THE WAVE FIELD AROUND A VERTICAL CYLINDER IN A CHANNEL

P. McIver
Loughborough University of Technology, U.K.

Introduction

In two recent papers [1,2] the diffraction of waves by one or two vertical cylinders standing in a channel was considered in some detail. In particular, the authors calculated a non-dimensional pressure ratio P, a function of position on the cylinder surface, defined as the modulus of the pressure when the cylinder is in the channel divided by the value when the cylinder is in open water. If the channel walls had no effect on the pressure field then P would be unity for all positions on the body. As is only to be expected, this is not the case. The mean values of P around the cylinder are often significantly displaced from unity with considerable oscillations about the mean also observed. The influence of the channel walls on the pressure field is clearly considerable, "but not in a way that is easily predictable"[2]. It is the purpose of the present work to interpret these numerical results with the aid of an approximate solution derived under the assumption that the waves are much longer then a typical body radius. Curves are presented that allow prediction of the effects observed in [1,2].

A body in open water

Firstly consider the case of a body of arbitrary cross section, but uniform throughout the depth, standing in open water. Let S be the cross-sectional area and a a typical body dimension. In the scattering problem, the depth dependence may be factored out and the motion is governed by the two-dimensional Helmholtz equation. Choose Cartesian coordinates, x and y, in a horizontal plane. For later convenience a reference point within the body is taken to be at $(x,y) = (0,d)$ and standard polar coordinates, r and θ, are defined with origin O at this reference point. Thus, the body is offset from the origin of the Cartesian system. A wave with potential e^{ikx} is incident on the body. Under the assumption that $ka = \epsilon \ll 1$, it may be shown that at large distances from the body, and to leading order in ϵ, the scattered wave field is

$$\phi_S = -\epsilon^2 \left(\frac{iS}{4a^2} H_0(kr) + \frac{\pi\mu}{2} H_1(kr)\cos\theta + \frac{\pi\lambda}{2} H_1(kr)\sin\theta \right). \tag{1}$$

Here H_n denotes a Hankel function of the first kind and μ and λ are 'dipole' coefficients corresponding to a uniform flow past the body in the x direction. Thus, if $\chi(r,\theta)$ is the response of the body to a uniform flow in the x direction then

$$\chi \sim a\mu \frac{\cos\theta}{r} + a\lambda \frac{\sin\theta}{r} \quad \text{as} \quad \frac{r}{a} \to \infty. \tag{2}$$

For a single circular cylinder of radius a, $S = \pi a^2$ and $\chi \equiv a\cos\theta/r$.

A body in a channel

Channel walls are now introduced at $y = \pm b$. The body is at $(x, y) = (0, d)$ and so may be offset from the channel centre-line. No further assumptions are made, so that for example if $kb = O(1)$ then $a \ll b$. The effects of the channel walls may be interpreted in terms of images (although the solution to be described is more easily derived in another way). Let (r_j, θ_j) be polar coordinates with origin O_j at image j, $j = 0$ corresponds to the body itself (in which case the subscript will be omitted). The result of introducing the channel walls on the scattered potential (1) may be found from summing over the image system. It may be shown that near the body the image Hankel functions may be expressed in the form

$$\sum_{j\neq 0} H_0(kr_j) = \sum_{m=0}^{\infty} (\alpha_{0,2m} \cos 2m\theta J_{2m}(kr) + \beta_{0,2m+1} \sin(2m+1)\theta J_{2m+1}(kr))$$

$$\sum_{j\neq 0} H_1(kr_j)\cos\theta_j = \sum_{m=0}^{\infty} (\alpha_{1,2m+1} \cos(2m+1)\theta J_{2m+1}(kr) + \beta_{1,2m} \sin 2m\theta J_{2m}(kr))$$

$$\sum_{j\neq 0} (-1)^j H_1(kr_j)\sin\theta_j = \sum_{m=0}^{\infty} (a_{1,2m} \cos 2m\theta J_{2m}(kr) + b_{1,2m+1} \sin(2m+1)\theta J_{2m+1}(kr)) \tag{3}$$

where J_m is a Bessel function and the complex expansion coefficients α_{nm}, β_{nm}, a_{nm} and b_{nm} have integral representations which are straightforward to evaluate numerically. These coefficients depend on the two non-dimensional parameters kb and kd. If the body offset $d = 0$, β_{nm} and a_{nm} are identically zero.

