Ship Waves Computations

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In the present paper the steady free surface potential flow past 3D bodies moving with constant forward speed is computed by means of a formulation involving the potential of single layer. Linear and fully nonlinear conditions at the free boundary have been implemented. The numerical method has been developed following the general features of the well known method proposed by Dawson [1]. In this work, starting from the encouraging results obtained in [2] for 2-D test cases, numerical computations have been performed for 3-D cases: for a submerged prolate spheroid the fully nonlinear formulation has been used, while in the case of floating bodies (Wigley hull and Series 60-$C_B = 0.6$) the linear problem has been solved.

The velocity potential ϕ satisfies Laplace's equation in the fluid field and:

(1) $$\phi_n = 0 \qquad on \quad the \quad body$$

(2) $$z = \eta(x,y) = \frac{Fr^2}{2}(1 - \nabla\phi\cdot\nabla\phi) \qquad on \quad the \quad free \quad surface$$

(3) $$\frac{Fr^2}{2}\nabla\phi\cdot\nabla(\nabla\phi\cdot\nabla\phi) + \phi_z = 0 \qquad on \quad the \quad free \quad surface$$

(3') $$Fr^2\phi_l^2\phi_{ll} + \phi_z = 0 \qquad on \quad the \quad free \quad surface$$

(4) $$\lim_{x\to-\infty} |\nabla\phi| = 1$$

where $\frac{\partial}{\partial l} = \mathbf{l}\cdot\nabla$: l is a parameter defined along a streamline lying on the free surface. Formula (3'), describing 2-D and 3-D problems as well, is simple, elegant and suitable for numerical implementation, though some Authors discussed about its validity[3]; anyway it is easy [2] to show the equivalence between (3') and (3).

The linear problem has been obtained assuming as basis flow the double model one, and neglecting all the quadratic terms of the free surface potential, following Dawson[1]. The boundary conditions have been implemented by two different approximations for the second derivative ϕ_{ll}. The first is Dawson's finite differences (FD) upwind operator [1], while the second is based on analytical derivation (AN); in the last case the upstream condition $\phi_{ll} = \phi_z = 0$ must be imposed.

The discretization is based on the collocation method; in the linear case the free surface panels are arranged on $z = 0$, while in the nonlinear problem the actual free surface is 'followed', step by step. More details about the numerical iterative procedure can be found in [2].

In fig. 1 free surface contours of the linear and nonlinear numerical simulation for the steady forward motion of a submerged spheroid are shown. In fig. 2, the wave resistance of the same body is compared with experimental results [4]: the nonlinear solution is very close to the experimental data. The influence of the grid spacing on the Wigley hull wave resistance has been studied for the Neumann-Kelvin problem: in fig. 3 the results for four free surface grids successively refined in the transverse direction are shown. The wave resistance computed with the finer one seems to be the convergence value for nearly all the tested Froude numbers. Fig.4 shows the Wigley's wave resistance computed with different schemes and compared with experimental values [5]. As expected, discrepancies between the wave patterns computed with Neuman-Kelvin and Dawson formulations (fig. 5) are very small, because of the hull slenderness. This is *not* the case for the Series 60 model also with a low C_b (0.6). In fig. 6 a detailed comparison has been made with the Series 60 experimental wave pattern for Fr=0.316 [6]. The three discrete models show significant differencies in the computed wave patterns: Dawson AN solution seems to be the most accurate since the complexity of wave pattern is well catched; throats and crests are in the right position. Dawson FD scheme predicts a larger Kelvin angle due to the numerical dispersion of finite difference approximation, while Neumann Kelvin solution appears to be less accurate and too smoth. Finally the wave resistance is reported in fig.7. The experimental data are taken from [7]. In all the Series 60 computations 1000 panels are used to discretize the ship while 1800 panels are used on the free surface.

References

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3. **Raven H. C.**, Variations on a Theme by Dawson, *17 Sym. on Naval Hydro.*, The Hague, 1988.
4. **Farell C., Guven O.**, On the Experimental Determination of the Resistance Components of a Submerged Spheroid, *Jou. Ship Res.*, Vol.17, n.2, 1973.
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7. **Kim Y.H., Jenkins D.**, Trim and Sinkage Effects on Wave Resistance with Series 60, $C_B = 0.6$, *DTNSRDC/SPD - 1013 - 01*, 1981.

