}$ reduces to $-\rho U \partial \psi^{(2)} / \partial x$.

Applying conservation of mass, Grue and Palm (1990) found that (8) reduces to

$$\mathbf{F} = \rho \int_{C_\infty} \frac{1}{2g} \left(\overline{\left(\frac{\partial \phi^{(1)}}{\partial t}\right)^2} - U^2 \overline{\left(\frac{\partial \phi^{(1)}}{\partial x}\right)^2} \right) \mathbf{n} ds - \rho \int_{S_\infty} \left(\frac{1}{2} \overline{(\nabla \phi^{(1)})^2} \mathbf{n} - \overline{\nabla \phi^{(1)} \frac{\partial \phi^{(1)}}{\partial n}} \right) dS$$
$$+ \rho U \mathbf{j} \left(\int_{C_\infty} \overline{\zeta^{(1)} \frac{\partial \phi^{(1)}}{\partial s}} ds + \int_{S_\infty} \frac{\partial \psi^{(2)}}{\partial s} ds dz \right) \tag{10}$$

where C_∞ denotes the waterline of S_∞ and $\zeta^{(1)}$ the first order free surface elevation. Obviously the last term of (10) is zero if the velocity circulation in the fluid is zero. It then turns out that the terms containing $\psi^{(2)}$ vanish in the expression for the horizontal forces. The physical reason for this is that in the linear momentum flux the contributions at infinity from the pressure term and the velocity term cancel each other. This is not true for the forces and the moment by the near field method, a result which is obvious since the momentum flux due to the velocity term is identically zero integrated over a rigid body.

In some special cases, however, as for example a boat sailing at a non-zero angle of attack, it is expected that a steady velocity circulation is present, giving rise to the lifting force in formula (10).

For the steady second order yaw moment we obtain

$$M_z = -\rho \int_{S_\infty} \overline{\frac{\partial \phi^{(1)}}{\partial \theta} \frac{\partial \phi^{(1)}}{\partial R}} dS - \rho U \int_{C_\infty} \overline{\left(y \frac{\partial \phi^{(1)}}{\partial R} - \frac{x}{R} \frac{\partial \phi^{(1)}}{\partial \theta} \right) \zeta^{(1)}} ds$$
$$- \rho U \int_{S_\infty} \left(y \frac{\partial \psi^{(2)}}{\partial R} - \frac{x}{R} \frac{\partial \psi^{(2)}}{\partial \theta} \right) dS + \rho U^2 \int_{C_\infty} y n_1 \overline{\zeta^{(2)}} ds \tag{11}$$

where R is the radius of the cylinder, θ the polar angle and $\zeta^{(2)}$ is a second order free surface elevation. The formula (11) is valid for arbitrary U and water depth. We notice that the two first terms are products of first order quantities. The last term will be neglected, being of $O(U^2)$. By an integral transform it may be shown that the third term, for small values of U, may be recast into the form

$$-\rho U \int_{S_\infty} \left(y \frac{\partial \psi^{(2)}}{\partial R} - \frac{x}{R} \frac{\partial \psi^{(2)}}{\partial \theta} \right) dS = -\rho U \left(\int_{S_B} \psi^{(2)} n_2 dS - \int_{S_F + S_B} y \frac{\partial \psi^{(2)}}{\partial n} dS \right)$$
$$= -\rho U \int_{S_F + S_B} \Psi_s \frac{\partial \psi^{(2)}}{\partial n} dS \tag{12}$$

Here

$$\Psi_s = \Psi - y \tag{13}$$

is the steady velocity potential if the body is moving along the positive y-axis (corresponding to χ_s when the body is moving along the positive x-axis).

We note that U is a prefactor of (5), (6) and (12). For small forward speed we may then evaluate these integrals with $\partial\psi^{(2)}/\partial n$ obtained for $U = 0$. From the body boundary condition we have

$$\frac{\partial\psi^{(2)}}{\partial n} = V_n^{(2)} \quad \text{on } S_B \tag{14}$$

where $V_n^{(2)}$ reads

$$\begin{aligned} V_n^{(2)} = & - \overline{\mathbf{n}\cdot[(\boldsymbol{\xi}^{(1)} + \boldsymbol{\alpha}^{(1)}\times\mathbf{x})\cdot\nabla]\nabla\phi^{(1)}} \\ & + \overline{(\boldsymbol{\alpha}^{(1)}\times\mathbf{n})\cdot[(\frac{d}{dt}\boldsymbol{\xi}^{(1)} + \frac{d}{dt}\boldsymbol{\alpha}^{(1)}\times\mathbf{x}) - \nabla\phi^{(1)}]} \end{aligned} \tag{15}$$

Here, $\boldsymbol{\xi}^{(1)} = Re\{(\xi_1, \xi_2, \xi_3)e^{i\sigma t}\}$ and $\boldsymbol{\alpha}^{(1)} = Re\{(\xi_4, \xi_5, \xi_6)e^{i\sigma t}\}$ denote respectively the first order translations and rotations of the body. σ denotes the frequency of oscillation.

