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The Nonlinear Diffraction Forces on a Submerged Spheroid

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1 Introduction

The accurate prediction of surface wave effects on a submerged three-dimensional body is of practical importance for the design and operation of submarines and underwater vehicles. Due to the nonlinearity of the free surface and the large computational burden for three-dimensional wave-body interaction calculations, there are as yet no effective methods for the computation of the important high-order forces on the body and the associated effects on the free surface.

At the previous workshop (Liu & Yue 1991), we presented a high-order spectral method for the study of nonlinear wave-body interactions in two dimensions. By expanding the singularity distributions on the free surface and the body in orthogonal spectral representations, we achieved exponential convergence of the results with respect to both the body and free-surface unknowns and the (arbitrary) order, M, of the perturbation. Furthermore, with the use of fast transform techniques, the computational effort grew only approximately linearly with the number of unknowns and the order M. Numerical results were given for the nonlinear (in fact fourth-order) horizontal drift force on a submerged circular cylinder. These were found to be in good agreement with experimental measurements.

Motivated by the success in two dimensions, we extend the high-order spectral method in this work to the study of nonlinear wave interactions with three-dimensional submerged bodies. Of special interests here are the steady horizontal force (in beam seas) and the vertical drift force and mean pitch moment (in head seas) on a near-surface spheroid. These, under more general conditions, are directly related to the slowly-varying excitations on the body.

2 Formulation

The arbitrary-high-order spectral method for wave-body interactions is formulated under the usual assumptions of potential flow. The essential steps of the formulation for the general three-dimensional problem follow that for the two-dimensional case (Liu & Yue 1991). The key ideas are the use of the surface potential $\Phi^S(\mathbf{x},t) = \Phi(\mathbf{x},\eta(\mathbf{x},t),t)$ (where $z=\eta$ is the free-surface elevation) in Zakharov's (1968) form of the nonlinear free-surface boundary conditions; and the perturbation expansion of the potential Φ in $\varepsilon=O(\Phi^S,\eta) \ll 1$ up to an *arbitrary* order M: $\Phi(\mathbf{x},z,t) = \sum_{m=1}^{M} \Phi^{(m)}(\mathbf{x},z,t)$.

For the wave-body problem, each perturbation potential $\Phi^{(m)}$ is considered to result from the combined influence of a dipole distribution $\mu^{(m)}$ on the free surface and a source distribution $\sigma^{(m)}$ on the body surface. Assuming doubly periodic boundary conditions in space, we expand $\mu^{(m)}$ in double Fourier series, and $\sigma^{(m)}$ in a Fourier-Chebyshev expansion:

$$\mu^{(m)}(x,y) = \sum_p \sum_q \mu_{pq}^{(m)} e^{i(px+qy)} , \tag{1}$$

$$\sigma^{(m)}(\varphi,\theta) = \sum_k \sum_\ell \sigma_{k\ell}^{(m)} e^{ik\varphi} T_\ell(1 - 2\pi/\theta) , \tag{2}$$

where φ, θ are respectively the polar and azimuthal angles of a point on the body surface with respect to its center. For this general three-dimensional problem, exponential convergence with respect to both the number of free surface and body modes is preserved. The computational burden remains only a linear function of the total number of modes and the order M.

3 Results

We consider, as an illustration, the nonlinear diffraction of Stokes waves by a submerged spheroid. The major and minor axes of the spheroid are a and b respectively, and its center is submerged a distance h beneath the mean free surface. The high-order spectral method is applied starting from initial values, and the linear and nonlinear steady (drift) and harmonic force and moment coefficients are obtained from harmonic analysis of the time histories after steady-state (limit cycle) is reached, typically in $3 \sim 4$ fundamental wave periods.

As an initial validation, we compare our M=1 computations to the linear results of Wu & Eatock-Taylor (1987) who solved the first-order diffraction problem analytically in terms of spheroidal coordinates. We obtain agreements up to four decimals and confirm the exponential convergence with respect to the number of free-surface Fourier and body Fourier and Chebyshev spectral modes. For a check of the high-order quantities, comparisons are also made for the second-order forces on a submerged sphere to the frequency-domain panel method of Kim & Yue (1989). The results are again excellent.