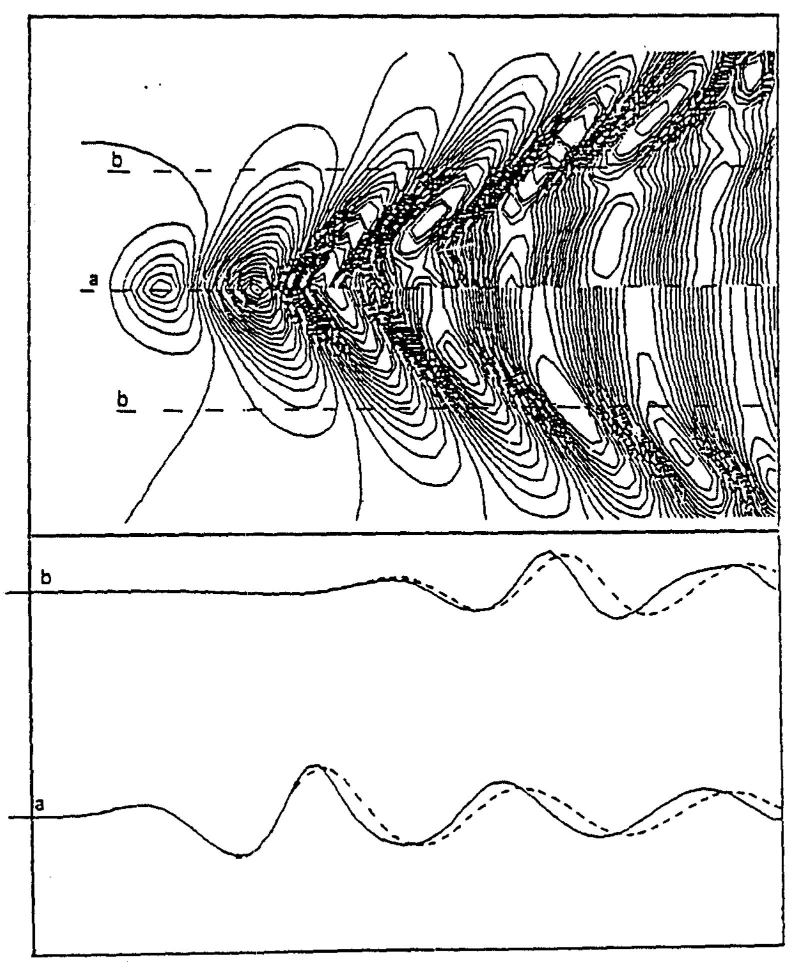

Fig.1 Numerical wave pattern for a submerged spheroid (*semiaxes ratio* = .2, Fr = .5, h/L = .22): linear (bottom half), nonlinear (top half); longitudinal cuts: linear (dashed), nonlinear (solid).

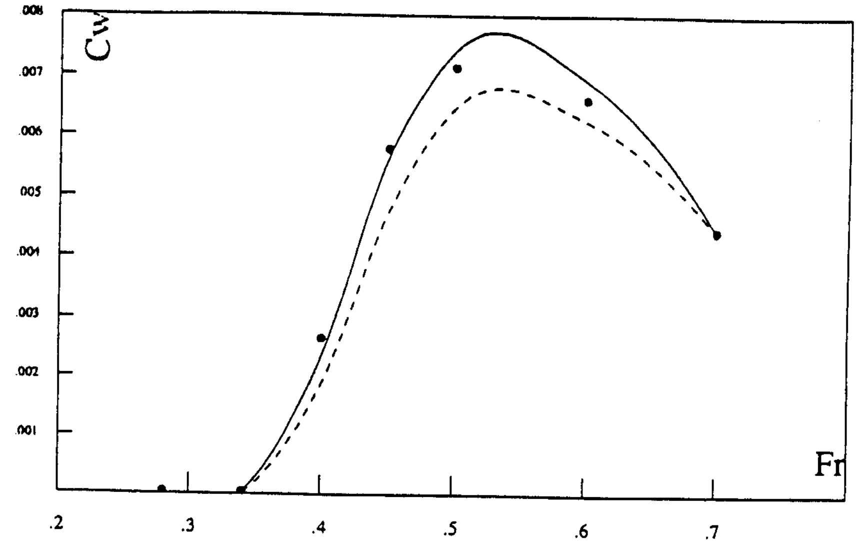

Fig.2 Experimental (•) and numerical wave resistance of the submerged prolate spheroid: linear (dotted) and nonlinear (solid).

