KELVIN SINGULARITY APPROACH TO COMPUTE FREE-SURFACE FLOWS ON YAWED AND HEELED BODIES

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1 Introduction

With the development of fast computers, steady flow computations for surface-piercing bodies, either in symmetrical flow or in yawed flow, can be achieved using panel methods. Two kinds of elementary singularities are available: the classical "Rankine" aerodynamic ones which must be distributed not only on the body but also on the free-surface (truncated at finite distance) or the "Kelvin" ones satisfying directly a linearized form of the free-surface boundary condition on the undisturbed location of it. During long years, the Rankine methods have been almost the ones used for steady flows but more recently some authors have pointed out the interest of the Kelvin singularities (Newman [1], Noblesse [2]); studies using these last methods have been presented recently (Maniar and alii [3]), Ba and alii [4]).

The work presented here is devoted to yawed flow and has been extended to flows with yaw and heel angles with use of Kelvin singularities. Various numerical schemes have been developped to give tools with quite different degrees of complexity and needs in computational time. The first one, lifting-line scheme, gives results with very short computational times, even on a simple personal computer. The second one, lifting-surface, very similar to aerodynamic vortex-lattice method, gives much more precise results but needs much longer computational time, particularly for heeled bodies. Work is on progress to use this scheme, not only for simple planshape bodies but also on real boats as sailing ones; results for real boats, Antiope and closely related to "Standfast 43" are presented. Finally, a last scheme, complete panel method is being developped to estimate the errors done in the previous ones when neglecting the body thickness. First results obtained on rectangular bodies are presented.

2 Formulation

2.1 Problem to solve

The free-surface flow past a body at small angle of attack α is studied. The fluid is incompressible and the flow is assumed irrotationnal except for a sheet composed of the lifting body and its wake. A frame of reference $(0, \vec{x}, \vec{y}, \vec{z})$ fixed with respect to the body is used. $O\vec{z}$ is vertical and the undisturbed free-surface is in the plane $(O, \vec{x}, \vec{y})$. The $0\vec{x}$ axis is parallel to free stream velocity and of opposite direction (fig. 1). F is the Froude number based on waterline length. So the pertubation velocity satisfies the Laplace equation in the fluid domain assumed to be bottomless, the body condition, the Kelvin linearized free surface condition, and a radiation condition at upstream infinity. Different forms of the third Green formula are used in the following numerical schemes with use of the classical Green function.

2.2 Lifting-line scheme

The body, assumed to be of high aspect ratio, is replaced by a line, with or without heel angle with respect to a vertical line, supporting the leading edge of a doublet sheet of semi-infinite extent towards downstream infinity with intensity $\Gamma(z_1)$ where z_1 is the coordinate along the lifting-line (fig. 2). The normal velocity on the line, along $O\vec{y}$ axis, $v(z_1)$ can be then computed; the body and Kutta-Joukowsky conditions are now replaced by the classical Prandtl equation:

$$\Gamma(z_1) = kU_\infty C(z_1)\left[\alpha(z_1) - \frac{v(z_1)}{U_\infty}\right]$$

where k is the two-dimensional lift slope and $C(z_1)$, the chord distribution. By writing this equation on n points z_{1i} on the lifting line, a linear set of equations on unknowns $\Gamma(z_{1i})$ is obtained. For the solution, the boundary condition at the bottom $\Gamma(-\ell) = 0$ is used but not $\Gamma(0) = 0$, because the body is surface-piercing. Instead, we insure that the circulation is null on a control point above the free-surface $(-z_{11})$, symmetrical of the first point under the plane z=0 one. It has been checked that this condition gives correct distribution circulation close to the free-surface both for $F \to 0$ or ∞ (cf. Villeger and alii, [5]). Computations are simple because in the expression of v, the complex integral exponential due to the Green function is reduced to a real exponential when computed on the lifting line . This scheme has been also used to compute free-surface elevations.

