The RAPID solution of steady nonlinear free surface problems

Hoyte C. Raven
Maritime Research Institute Netherlands (MARIN)
P.O.Box 28, 6700 AA Wageningen, Netherlands

In the calculation of the steady potential flow around a surface-piercing body, the free-surface boundary conditions (FSC's) are usually linearized. In previous studies reported at the 4th and 5th Workshop and in [3], I have examined the merits of different forms of linearization (Neumann-Kelvin, Dawson and Eggers) by comparing the resistance predictions, estimating the terms neglected in the linearization and evaluating the residual normal velocity and pressure on the calculated free surface. Important conclusions were that substantial errors are made in the linearization, even if the results seem to be accurate; that these errors are often dominated by the terms representing the transfer of the boundary condition from the actual to the undisturbed free surface; and that the Taylor expansions used for this transfer are converging only slowly. For full hull forms at low speed a negative wave resistance is often predicted, which can be explained by an energy flux through the calculated free surface caused by the linearization. Another deficiency of linearized methods is the fact that the hull form above the undisturbed waterline is not correctly taken into account, which introduces errors for hulls with strongly sloping sections, partly or slightly submerged bulbous bows and so on. Therefore, the linearization is to be abandoned if a greater accuracy and reliability of the predictions is desired.

A few nonlinear methods for the wave-resistance problem have been proposed already. Ni [4] set up a method using higher-order source panels on the free surface, the location of which was iteratively updated. Initially the method was overly sensitive to numerical details, but a later paper suggest an acceptable stability. In a very similar method Kim and Lucas [6] needed artificial damping on the free surface to be able to get a converged result. Jensen et al [5] came up with a method containing many original details, based on a distribution of point sources above the free surface. Also in this method, convergence of the iteration appeared to be the weak point.

Recently I have developed a new nonlinear method, for which, in view of the lessons learnt from the study on linear methods, a discretization by singularities located at a distance above the free surface was selected. Such singularities outside the fluid domain have a number of advantages; in the first place the fact that the source locations need not be adapted in each iteration; thus many geometric evaluations that otherwise must be done in each iteration, can now be skipped, and a part of the influence coefficients remains unaltered. Other benefits of the method are a greater smoothness of the induced velocity field, and desingularization of the expressions for the induced velocities. On the other hand, possible disadvantages are the poorer numerical conditioning, the arbitrariness in the choice of the elevation, and the fact that the analytic continuation of the potential up to the level of the singularities not always exists. On several of these aspects some information is available from previous papers, but hardly for the particular type of boundary condition imposed here.

Numerical Dispersion of Raised-Source Methods

In an initial study, a prototype code has been written for solving the *linear* wave resistance problem using singularities raised above the free surface. Since from the first results the numerical dispersion of this method appeared to differ from that of the standard DAWSON-approach, this has been subsequently analysed.

The analysis is similar to that proposed by Sclavounos and Nakos [2], and is restricted to a 2D case for simplicity. First we consider a *continuous* source distribution with infinite extension, at a distance y_f, above the undisturbed free surface. The velocity components u, v on the undisturbed free surface $y = 0$ are expressed as boundary integrals, and substituted in the Kelvin condition $u_x + k_0 v = R.H.S.$. Here the right hand side incorporates the effect of sources not on the free surface (e.g. the body). The result is a boundary integral equation for σ, which can be represented by: $\mathcal{W}_1\sigma = R.H.S.$.

The properties of the linear operator $\mathcal{W}_1$ are studied in Fourier space. The Fourier series representation

$$\sigma(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \tilde{\sigma}(k) e^{ikx} dk$$

is substituted in the boundary integral equation. We then find

$$\tilde{\mathcal{W}}_1(k)\tilde{\sigma}(k) = R.\tilde{H}.S., \qquad \tilde{\mathcal{W}}_1(k) = -\frac{1}{2}e^{-ky_{fs}}(1 - k/k_0).$$

The zero of the Fourier transform of the operator, $\tilde{\mathcal{W}}_1$, determines the wavelike behaviour of the solution; this has the wavenumber $k = k_0$ as expected.