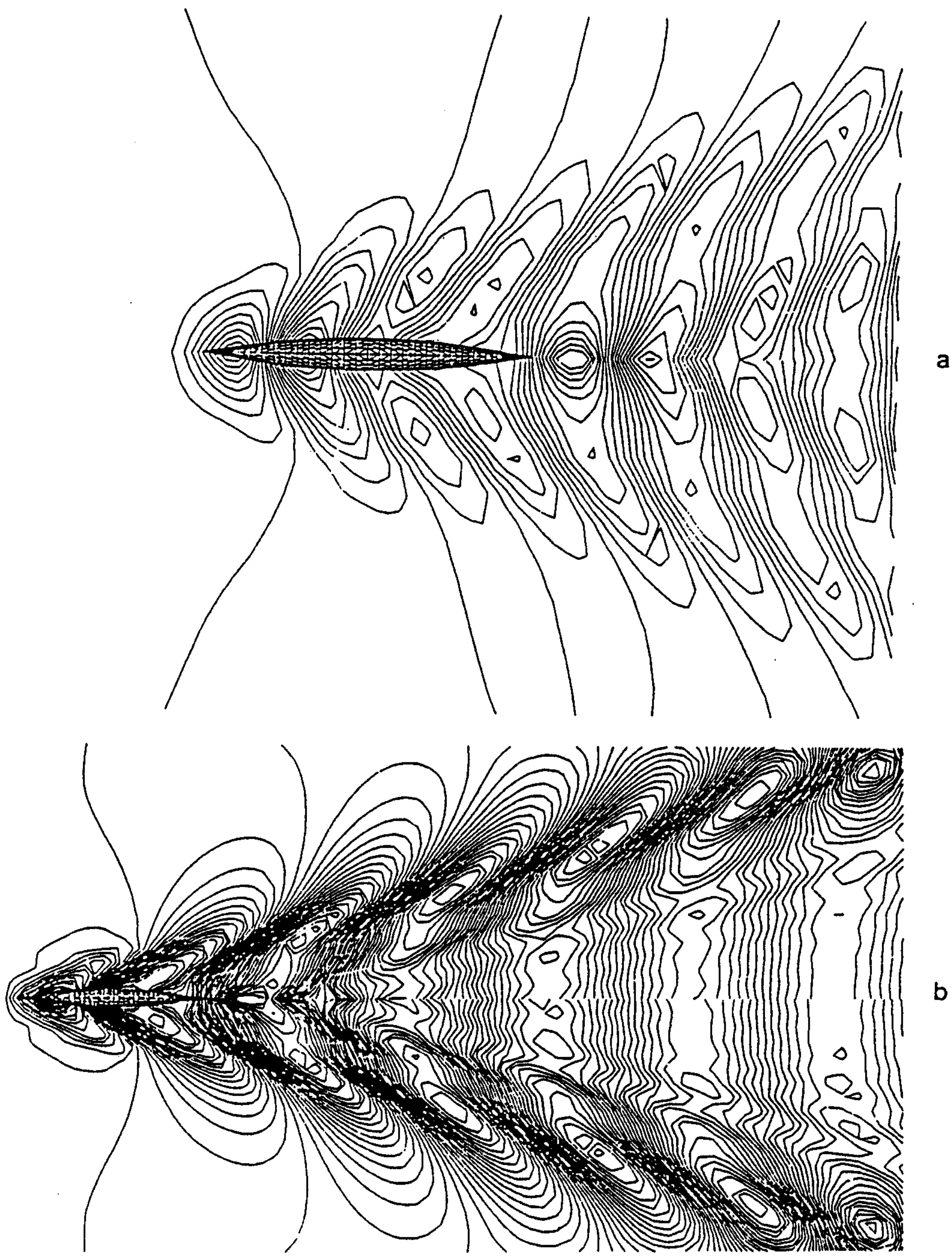

Fig.3 Computed waves for the Wigley hull: a) Fr = .3, b) Fr = .5; Neuman-Kelvin formulation (bottom half), Dawson linear formulation (top half).