From a calculation of the near-field potential, it may be shown that the non-dimensional pressure ratio, defined in the introduction, is

$$P = 1 + \epsilon^2\left\{\frac{S}{4a^2}\mathrm{Im}\,\alpha_{00} - \frac{\pi\lambda}{2}\mathrm{Re}\,a_{10}\right\} + \epsilon^3\left\{C - \left(\frac{\pi\mu}{4}\mathrm{Re}\,\alpha_{11} + \frac{S}{4a^2}\mathrm{Re}\,\alpha_{00}\right)\left(\frac{x}{a} + \chi\right) - \left(\frac{\pi\lambda}{4}\mathrm{Re}\,b_{11} - \frac{S}{8a^2}\mathrm{Im}\,\beta_{01}\right)\left(\frac{y-d}{a} + \tau\right)\right\}. \tag{4}$$

Here C is a constant that is identically zero for geometries involving circular cylinders and τ is the response to a uniform flow in the y direction and is defined in a similar way to χ above. Referring back to equations (3) the expansion coefficients appearing in (4) may be interpreted as follows. The coefficient α_{00} is the sum of the image H_0 functions evaluated at O, the origin within the body, and the order ϵ^2 term gives the additional mean pressure field due to the image system. The three coefficients β_{01}, α_{11} and b_{11} can be interpreted as the strength of the uniform flow generated at the body by each of the image sets in (3). The term at order ϵ^3 involving α_{00} arises from an interaction between the images and the open-sea solution and has no simple interpretation. These last four terms give the variation about the mean of P as a function of position on the body.

Results

The real and imaginary parts of α_{00} and α_{11} as a function of kb are plotted in figures 1 to 4 for $d = 0$, corresponding to a centrally placed body. The oscillations about zero of these quantities as a function of kb explain the behaviour observed in [1,2]. The mean pressure variation at the body due to the channel walls is given by the order ϵ^2 term in (4) and the apparently unpredictable variations in the mean found in figure 4 of [1] and figures 3 and 5 of [2] are explained qualitatively by reference to figure 2. The present theory tends to underestimate the magnitude of the difference from unity of the mean value of P while correctly predicting the sign.

For two identical bodies the cross-sectional area is doubled and the dipole coefficients approximately doubled. Hence, from (4), the mean deviation from the open sea value and the amplitude of the fluctuations about the mean will also be doubled and this is consistent with figure 2 of [2].

One of the points made in [2] is the slow decay of channel effects with increasing width and this is born out by figure 5 where $\operatorname{Im}\alpha_{00}$ is plotted for a larger range of kb. For comparison a curve of $(kb)^{-1/2}$ is also included; it may be demonstrated that this is the dominant behaviour of the expansion coefficients in (3) as $kb \to \infty$.

The effects of moving the body off the centre line of the channel show little pattern. The imaginary part of α_{00} is plotted as a function of offset d/b for three values of kb in figure 6. The results for $kb = 3$ may be used to compare with figure 6 of [1] and again qualitative agreement is found.

For $d = 0$ and $kb < \pi$, $\operatorname{Re}\alpha_{00} = 1/kb - 1$ and $\operatorname{Re}\alpha_{11} = 2/kb - 1$ so that for a centrally placed circular cylinder (4) reduces to

$$P = 1 + \epsilon^2 \frac{\pi}{4} \operatorname{Im}\alpha_{00} + \epsilon^3 \frac{\pi}{4}\left(2 - \frac{3}{kb}\right)\cos\theta \tag{5}$$

(no equivalent reduction appears to be possible for $kb > \pi$). The deviation of P from its mean value is given by the order ϵ^3 term in (5) and this is zero for $kb = 1.5$. For $kb < 1.5$ P will increase as θ increases from zero and for $kb > 1.5$ P will first decrease with increasing θ. This is entirely consistent with figure 4 of [1]. Observing that $\operatorname{Im}\alpha_{00}$ has a zero near $kb = 2$ it might be summised that, for a single cylinder, laboratory measurements of the pressure will coincide most nearly with the open sea values for kb somewhere in the range 1.5 to 2.

Acknowledgment

I am grateful to Dr. Gareth Thomas for comments on an earlier draft of this abstract.

References

1. G.P. Thomas "The diffraction of water waves by a circular cylinder in a channel" *Ocean Engng.*, **18**, 17-44, 1991.
2. B.P. Butler and G.P. Thomas "The diffraction of water waves by an array of circular cylinders in a channel" *Ocean Engng.*, to appear.