We now carry out the simulations for the spheroid to high order M, and obtain the zeroth- (drift), first-, second-, and third-harmonic forces and moments on the body. For a slender spheroid in head seas, strip theory predicts the presence of a vertical drift force but no mean pitch moment up to second-order. Using a linear three-dimensional panel method, Lee & Newman (1991) show the existence, in head seas, of a non-zero mean pitch moment due to three-dimensional effects which is generally bow-down. In this study we find that, when effects of higher-order potentials are included, the mean pitch moment surprisingly may change sign from bow-down to bow-up as the incident wave steepens or when the body submergence is decreased. Detailed analyses indicate that the bow-up contributions are mainly due to quadratic interactions among high-order potentials which are formally fourth-order in magnitude. Thus the three-dimensional and nonlinear effects on the mean pitch moment are generally of opposite signs and, for given ambient head waves, there is a particular submergence at which the mean pitch moment is identically zero. A typical set of results is shown in figures 1 and 2.

For beam seas, the second-order horizontal drift force does not vanish in contrast to strip theory predictions (for circular sections). In the presence of three-dimensional effects, a *positive* second-order drift force can be expected in principle. When strong nonlinear interactions are involved (due to steep incident waves or shallow submergence), it is known (see Liu & Yue 1991) that there can be a negative drift force even on a two-dimensional submerged circular cylinder. For high aspect ratio spheroids, this effect can of course also be anticipated. Thus, as with the mean pitch moment in head seas, three-dimensional and (higher-order) nonlinear effects produce opposing horizontal drift forces on the spheroid. Figures 3 and 4 show this for a typical case, where again for a given wave steepness, there exists a submergence at which the steady horizontal force vanishes.

More detailed results for the forces and moments on a submerged spheroid will be presented and discussed at the Workshop. We remark that the present method can be generalized in a direct manner to include forward speed. Selected results for the case with forward speed will also be shown.

4 References

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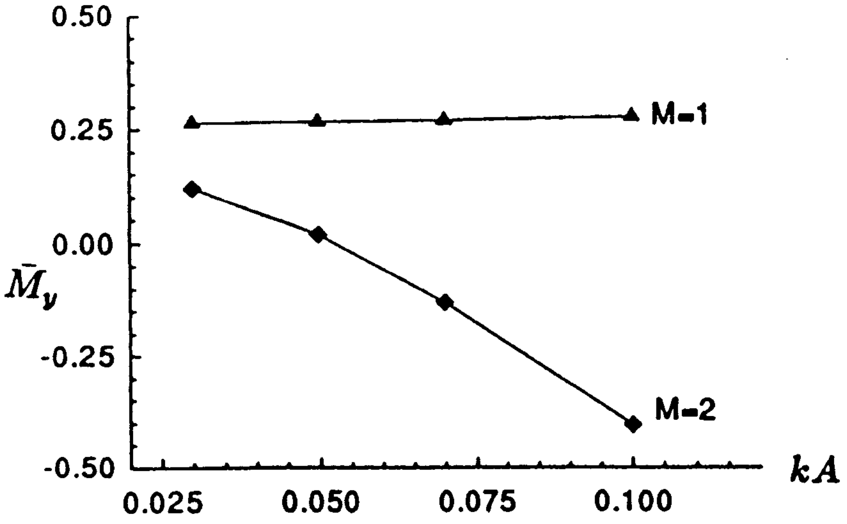

Fig. 1 Mean pitch moment coefficient ($\bar{M}_y/\rho g v b (kA)^2 e^{-2kh}$, $v = \frac{4}{3}\pi a b^2$) on a spheroid, $a = 10b, h = 2b$, as a function of incident wave slope kA.

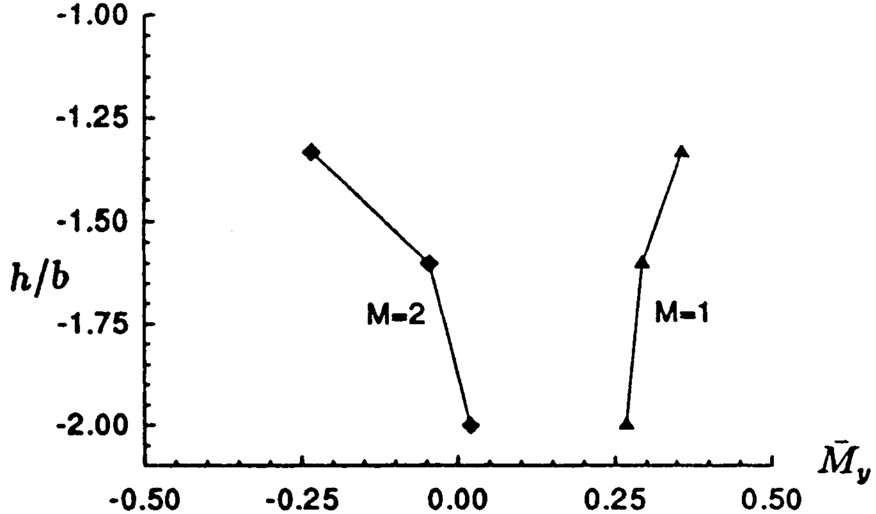

Fig. 2 Mean pitch moment coefficient as a function of submergence h/b ($a = 10b, kA = 0.05$).

