The radiation and scattering of surface waves by a vertical circular cylinder in a channel

by

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We consider the problems of the radiation and scattering of surface gravity waves by a vertical circular cylinder placed on the centreline of a channel of width $2d$ and depth H, and either extending from the bottom through the free surface or truncated so as to fill only part of the depth. These problems, which are important due to the need to know how the side walls of a wave tank affect the results of experiments on relatively large models, are solved for arbitrary incident wavenumber k, by constructing appropriate multipoles for cylinders placed symmetrically in channels and then using the body boundary condition to derive a set of infinite systems of linear algebraic equations. This method is superior to the more usual approach of using a set of image cylinders to model the channel walls, in particular the occurrence of modes other than the fundamental when $kd > \pi/2$ is accurately modelled and the correct form predicted for the far-field.

We begin by examining the problem of a plane wave incident on a vertical circular cylinder that extends throughout the fluid depth and is placed symmetrically in a wave tank. Due to the constant depth variation in this problem and the symmetry of the geometry, the problem is equivalent to the two-dimensional acoustic scattering of a wave normally incident upon an infinite array of equally-spaced identical circular cylinders, a problem with a very long history. There are many methods of solution for this problem but the most often used is a direct method of solution (Spring & Monkmeyer 1975). The main idea of the direct method is to express the total velocity potential as a sum of an incident wave and a general circular wave emanating from each cylinder in the array. In the case of a plane wave normally incident on an infinite row of identical cylinders all these circular waves will be identical and thus the body boundary condition need only be applied on one cylinder. Using Graf's addition theorem for Bessel functions these circular waves can be expressed in terms of coordinates centred on one particular cylinder and then the boundary condition can be applied on that cylinder giving rise to an infinite system of linear algebraic equations. In order to solve such a system numerically a truncation procedure must be used and this corresponds to using only a finite number of circumferential modes to represent the cylindrical waves.

It is well known that at any given wavenumber a finite number of propagating modes can exist in a channel and that as the wavenumber increases so does the number of modes. In fact if k is the wavenumber and the channel width is $2d$ then if $(j-1)\pi < kd < (j-\frac{1}{2})\pi$, $(j \geq 1)$ j modes are possible. Thus if an incident wave is scattered by a cylinder in a channel there will be up to j reflected and j transmitted propagating modes. Calculating the correct far-field behaviour using the direct method discussed above is virtually impossible since the velocity potential is descibed in terms of an infinite sum of circular waves, each centred at a different point, each one of which is only known approximately from the solution to a truncated system of equations.

Another difficulty which arises when using this method is the occurrence of slowly-convergent Hankel series as is described in Thomas (1991) though a careful treatment involving integral representations can alleviate the problems. Finally the method becomes fairly unwieldy when used to solve problems involving truncated cylinders and Yeung & Sphaier (1989a,b) found it necessary to neglect the (albeit small) interference effects caused by non-propagating modes.

In this paper we will consider problems involving circular cylinders in channels using a fundamentally different method which correctly predicts the far-field behaviour, avoids the treatment of slowly convergent series and is in principle, exact. The method is based around the construction of suitable multipoles for channel problems. Thus when considering a channel of water of depth H and width $2d$ suitable multipoles will be solutions of Laplace's equation in $-\infty < x < \infty$, $|y| \leq d$, $-H \leq z \leq 0$ which are singular at $(x, y) = (0, 0)$ and which satisfy the condition of zero normal velocity on $|y| = d$. They must also look like outgoing plane waves as $|x| \to \infty$ or else be exponentially small there. The key to the construction of these multipoles is the derivation of suitable integral representations for solutions to Laplace's equation in a laterally unbounded fluid which can then be modified to take account of the channel walls. The procedure is fairly complicated but picks out the correct far-field behaviour in a natural way.

Once the multipoles have been constructed a wide class of problems can be solved in a straightforward manner including both scattering and radiation problems, problems involving cylinders which occupy the whole depth of fluid and problems involving truncated cylinders, either occupying $-H \leq z \leq -D$ or $-D \leq z \leq 0$, $(D < H)$.

References

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DISCUSSION

MARTIN: You showed a graph of the sway added-mass coefficient, with a singularity at the trapped-mode frequencies. What happens if you truncate the cylinder?
LINTON: When the cylinder is truncated so that it extends from the free surface part way to the bottom the qualitative behavior of the sway added mass is the same as for the non truncated cylinder. If the draft of the cylinder is D, then as K changes from H, the total depth, to O the value of the wavenumber, kd, at which the sway added mass is singular change smoothly from the value for the nontruncated cylinder to $\pi/2$.

MILOH: Near the cut-off frequency viscous damping becomes important and so are the non-linear terms. Can you give us some estimates of these two effects?
LINTON: The short answer is no. My aim is to fully understand the linear problem for arbitrary bodies in channels and I have chosen circular cylinders as a starting point as we can go a long way towards solving this problem analytically.

PAWLOWSKI: You mentioned that at present you concentrates on understanding the linear aspects of the problem, which obviously is quite important for people who do experiments. Do you plan an extension of the work to deal with non-linear aspects of the some physical situation?
LINTON: The reply is the same as to the question of Miloh.

YEUNG: You have mentioned that the method of images may not obviously yield the far-field wave behavior in the channel. This is not true. To my recollection, the Hankel series (i.e. the slowly convergent portion) was summed analytically and the resulting integral form in an Appendix of the JEM paper (Yeung & Sphaier, 1989a) I can be shown to yield such wave field. In this same work, we found that if the body size approaches about half-width of the tank, the resonance peak of the added mass (in heave) disappears. I am most gratified to see that this feature is confirmed by your work. Finally, I want to mention that it is nice to have expressions of such multipoles in a channel.
LINTON: With reference to the farfield behavior of the solution, it is my opinion that the multipole method is a more natural method of solution for this problem and the behavior in the farfield can be obtained in a much simpler manner as the residues of various integrals.

NEWMAN: How does the multipole method converge when the cylinder diameter approaches the channel width?
LINTON: The expansions of the multipoles in polar coordinates are valid over the range $0<r/d<2$, i. e. up to the first image singularity in the wall. The cylinder diameter approaching the wall is equivalent to $a/d \rightarrow 1$ and this is well within the region of validity of the expansions. As a result no problems of convergence are encountered in this limit.

DISCUSSION

TUCK: I take it the added mass actually becomes infinite at the trapped-wave frequency. If so, what happens to its value if the body is made ***slightly*** non-symmetric or is moved off-center, so that the problem is not exactly anti-symmetric. It would seem rather surprising if a small change in geometry could produce a large change in the added mass.

LINTON: The singularity in the added mass coefficient μ is caused by a singularity in the complex force coefficient $q(\omega) = \nu(\omega) + i\,\mu(\omega)$ on the real ω-axis. For an off center cylinder this singularity is close to, but not on, the real ω-axis. This results in large spikes in the damping and added mass coefficients. As the cylinder approaches the centerline the spike in the damping coefficient gets higher and narrower until when the cylinder is on the centerline the damping is identically zero ($kd<\pi/2$). The spike in the added mass coefficient however becomes a singularity in this limit.

EVANS: It is perhaps worth making the point that the singularity in sway added mass at the trapped mode frequency may be overlooked because of the closeness of the first cut-off frequency for the channel. This is particularly true for small or truncated cylinders. It is only for large cylinders extending throughout the water depth that the trapped mode frequency is markedly distinct, and of course below, the cut-off frequency.