2.3 Lifting-surface scheme

The longitudinal component of pertubation velocity u (parallel to $O\vec{x}$ axis), also harmonic, can be computed from the 3rd Green formula. By integration from upstream infinity to field point, the potential is obtained and by derivation with respect to y_1', the normal velocity. The body condition leads to an integral equation on the unknown doublet distribution:

$$\iint_{S^+} \gamma(M')K(M,M')dS_{M'} = U_\infty \alpha$$

where S^+ is the projection of upper-pressure side of body plane $z = 0$ or $z_1 = 0$ (plane parallel to free stream velocity and leaned at heel angle β with respect to a vertical plane). S^+ is divided into $Nx \times Nz$ panels (fig. 3). Semi-infinite doublet sheets, extending towards downstream infinity are distributed on every panel. A linear set of equations is obtained by writing the body condition on each panel, the unknown being the doublet intensities. The continuity of doublet strengths through the trailing edge assure the satisfaction of Kutta-Joukowsky condition. Computation of the shape of the free-surface close to the body has been also done.

2.4 Panel method

The third Green formula leads to an integral equation whose unknowns are source intensity σ on the body S and doublet intensity μ on the body mean camber surface Σ_1 and a plane wake parallel to free-stream velocity Σ_2. The body condition can be written:

$$\frac{\sigma(M)}{2} - \frac{1}{4\pi}\iint_S \sigma(M')\frac{\partial}{\partial n_M}g(M,M')dS_{M'} + \frac{1}{4\pi k_0}\int_{WL}\sigma(M')\vec{n}.\vec{x}|_{M'}\frac{\partial}{\partial n_M}g(M,M')d\ell_{M'}$$
$$-\frac{1}{4\pi}\iint_{\Sigma_1 U \Sigma_2}\mu(M')\frac{\partial}{\partial n_M}\frac{\partial}{\partial n_{M'}}g(M,M')dS_{M'} = -\vec{U}_\infty.\vec{n}_M \qquad \text{for} \qquad M \epsilon S$$

where g is the Green function and k_o is the wave number. $\vec{n}_M$ is the outward normal to the body S.

The line integral on doublet distribution vanishes because doublets are distributed on a zero thickness plane; only the integral on the water-line WL dealing with source distribution is non zero. A linear system is obtained by writing the body condition on each body surface panel ($Nx \times Nz$ equations) and the Kutta-Joukowsky condition (considered as a body condition just downstream of the trailing edge) on the Nz horizontal strips of the wake (fig. 4); the unknowns are $Nx \times Nz$ source strengths (on each surface panel) and the Nz doublet intensities (one for each strip). As the computational time is much more higher than for the previous method, it has been used only to estimate the error done by neglecting the body thickness.

3 Numerical results

As examples of lifting-surface computations, side-force coefficients on well-known Antiope sailing boat (Lechter [6] with free heel angle or Kirkman and alii[7] with fixed heel angle), are plotted figure 5 versus leeway angles at various heel angle at Froude number F=.3. The unknown doublet intensities are located on the keel and intensities on the rest of the hull are linked to the previous ones. The agreement is quite acceptable, the difference between computations and measurements can probably be attributed to body thickness. Figure 6 presents similar computations for sailing boat with fully detached keel derived from the "Standfast 43" (Gerritsma [8]) also for F=.3 and with free heel angle. In both computations, heel effect seems to be underestimated by computations done with the static heel angle.

4 Conclusion

We have presented 3 methods of computations for free-surface lifting flows using Kelvin singularities. The first one considers the body as a simple lifting-line and the second, lifting surface, ignores body thickness; both uses only semi-infinite doublet sheets. Focus has been brought on heel angle effect on numerical results, particularly first results on real sailing boats are presented.

The lifting-line scheme can predict heel angle effect but is not able to describe the variation of efforts with Froude number (maximum of lift and side-force close to Froude number # 0.6); nevertheless, it can be considered as a useful tool, for example, in a pilot study because computational time is quite negligeable. The lifting-surface scheme gives much more precise results but needs longer computational times particularly when heel angle is present; it underestimates side-force coefficients due to the zero-thickness assumption. For free-surface elevation computations, results are quite similar for both models, but computational time needed for lifting-line is ten times shorter than for lifting-surface but are not suitable for swept bodies. Finally, a panel method has been developed only to estimate the effect of body thickness on side-force computations.