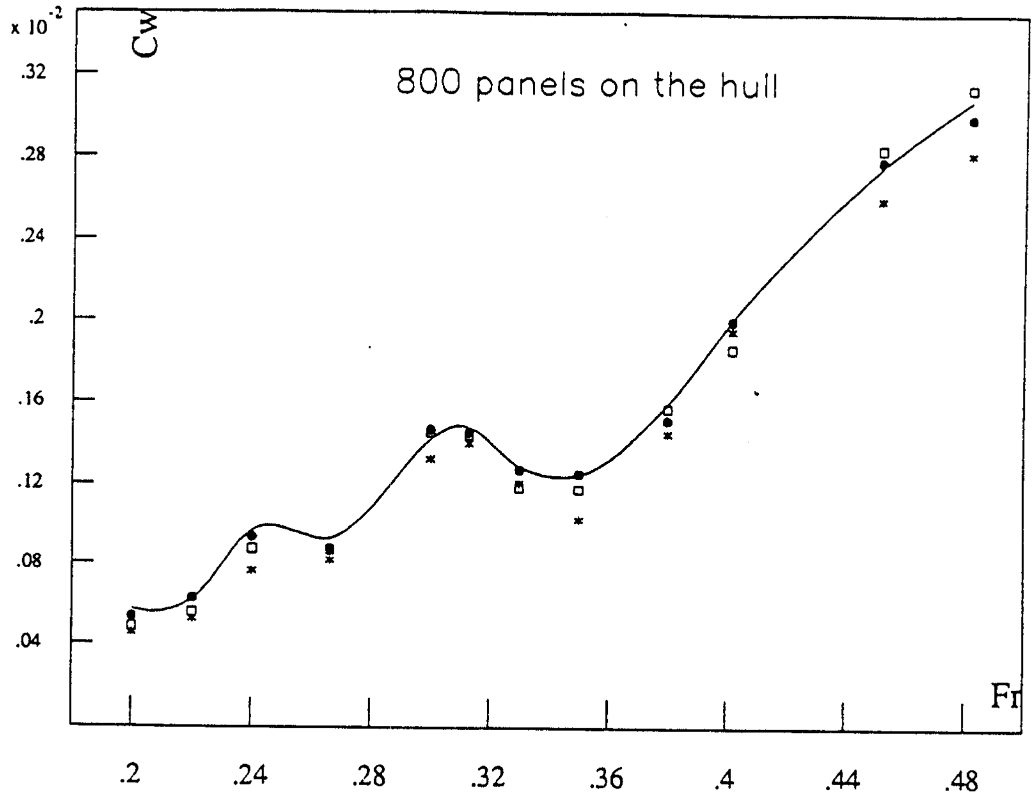

Fig.4 Wave resistance for the Wigley hull (Neuman-Kelvin formulation): free surface grid refinement in trasverse direction;

∗ dy/dx = 1.0 - 1440 panels ;
□ dy/dx = .75 - 1920 panels ;
• dy/dx = .60 - 2400 panels ;
— dy/dx = .54 - 2688 panels.

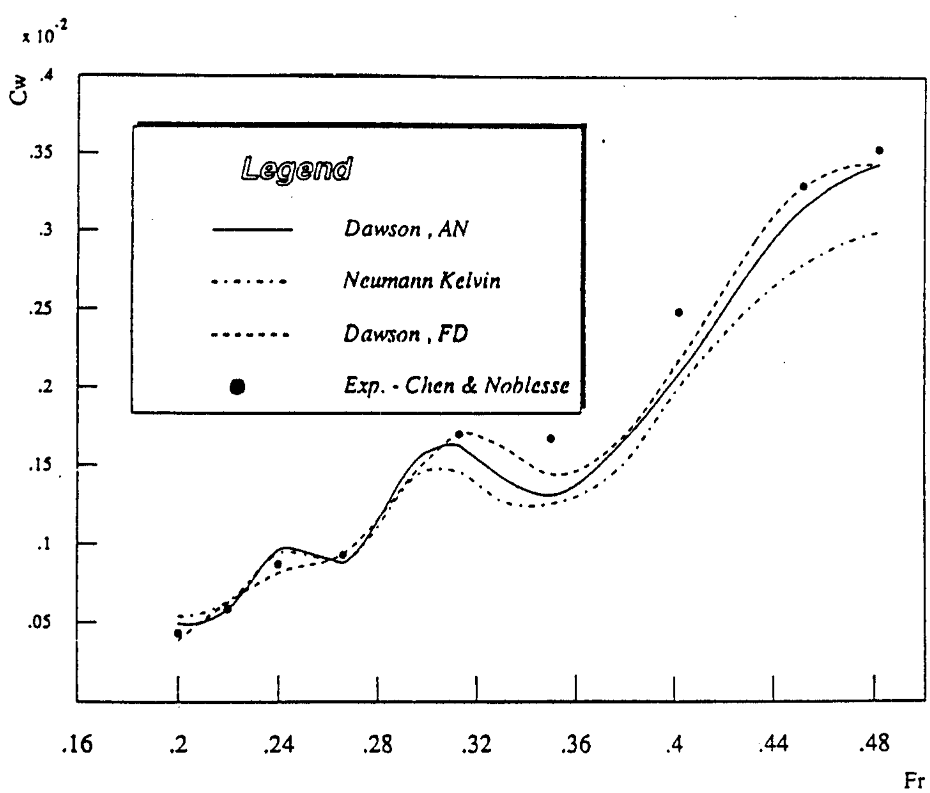

Fig.5 Wave resistance for the Wigley hull (2400 panels on the free surface and 800 on the body): some comparisons.