At the free surface we have

$$\frac{\partial\psi^{(2)}}{\partial z} = -\frac{1}{g}\overline{\frac{\partial}{\partial t}\nabla\phi^{(1)}\cdot\nabla\phi^{(1)}} + \frac{1}{g^2}\overline{\frac{\partial\phi^{(1)}}{\partial t}\frac{\partial^3\phi^{(1)}}{\partial z\partial t^2}} + \frac{1}{g}\overline{\frac{\partial\phi^{(1)}}{\partial t}\frac{\partial^2\phi^{(1)}}{\partial z^2}} \quad \text{on } z = 0 \tag{16}$$

The two first terms in the right hand side of (16) vanish. Introducing $\phi^{(1)} = Re(\phi e^{i\sigma t})$ for the first order potential, we obtain for the third term

$$\frac{\partial\psi^{(2)}}{\partial z} = -\frac{\sigma}{2g} Im(\phi\frac{\partial^2\phi^*}{\partial z^2}) \quad \text{on } z = 0 \tag{17}$$

Here a star denotes complex conjugate. In the general case we note that $Im(\phi\partial^2\phi^*/\partial z^2)$ vanishes far away from the body since we have there that

$$\phi(x, y, z) = e^{Kz}\phi(x, y, 0) \tag{18}$$

where K is the wave number. This conclusion is, however, not true close to the body, except in the special case when the body is restrained and have vertical walls extending deeply in the fluid. In this case the steady second order velocity is zero in the entire fluid region. Furthermore, we observe that the $\psi^{(2)}$-field is generated by the presence of first order evanescent modes in vicinity of the body, due to linear body motions or non-vertical body boundaries.

Introducing (14), (15) and (17) into (5), (6) and (12) we observe that $\psi^{(2)}$ is replaced entirely by products of first order quantities. The free surface integrals on the right hand sides of (5), (6) and (12) are easily evaluated numerically since the integrands decay rapidly away from the body.

The term (12) is in numerical examples found to contribute with up to 50% of the forward speed part of the yaw moment, while the rest is from products of two first order quantites. The $\psi^{(2)}$-contribution is always found to increase the effect of the forward speed. The effect of $\psi^{(2)}$ is found to be small in the near field method.

References

[1] Grue, J. and Palm, E., Mean forces on floating bodies in waves and current. *Abstr. 5th Int. Workshop on Water Waves and Floating Bodies, Manchester*, UK. 1990.

DISCUSSION

TUCK: Can I make sure I understand some interesting symmetry properties of your results. Namely, if you have a fore-and-aft symmetric ship moving in (exactly) beam seas, there is a large *yaw* moment, proportional to the forward speed and the square of the wave amplitude?
GRUE & PALM: Yes.

KRING: If you have found second order effects to be significant, what is your intuition concerning higher-orders?
GRUE & PALM: The steady drift force and moment are to leading order quadratic in the incident wave amplitude. Their magnitude are in several offshore problems found to be large and of great practical importance. Examples are the added resistance, wave drift forces at zero speed and the mean yaw moment acting on floating ships and platforms. For moderate wave amplitudes it is believed that the 4th order contribution, which is the next high-order contribution, will be small. For large wave amplitudes, however, perturbation methods do not longer apply, and high-order methods are important.

CLARK: For floating bodies of deep draft and vertical sides, such as a T.L.P, can we safely ignore the contribution to wave drift damping coefficient for Yaw Moment from the potential $\psi^{(2)}$?
GRUE & PALM: If the mean wave period of the spectrum is smaller than ≈ 7 sec, the effect of $\psi^{(2)}$ can be neglected. On the other hand, if the mean wave period exceeds 7 s we observe in the calculations for a TLP that first order motions are present, introducing the effect of $\psi^{(2)}$ which then becomes important. This is further discussed by Grue & Palm (1992): "Mean yaw moment on floating bodies advancing with a forward speed in waves", BOSS 192