References

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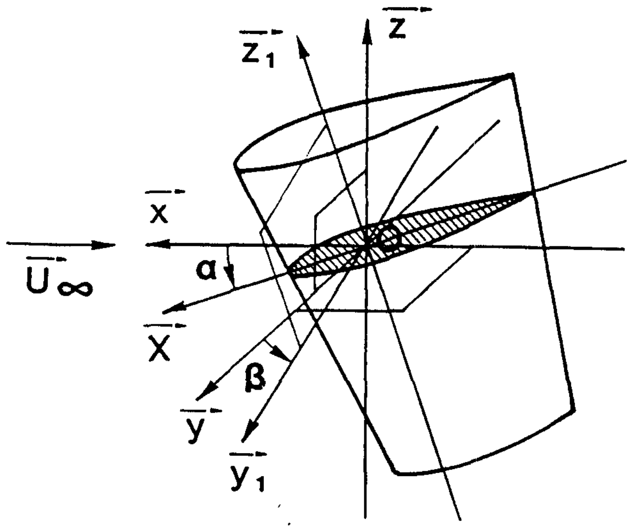

FIG. 1 - SCHEME OF FLOW

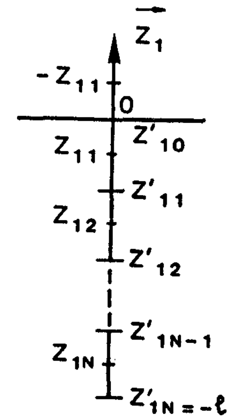

FIG. 2 - LIFTING-LINE

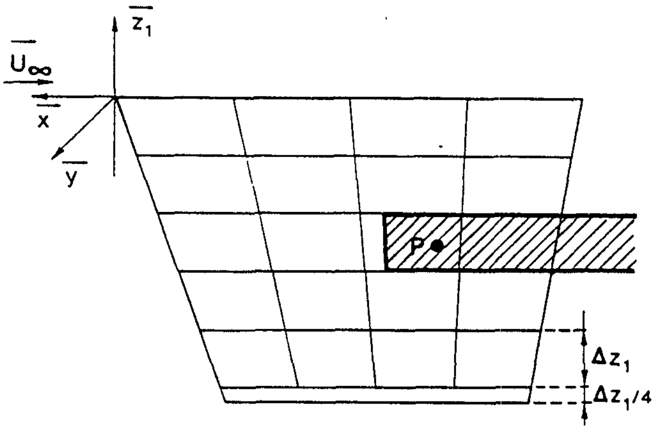

FIG. 3 - LIFTING-SURFACE

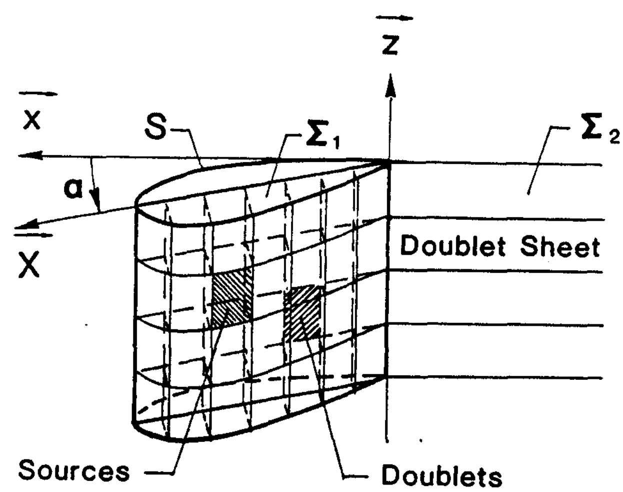

FIG. 4 - REPARTITION OF SOURCE AND DOUBLET DISTRIBUTION

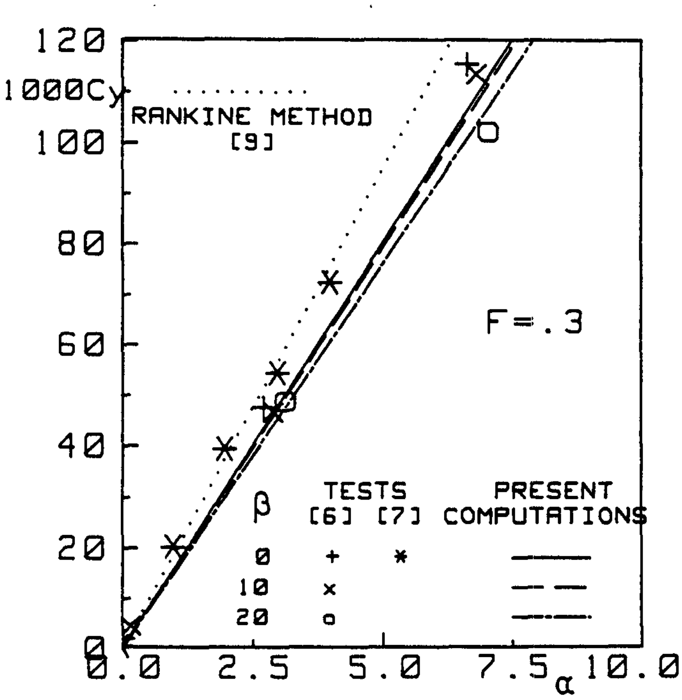

FIG. 5 - SIDE FORCE COEFFICIENT FOR SAILBOAT ANTIOPE (LIFTING SURFACE)