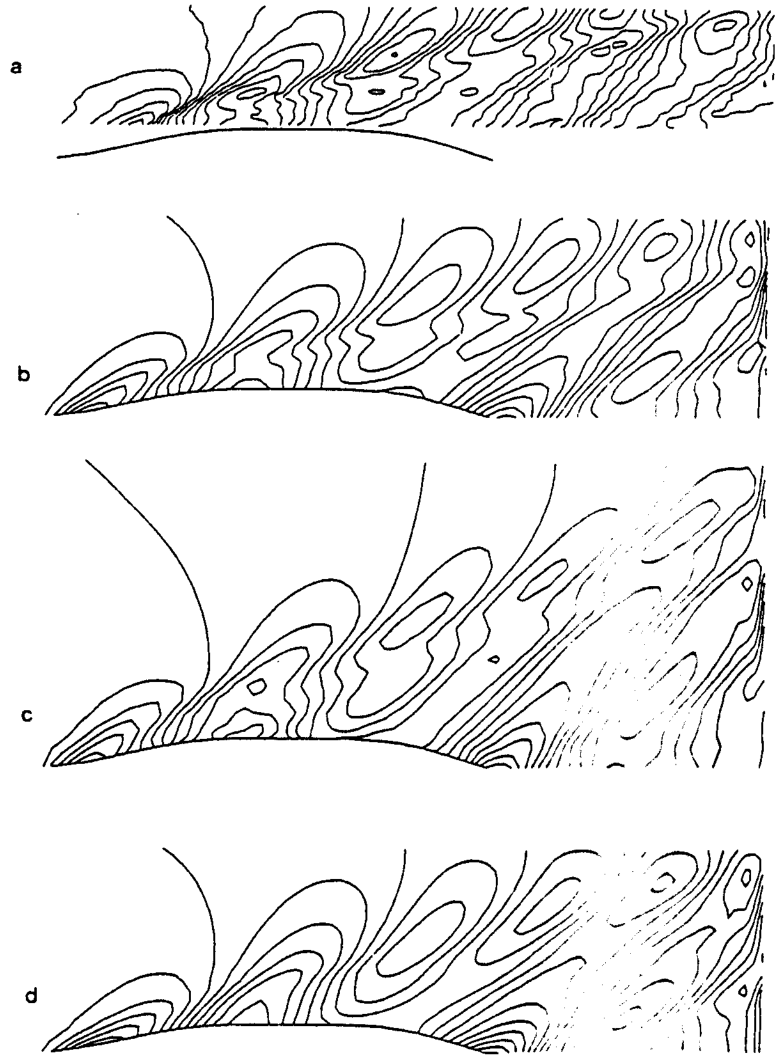

Fig.6 Series 60 hull waves (Cb=0.6, Fr=.316, $2\pi Fr^2/dx$ = [illegible] a) experimental results; b) Dawson formulation, AN; c) Dawson formulation, FD; d) Neuman-Kelvin formulation, AN.