We now consider the *discretized* problem with constant-strength source panels of uniform size. Infinite sums over the free surface panels now replace the integrations over the continuous source distribution, and discrete Fourier transforms of the source strength and operator are used. In analogy to Dawson's method the term u_x in the Kelvin condition is implemented by a difference scheme for the first derivative of the induced velocity. Therefore, also the discrete Fourier transform $ik\hat{\mathcal{R}}_1$ of the difference operator enters. The resulting transform of the complete operator is:

$$\hat{\mathcal{W}}_1(k) = -\frac{1}{2\pi}[\Sigma_2(k\Delta x, \alpha) - \frac{k\hat{\mathcal{R}}_1(k)}{k_0}\Sigma_1(k\Delta x, \alpha)].$$

Here, Δx is the panel length, and $\alpha = y_{fs}/\Delta x$. The root of the operator is:

$$k/k_0 = \frac{\Sigma_2}{\hat{\mathcal{R}}_1\Sigma_1}.$$

We shall now first leave out of account the errors introduced by the difference scheme, substituting $\hat{\mathcal{R}}_1 = 1$. Since both Σ_1 and Σ_2 are real, the discretization of the source distribution in constant-strength panels therefore introduces numerical dispersion but no numerical damping. The infinite sums Σ_1 and Σ_2 must be evaluated numerically. The *order* of the dispersion error cannot simply be deduced, but for a number of panel elevations and source panel sizes its value has been evaluated. Table I shows that the raised-panel method is surprisingly accurate for panel elevations exceeding half the panel size.

The same sort of analysis was carried out for the Dawson method with source panels on the undisturbed FS, both with and without the dispersion correction described in [1]. As shown in Table I, without dispersion correction considerable errors occur; the correction eliminates these, but only for the transverse wave components. E.g. for components at an angle of 60°, the wave number error is 29 % with dispersion correction, and 38 % without, if 20 panels per transverse wave length are used. The raised-panel method, which does not have such a dependence on the wave direction, is found to be far superior.

	$\alpha = 0.5$	$\alpha = 1.0$	$\alpha = 1.5$	$\alpha = 2.0$	Standard	Corrected
$k_0\Delta x = 0.1\pi$	0.9937	0.9997	1.0000	1.0000	1.0744	1.0003
$k_0\Delta x = 0.2\pi$	0.9819	0.9986	0.9999	1.0000	1.1608	1.0026
$k_0\Delta x = 0.3\pi$	0.9611	0.9957	0.9996	1.0000	1.2623	1.0090

Table 1: *Wavenumber ratio k/k_0 for raised-panel and standard method*

Also a raised point-source method has been analysed. In a 2D case this reaches the same level of accuracy as the raised-panel method, although for a somewhat larger elevation. It could be shown that the numerical dispersion due to the source discretization was still of $\mathcal{O}(\Delta x)$, but with a very small coefficient. It was realized however, that this result is not necessarily representative of the performance of a 3D point-source method in 2D applications: the fact that also in transverse direction the distribution is discretized in point sources introduces additional errors in the induced velocities that do not occur for the raised-panel method. Evaluating the resulting double summations in the induced-velocity expressions we find that the 3D raised point-source method is substantially less accurate unless the aspect ratio of the 2D source array is kept close to unity. E.g. for $\Delta z/\Delta x = 2$, the wave length error is 2.5 % ; for $\Delta z/\Delta x = 3$, the error even amounts to 11.6 % . Since keeping the aspect ratio close to unity generally requires a greater number of sources than usual and thus increases the calculation time, this is a significant disadvantage of the point-source approach.

Up to this point we have left out of account the numerical errors introduced by the difference scheme. Since $-\mathcal{R}_1$ has real and imaginary parts, both dispersion and damping result. But the dispersion contribution is only of $\mathcal{O}(\Delta x^4)$ for the particular difference scheme used in our method; the leading-order dispersion thus results from the source discretization and not from the difference scheme. Therefore the errors due to the discretization and due to the differencing may be dealt with separately, as has been done in this study.

These conclusions have all been confirmed by numerical experiments for 3D surface-piercing hulls. In particular, with the raised source panel method the wave profile along the hull agrees accurately with the DAWSON-result; but at a lateral distance from the path of the ship a significant phase difference is found, caused by the fact that DAWSON contains a numerical dispersion error for diverging components. Fig.1 shows that this forward shift in the DAWSON-prediction (with dispersion correction) decreases upon panel refinement, while for the raised-panel method the phase of the waves is absolutely insensitive to the panel density due to the virtual absence of numerical dispersion. For the raised point-source approach in fact the aspect ratio of the 2-D source array had the predicted effect on the phase of the waves.

