

Axisymmetric Nonlinear Unsteady Free-Surface Flow Due to a Submerged Sink; The Drainage Problem

T. Miloh¹ & P.A. Tyvand²

¹ Tel-Aviv University, Tel-Aviv 69978, ISRAEL

² Agricultural University of Norway, P.O.Box 65 1432 As-NLH

Abstract

The paper presents an analytic treatment of the initial temporal evolution of a free-surface (or interface) flow due to a submerged 3-D point sink (or source). The fluid is considered to be inviscid and incompressible, the flow is assumed irrotational and surface-tension effects on the interface are considered to leading order. The problem of a moving (or stationary) point singularity, submerged at a depth $h(t)$ below the undisturbed free surface in an otherwise unbounded expanse of fluid, is treated first. The singularity strength is assumed to be suddenly (impulsively) turned on at $t=0$ and a nonlinear initial boundary value problem, alá Peregrine, is formulated and solved to third-order using a Taylor series in time (see e.g. Miloh 1991, Tyvand 1991). In the preceding consistent analysis, gravity will enter into the problem only in the second-order term of the velocity potential and in the third-order free-surface elevation on the axis of symmetry. At this particular value of the Froude number nonlinear effects are in equilibrium (balance) with gravity at the origin (i.e. the intersection of the free-surface with the line of symmetry). The Froude number can be defined in terms of the total input of the sink and the instantaneous submergence depth of the sink h , as,

$$F = \frac{Q}{4\pi(gh^5)^{1/2}} \quad (1)$$

which, may be also compared against the 2-D corresponding value of $Q/(2\pi\sqrt{gh^3})$. The critical Froude number for a point sink (ignoring surface-tension) found in this paper is

$$F_c = 1/(15)^{1/2} \quad (2)$$

which again, may be compared against the analogous 2-D value of $F_c = 1/3$. For $F > F_c$ a local surface dip is formed first and then the free-surface is abruptly collapsing towards the sink. Far from the sink gravity is strong enough to overcome the nonlinear effects resulting in a "pull up" of the free-surface. This local phenomena of instantaneous dip formation, is often observed in draining tanks. In the second part of this paper the analysis of a free-surface collapse is extended for the case of a sink (drain) located at the bottom of a cylindrical container. In this case the free-surface elevation above the drain decreases with time. Following the numerical simulations of Zhou & Grabel (1990) and the experimental work of Lubin & Springer (1967), it has been demonstrated that for a small outward flux the free-surface remains almost horizontal, aside from small damped sloshing oscillations. When the mean free-surface elevation reaches a certain critical depth, a downward dip is formed rather quickly which leads to an immediate collapse of the free-surface unto the drain. Interpreting the numerical solution of Zhou & Grabel (1990) for the free-surface evolution in terms of the Froude number (1), shows excellent agreement with the theoretical prediction of the critical Froude number (2), thus suggesting a useful criterion for dip formation. Good agreement is also found by comparing our theoretical formula with the experimental results reported in Lubin & Springer (1967). A modification of the analysis to the case of time dependent (oscillating) source strength, two-layer model and surface-tension effects on the free-surface (interface), is also presented.

References

1. Lubin B.T & Springer G.S "The Formation of a Dip on the Surface of a Liquid Draining From a Tank" *J. Fluid Mech.* Vol. 29 pp. 385-390 (1967).
2. Miloh T. "On the Oblique Water-Entry Problem of a Rigid Sphere" *J. Eng. Math.* Vol. 25 pp. 77-92 (1991).
3. Tyvand P.A "Motion of a Vortex Near a Free-Surface" *J. Fluid Mech.* Vol. 225 pp. 673-686 (1991).
4. Zhou Q.N & Grabel W.P "Axisymmetric Draining of a Cylindrical Tank with a Free-Surface" *J. Fluid Mech.* Vol. 221 pp. 511-532 (1990).

DISCUSSION

PALM: In the practical applications of your problem, I believe you often will have a (stochastic) velocity circulation present, which may lead to relative large velocity as the curves contract. Does such a circulation influence essentially on your result?

MILOH & TYVAND: First let me say that in our "crude" experiment we did not observe any severe rotational motion. Such a motion is clearly not presented in the numerical simulations of dip formation, which assume the motion to be irrotational. Our present theoretical model can not of course accommodate for the presence of velocity circulation or residual vorticity in the fluid, which according to Marris is known to grow exponentially with time. What we are going to do next is to modify the analysis to include the effect of a thin vortex tube of uniform strength which is present at $t = 0$ and which extends vertically from the sink to the free-surface. I believe that the critical value for the Froude number will depend now on the initial vortex strength (which is assumed to be given).

SCHULTZ: If I recall, Zhou & Graebel sometimes showed the formation of a jet over a (distributed?) sink. Could your small time expansion predict this?

MILOH & TYVAND: It is true that Zhou & Graebel obtained sometimes a jet in their numerical simulation just above the sink (axis at symmetry). The value which we found for the critical Froude number corresponds to an instability condition of the free-surface, be it a dip or a jet. A dip formation at the free-surface is more likely to be observed in nature than a jetting phenomenon.