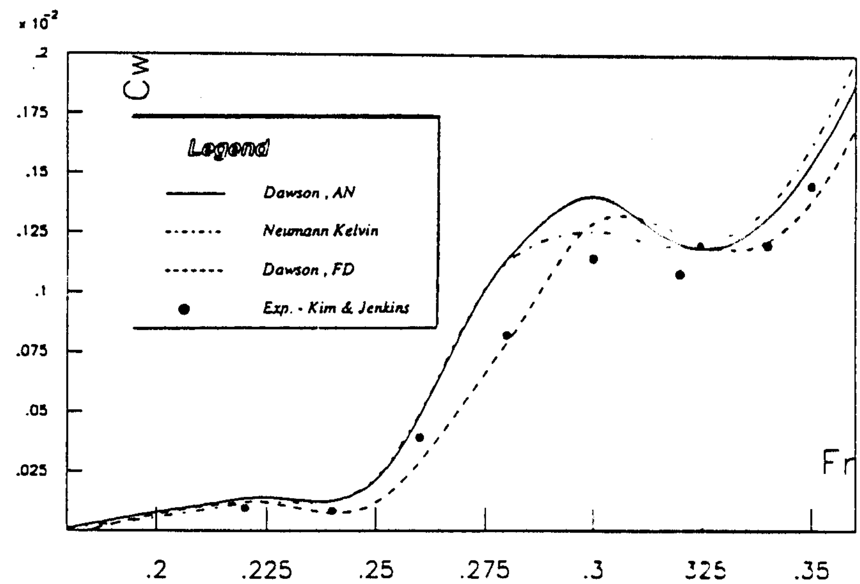

Fig.7 Wave resistance for the Series 60 (Cb=0.6, $2\pi Fr^2/dx$ = 31.).

DISCUSSION

BERTRAM: Could explain in more detail how you solve the nonlinear equation on the free surface?

LALLI & al.: We remind that:

$$\phi(x,y,z) = Ux + \varphi(x,y,z);$$

$$\varphi(x,y,z) = \int_{\partial B} \frac{\sigma(\xi_1,\xi_2,\xi_3)}{r} d\Sigma + \int_{S} \frac{\sigma(\xi_1,\xi_2,\xi_3)}{r} d\Sigma$$

where $P(\xi_1,\xi_2,\xi_3)$ are points of the hydrodynamic field boundaries.

The body surface ∂B and part of the free surface S are discretized by means of plane elements; the simple layer density σ is assumed to be constant on every boundary element.

The fully nonlinear problem is solved by an iterative algorithm, in which the free boundary S is updated step by step; to initialize the procedure, the potential flow is computed with a linear formulation. The iterative scheme consists of two cycles. An 'internal' one (index j), in which the following nonlinear system is solved iteratively:

$$^{(j)}\phi_{ni} = 0 \text{ on } \partial B$$

$$^{(j-1)}\phi_{li}^2 \;[^{(m)}\alpha_i \frac{\delta}{\delta l}\, ^{(j)}\varphi_{xi} + {}^{(m)}\beta_i \frac{\delta}{\delta l}\, ^{(j)}\varphi_{yi} + {}^{(m)}\gamma_i \frac{\delta}{\delta l}\, ^{(j)}\varphi_{zi}\,] + \frac{^{(j)}\varphi_{zi}}{Fr^2} = 0 \text{ on S.}$$

When the solution of this system satisfies the required accuracy, in the 'external' cycle (index m) the free surface is updated by the dynamic boundary condition, until the convergence is reached. An under relaxation is used in both cycles: the value of the parameters must decrease as the Froude number grows. The finite differences operator $\frac{\delta}{\delta l}$ (with $l = (\alpha,\beta,\gamma)$) is the one proposed by Dawson. Although the implementation of the term ϕ_{ll} by analytic derivation gives very good results for the linear problem, in both 2D and 3D, in nonlinear calculations we have obtained, so far, some strange results: in this period, this delicate point is under examination.

Since the nonlinear system is solved with a certain accuracy at every step of the external cycle, such procedure is rather time consuming, but, as shown in [2], it is very robust and reliable: rather high Froude numbers flows can be simulated.

RAVEN: This paper contains very interesting information on the errors introduced by the difference scheme. It appears that the numerical dispersion largely disappears when using an analytical derivative instead. At first sight this seems to contradict the result of e.g. the analysis of Sclavounos & Nakos (ONR 88), that the leading-order dispersion ($O(\Delta x)$) comes from the use of constant strength source panels, not from the differencing scheme; doing away with the difference scheme would thus reduce the damping but not the dispersion (to leading order).

However, with analytical derivatives a different integral operator is used, which results in a different numerical dispersion due to the source discretization. I could not yet work out what the dispersion is, but it appears to be smaller than in Dawson's method. Thus the larger numerical dispersion with the FD scheme probably does not come from the FD scheme itself, but from the different integral operator connected to its use. This reconciles the contradicting results, and answers my own question posed after your lecture.