Thus the raised-panel method appears to have the smallest dispersion error, and also in other respects seems to be the most promising candidate to go on with.

R A P I D : RAised Panel Iterative Dawson

The next step was, of course, to set up the iterative procedure to solve the nonlinear problem. In each iteration the FSC is linearized with respect to a base flow and the current location of the FS collocation points. This linear problem is then solved, the collocation points are moved up or down to the newly calculated free surface, and the base flow is updated.

The method was exceptionally successful from the outset. No convergence problems were experienced for several test cases. The method is quite robust, and requires no artificial damping or smoothing of any kind. The residual errors in the free surface conditions are usually reduced by a factor of 2 to 4 per iteration; 5 to 15 iterations generally suffice.

Fig. 2 gives an example of the results. It compares the converged solution (after 12 iterations) for the wave profile of a tanker at $F_n = 0.155$, with the result of the first iteration (the Neumann-Kelvin solution). Although full hull forms are notoriously difficult for this sort of calculations, no problems were experienced. Also for flows displaying greater nonlinearities the convergence is usually automatic.

Calculation times are, of course, strongly dependent on the panel number, but usually range from 1 to 30 minutes on a CRAY-YMP machine. For example, for the tanker, with 800 hull panels and 2700 free surface panels, the calculation time was about 2000 sec. ; for a Series 60 model with 550 hull and 550 FS panels, the time required amounted to 90 sec. It is to be noted that these figures apply to a prototype code that is definitely not optimized in this respect.

Further work

Of course, many other studies have been carried out to make the method generally applicable and sufficiently validated. These concerned e.g. the dependence on the panel elevation and the number of panels on the free surface and the hull; the accuracy with which the exact boundary conditions are satisfied on the free surface; the properties of the solution near the waterline, in view of possible singularities there; and the accuracy of the evaluation of the resistance. Most of these studies have been completed by now, and they will partly be reported in [7].

References

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Figure Captions

Fig.1 : Longitudinal wave cut at a distance of $0.40L$, for 15, 30 and 60 panels per transverse wavelength. Wigley hull, $F_n = 0.40$. Kelvin condition imposed.
top: DAWSON, with dispersion correction. bottom: raised-panel method.
Fig.2 : Wave profile along the hull of a tanker; Nonlinear and Neumann-Kelvin result.