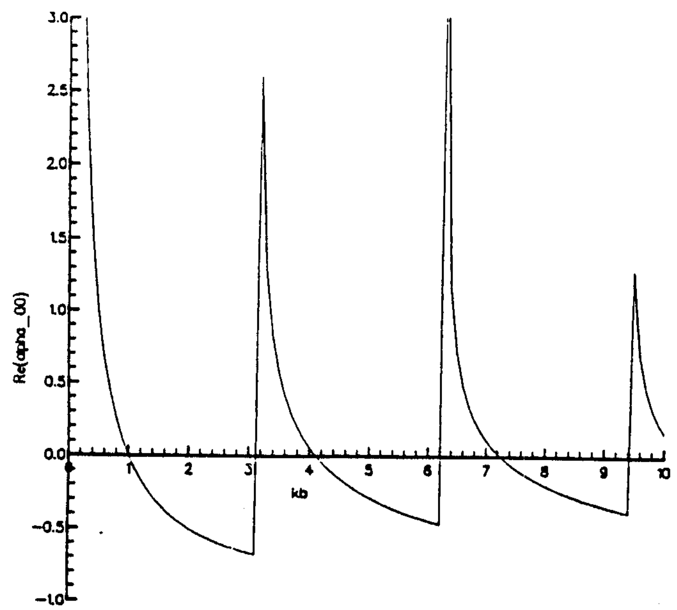

Figure 1

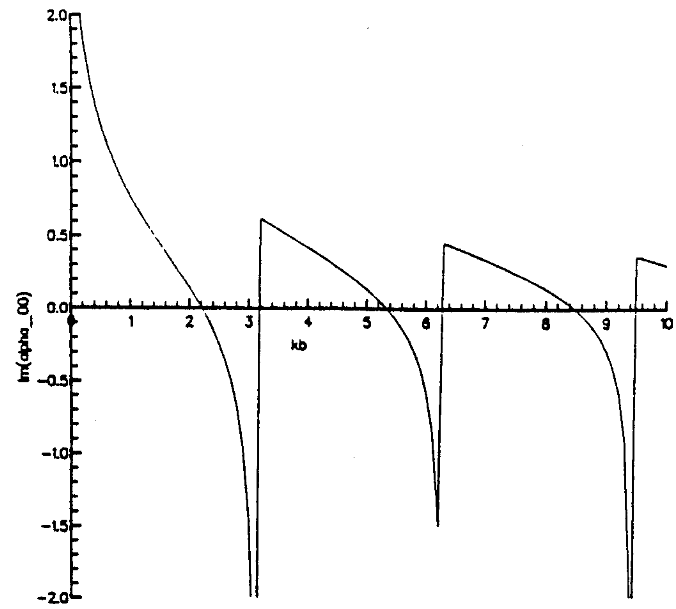

Figure 2

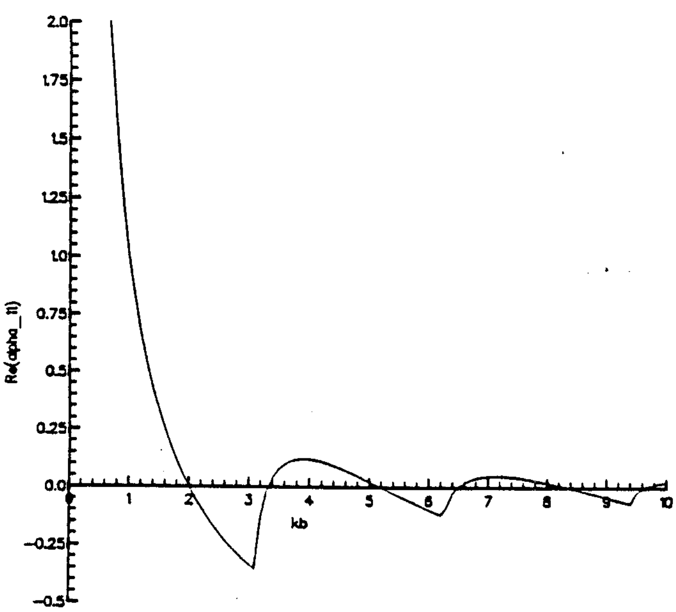

Figure 3

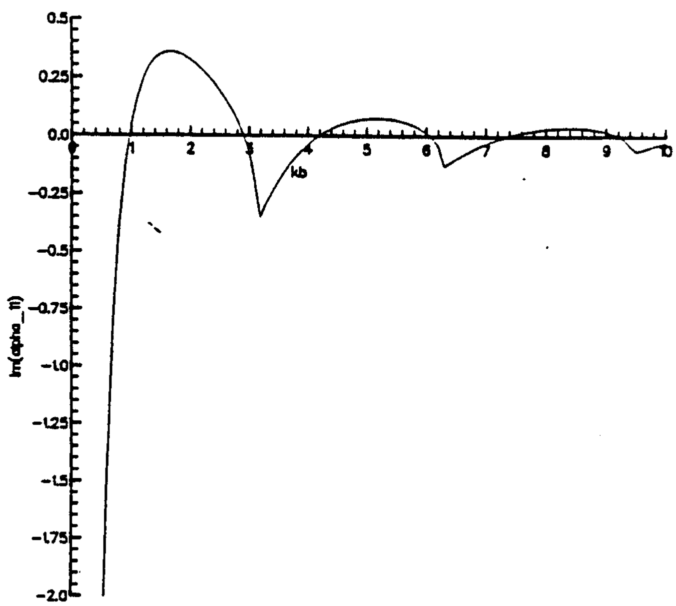

Figure 4

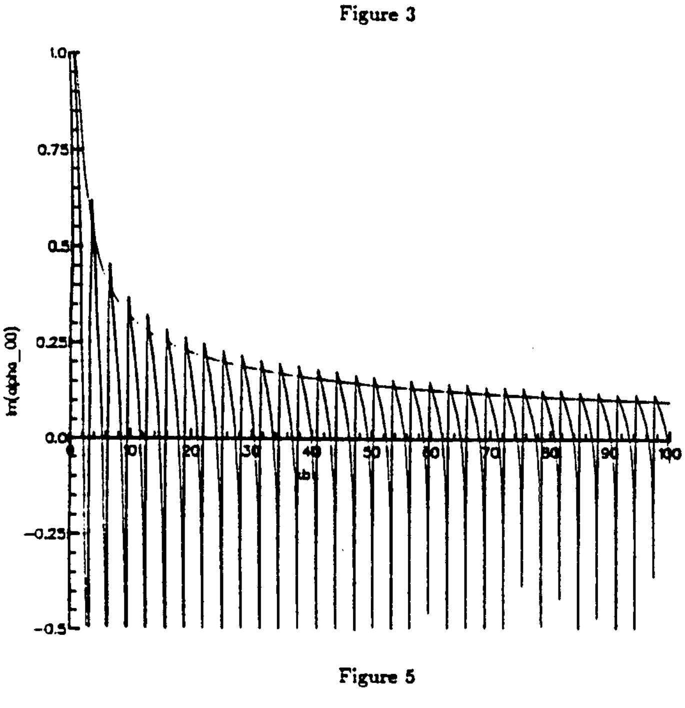

Figure 5

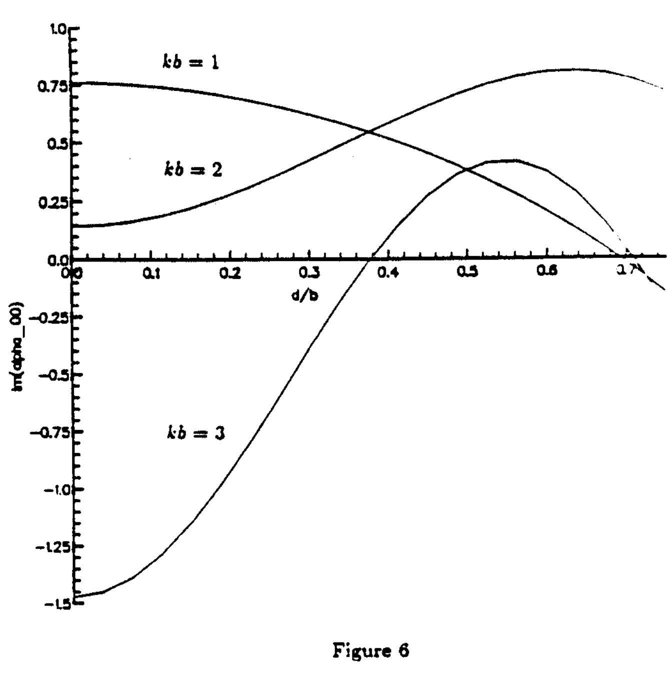

Figure 6

DISCUSSION

CLARK: It would appear that your analysis predicts that the overall force on a cylinder in a channel approaches the open sea value quite quickly as the channel width increases, but the same is not true for the pressure. What advice can one give to those wishing to make pressure measurements on a model in a wave tank.

P. McIVER: Based on my results, there doesn't seem to be a realistic strategy to reduce errors in pressure measurements over a wide range of frequencies apart from the obvious one of avoiding the cut-off resonances. As indicated at the end of the abstract, there is a small range of frequencies below the first cut off for which pressures will be close to their open sea values. There are similar point between all successive cut-off frequencies but, beyond the first interval, they probably occur too close to the cut-off resonances to be of practical value. Correction factors could be derived from the present work but I'm not sure how effective they would be for a complicated geometry.