LALLI & al.: We thank Dr. Raven for the comments. We do not believe our results to be in contradiction with the analysis of Sclavounos and Nakos (ONR 88). In fact their analysis concerns three different forms (W_1, W_2, W_3 in their paper) of the integral operator that arises from the Neumann-Kelvin problem, but none of their discrete operators seems to be the one we solve. Two of the three discrete operators ($\hat{W}_1$, $\hat{W}_2$) possess distinct numerical properties due to the use of finite difference approximation. In the third one ($\hat{W}_3$) the derivatives are obtained by analytic differentiation of a basic function of order m; furthermore, the double derivative appears under the integral sign. In the discrete scheme AN, although the continuous operator is exactly W_1, the discrete one is not $\hat{W}_1$, since no finite difference approximation is used for the convective term. However, using a piecewise constant variation of the unknown source density over the panel, numerical results obtained with the AN discrete scheme seems to possess an improved numerical dispersion with respect to those obtained with the FD scheme, which belong to the $\hat{W}_1$ type. This can be easily shown with a simple 2-D result.

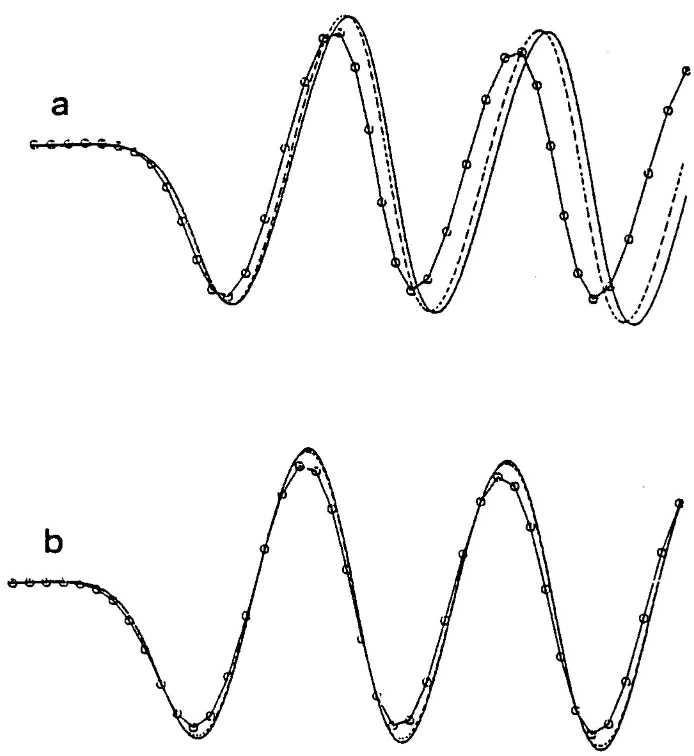

fig.1

DISCUSSION

In fig. 1 surface waves due to a submerged dipole (Fr=0.6) are shown. In both a) and b) the analytical solution is the solid line and the numerical results have been obtained with 12(—) and 60(- - -) panels per wave length. In a) the FD solution shows the well known underprediction of the wave length. In b) the AN solution with only 12 panels per wave length gives a reasonable agreement with the analytical one, while with 60 panels the numerical AN and the theoretical solutions are almost coinciding

TUCK: A very minor pedantic matter: I believe there is no such thing as a "Neumann-Kelvin boundary condition". There *is* a Neumann-Kelvin boundary-value problem, namely to solve Laplace's equation subject to a ***Neumann*** boundary condition on the hull and a ***Kelvin*** boundary condition on the plane of the undisturbed free surface.
LALLI & al.: We agree with Prof. Tuck. Sometimes in the speech a lapsus can occur.

EGGERS: There is an old quotation due to Oscar Wilde, saying that consistency is the excuse of the unimaginative; this seems to math with Dr. Bertram's heretic comments expressed earlier today. However, even if I do not want to raise a heated discussion, I cannot accept the statement that it is easy to show the correctness of Dawson's free surface condition, although that meanwhile it even has been accepted as a basic for thesis work on viscous effects in Sweden. In our community at least, there should be consensus that when extending his analysis to 3-D problems, Dawson has inadvertently disregarded a curvature effect on derivatives along a curve; it may well be that this effect is unimportant for calculations.
LALLI & al.: First of all, we confirm that the equivalence between the classical form of the exact nonlinear boundary condition:

(1) $$\frac{Fr^2}{2}\nabla\phi\,.\,\nabla(\nabla\phi\,.\,\nabla\phi) + \phi_z = 0$$

and the following form:

(2) $$Fr^2\,\phi_l^2\,\phi_{ll} + \phi_z = 0$$

can be demonstrated, both in 2D and in 3D. In fact, starting from (1):

$$\frac{Fr^2}{2}\sum_{i=1}^{3}\frac{\partial\phi}{\partial x_i}\frac{\partial}{\partial x_i}\,|\text{grad}\,\phi|^2 = Fr^2\,|\text{grad}\phi|^2\sum_{i=1}^{3}\frac{\partial\phi/\partial x_i}{|\text{grad}\,\phi|}\frac{\partial}{\partial x_i}\,|\text{grad}\phi|$$

and using the definitions:

$$\frac{\partial}{\partial l} \equiv \sum_{i=1}^{3}\frac{\partial\phi/\partial x_i}{|\text{grad}\,\phi|}\frac{\partial}{\partial x_i}\ ;\quad \phi_l \equiv \sum_{i=1}^{3}\frac{\partial\phi/\partial x_i}{|\text{grad}\,\phi|}\frac{\partial f}{\partial x_i} \equiv |\text{grad}\,\phi|$$

we get easily (2). On the other hand, starting from (2):

$$\phi_l^2\phi_{ll} = \nabla\phi\,.\,\nabla\phi\,\frac{\partial(\nabla\phi\,.\,\mathbf{l})}{\partial l} = \nabla\phi\,.\,\nabla\phi\left(\frac{\partial\nabla\phi}{\partial l}\,.\,\mathbf{l} + \nabla\phi\,.\,\frac{\partial \mathbf{l}}{\partial l}\right)$$

but:

$$\nabla\phi \cdot \frac{\partial \mathbf{l}}{\partial l} = -K\,\nabla\phi \cdot \mathbf{n} \equiv 0$$

for ***any*** value of the curvature K; finally, it is easy to show that:

$$\nabla\phi \cdot \nabla\phi\left[\nabla(\nabla\phi)\cdot\mathbf{l}\right]\cdot\mathbf{l} = \frac{1}{2}\nabla\phi \cdot \nabla(\nabla\phi \cdot \nabla\phi)$$

Dawson was not interested in the fully nonlinear problem, so he didn't write the condition in form (2), anyway he had the merit to suggest the introduction of derivatives along the streamlines lying on the free surface, that allow one to obtain a simpler formation, in both linear and nonlinear cases. We agree however that the calculations performed in [1] are not so easy to follow, mainly because Dawson does not consider clearly as separate problems the introduction of derivatives along streamlines and linearization, but we don't agree at all with the criticisms made by Jensen, Mi and Söding (Jensen et al., 1986) and by Raven [3]. In his paper, in the first equation of V paragraph (which seems to be the exact condition), Dawson omits the term $\frac{1}{2}\phi_z(\phi_x^2+\phi_y^2+\phi_z^2)_z$: this could be considered an error, although this term should be anyway neglected after linearization. But in all the calculations he performed to obtain the discussed equation (14) $(\Phi_l^2\phi_l)_l + g\phi_z = \Phi_l^2\Phi_{ll}$ there are no errors: the only hypothesis used (but not mentioned) is that the free surface potential ϕ' generates a flow along the basis flow streamlines: this statement is a consequence of the linearization. So we don't think that Dawson neglected any curvature effects. For example, in Jensen et al. (1986), we don't agree with the comments between equations (12) and (13): though $(\nabla\Phi)_l \neq \nabla(\Phi_l)$, nevertheless it is true that:

$$\frac{1}{2}\nabla\phi \cdot \nabla(\Phi_l^2) = \frac{1}{2}\phi_l\,(\Phi_l^2)_l = \Phi_l\,\Phi_{ll}\,\phi_l$$

hence (12) always transforms into (13).

One may of course discuss about the simplicity of Dawson linearization, and on some possible improvements, for instance by means of Taylor expansions (but Taylor expansion should be made around the double model free surface elevation, and not around z = 0 plane). Anyway, our opinion is that it is very important, in the wave resistance problem, to deal with the exact nonlinear condition, as many Authors now do: consequently, we concentrate our efforts on this goal, and we think it is not worthwhile to implement a more complicated linear formulation. Moreover, numerical results show that Dawson linear condition, very easy to treat numerically, is a significant improvement with respect to the classical Neumann-Kelvin problem.

We thank Prof.Eggers for his question, allowing us to clarify our statements; we also must acknowledge Jensen, Mi and Söding (Jensen et al., 1986) and Raven [3] for their stimulating comments, although we don't agree with them.

Ref.: Jensen G., Mi Z.-X., Söding H., "Rankine Source Methods for Numerical Solutions of the Steady Wave Resistance Problem", 16 Sym. on Naval Hydro., 1986.