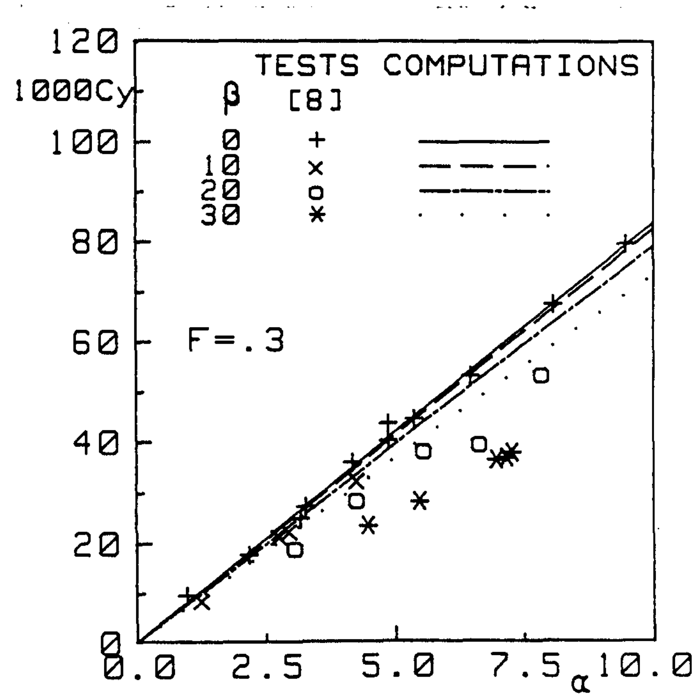

FIG. 6 - SIDE FORCE COEFFICIENT FOR SAILBOAT STANDFAST (LIFTING-SURFACE)

DISCUSSION

NEWMAN: Can you comment on the occurrence of the free-surface "jump" behind the trailing edge, which we observed to occur at a Froude number ≈ 0.7? (Ref.: Maniar, Newman and Xü, Naval Hydrodynamics Symposium, Ann Arbor, 1990).
GUILBAUD & al.: From our observations, no trailing-edge jump can be seen at F = 0.4 but jump was seen for F = 0.71 and F ≥ 2. For test at F = 0.6, observations were not quite clear; jump appears and vanishes. The flow seems to be unstable.

YEUNG: If possible, I wonder if you can clarify the treatment of "tip vortices" that might shed off at the lower end of the keel surface.
GUILBAUD & al.: We have presented here the first results obtained with a panel method using Kelvin singularities. We have still to check the results obtained: so it is only the first step by developing such a method with simple shape bodies (rectangular wings with NACA airfoils). Addition of panels on the lower end of the body or "tip vortices" shedding of this end will be studied in further developments of the method. Nevertheless, I think that these tip vortices are probably not of prime effect on the forces and moments coefficients on the body but certainly in the vortex sheet shape.

TUCK: Lifting surface theory does not ***neglect*** thickness. Rather it ***decouples*** or ***separates*** thickness from lifting effects at least for flows with certain symmetries. However, for the heeled case, there is a thickness-induced incidence, and hence thickness does affect lift. Thus there is an ***intermediate*** case between your zero-thickness lifting surface and the panel method, where thickness is included but the problem is not more difficult to solve than you lifting surface problem.
This is not a trivial observation because your panel method itself is subject to severe restrictions, because you assume a wake in the plane of the stream. Hence it is not clear that your panel method can capture thickness effects with any more accuracy than lifting surface theory would do.
GUILBAUD & al.: I agree with you first comment. For the restriction on the wake, the influence of its chosen direction will be part of future work. Nevertheless, experience of aerodynamic (cf B.E. Mazzonji *et al.*) shows that variation of wake direction between free-stream velocity and body bisection induces very low side force coefficient variation (less than 2% for 15° variation) for a single body. Probably, the same effect must be present for low Froude numbers