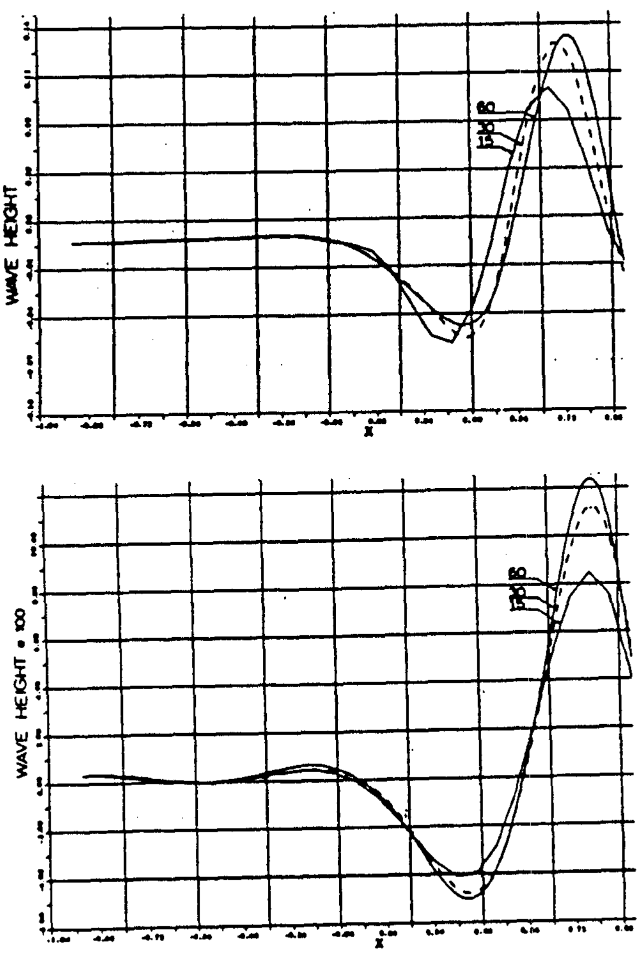

Fig. 1

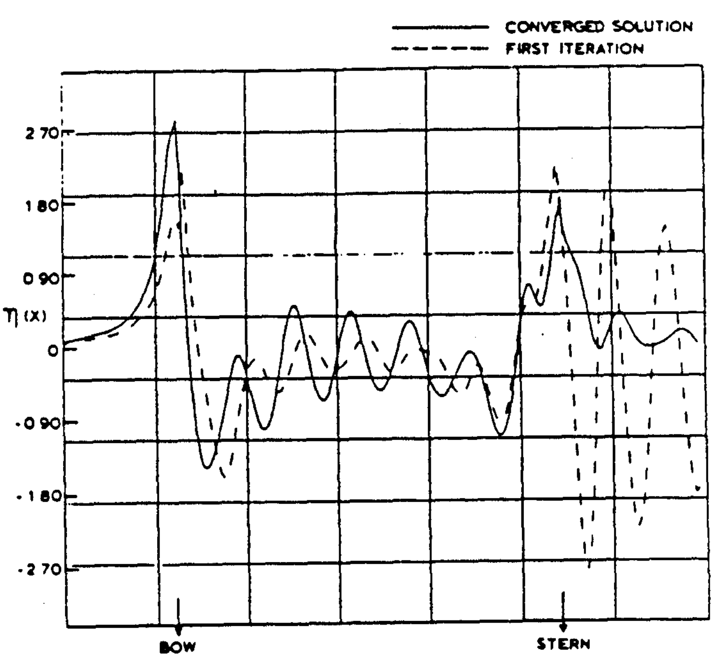

Fig. 2

DISCUSSION

LALLI: I have 2 questions:
1) In your preliminary abstract you mention a particular finite differences scheme characterized by a numerical dispersion of $O(\Delta x^4)$: can you describe such scheme?
2) In your analysis about numerical properties of raised panels method the results seem to show the solution to be independent from the vertical displacement of the panels, in a certain range; did you verify such desirable property also in the nonlinear case, by numerical experiments?
RAVEN:
1. This scheme, which was originally proposed by Piers (National Aerospace Laboratory, the Netherlands), reads as follows:

$$\frac{\partial u}{\partial x} = -\frac{1}{2Fn_h^4}\,\delta_2(u) + \left(1 + \frac{1}{6Fn_h^4}\right)\delta_3(u) + O(h^2)$$

where δ_2 and δ_3 are the standard 2- and 3-point backward difference schemes, and $Fn_h = U_\infty/\sqrt{gh}$, h denoting the panel length. It is easy to show that this scheme introduces a 4th order dispersion and 4th order damping for 2D waves.
2. The dependence of the nonlinear solution on the panel elevation is quite small within certain limits. A lower limit for the elevation is dictated by the increase of the numerical dispersion, and by practical considerations. The upper limit is imposed by the deterioration of the matrix conditioning. In practice a panel elevation of 0.3 Δx to 1.5 Δx will do.

CAMPANA: You mention that no convergence problems were experienced for several test cases. Did you observe any variation of the convergence rate when the Froude number or the number of panels grow?
RAVEN: The number of iterations needed does increase with increasing panel number, particularly with transverse refinement close to the waterline. This is increase is, however, only gradual. A reduced convergence rate for higher Froude number could occur, but I have not yet clearly observed that.

CAO: It is not clear to me how you implement the radiation condition. Do you use a "staggered grid" technique similar to that used by Jensen *et al.* or do you use a "Dawson type" finite difference technique? I wonder whether the finite difference technique is possible for the raised source method.
RAVEN: I do not apply Jensen's technique of shifting collocation points by one panel length. I use the same finite-difference scheme as in Dawson, with the same upstream conditions. It appears that this is a little bit less effective to impose the radiation condition in the raised-source method. Very slight upstream waves sometimes occur; a problem that I have to give some attention in the near future.