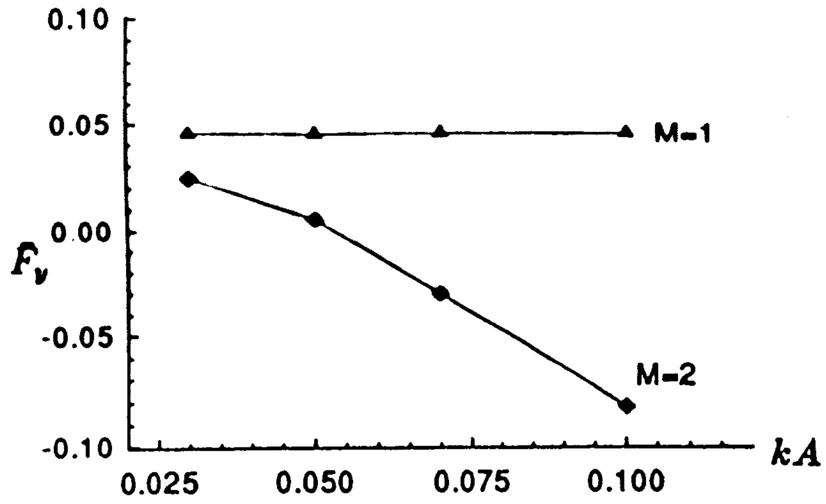

Fig. 3 Horizontal drift force coefficient ($\bar{F}_y/\rho g v (kA)^2 e^{-2kh}$) on a spheroid, $a = 10b, h = 2b$, as a function of incident wave slope kA.

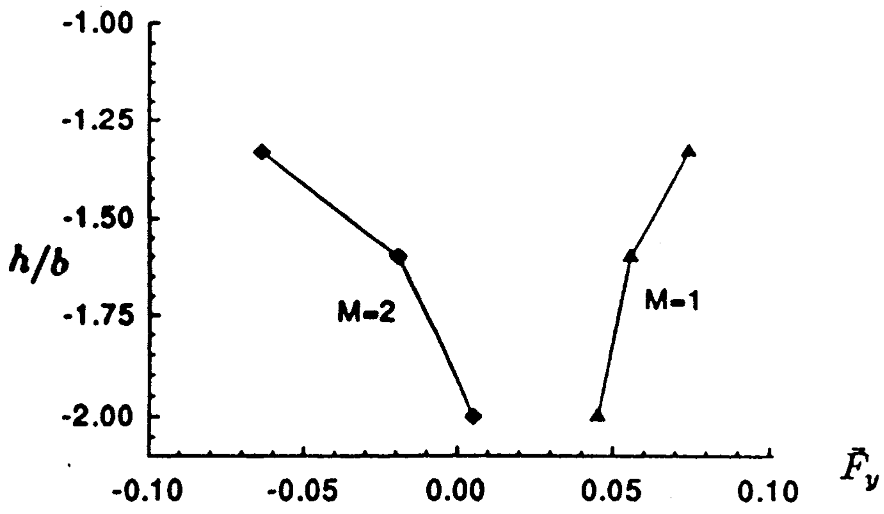

Fig. 4 Horizontal drift force coefficient as a function of submergence h/b ($a = 10b, kA = 0.05$).

DISCUSSION

GRUE: Is it a simple explanation why the coupling between the first order and third order potentials give the main contribution to the mean pitch moment when the wave slope is moderate? Since 4th order effects are comparable with 2nd order effects when κa is moderate, what can then be said about the 6th order effects which are not taken into account in the results presented.

LIU & YUE: For linear theory, the mean pitch moment on a spheroid is given by $M_{3D} = O(\varepsilon^2\alpha^4)$ which is positive, where ε is the surface wave slope and α is the slenderness. Thus, for a slender spheroid, $M_{3D} \to 0$ as $\alpha \to 0$. When nonlinear effects are included, the leading order contribution to the mean pitch moment is of fourth order (i.e., $M_{NL} = O(\varepsilon^4)$) which is found to be negative. For small α, M_{NL} may be comparable to or greater than M_{3D} and the total mean pitch moment is negative. Admittedly, even higher order ($O(\varepsilon^6)$) contributions are present but they are only higher-order corrections to M_{NL}.