On a bound for the frequency of trapped surface waves for a cylinder spanning a channel

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There has been substantial effort recently put into proving, for a variety of different geometries, the existence of trapped waves, that is, unforced time-harmonic motions which do not radiate energy to large distances. Thus it is known that such motions can exist in a deep channel which includes a cylinder spanning the channel, for various shapes of cylinder.

The converse problem of proving the absence of such trapped waves has received much less consideration, and the only relevant uniqueness proof for a channel spanned by a cylinder is that of McIver (1991). In an appendix to that paper, McIver demonstrates that no trapped-wave motions can exist for the case in which the cylinder is surface-piercing and is entirely contained between vertical lines through the free-surface intersections. This is exactly the same geometrical condition which John (1950) found would ensure uniqueness in water-wave radiation and scattering problems, in finite or infinite depth. (It must be noted, however, that John's criterion cannot be used to show that the absence of trapped waves for a cylinder in a finite-depth channel at all frequencies.) Both John and McIver achieved their uniqueness results by considering integrals of the potential along vertical lines down from the free surface.

John's work was extended by Simon and Ursell (1984), who established uniqueness for a wider class of two-dimensional radiation and scattering problems by considering integrals along non-vertical lines. The work to be presented here is the corresponding extension of McIver's work; although this extension does not rule out trapped waves at all frequencies for any geometry except that already considered by McIver, it does yield an easy lower bound for the ratio of the trapped-mode frequency to the 'cut-off' frequency, in finite or infinite depth.

References:

McIver (1991) Q.J. Mech. appl. Math. 44, 193-208

John (1950) Comm. pure appl. Math. 3, 45-101

Simon and Ursell (1984) J. Fluid Mech. 148, 137-154.

DISCUSSION

EVANS: Does your method work for trapped modes which are *anti*symmetric about the vertical plane perpendicular to the channel walls?
SIMON: One does no have to impose any particular symmetry on the geometry of the problem in order to use this method.

MARTIN: Do you need antisymmetry with respect to the midplane, $z = 0$?
SIMON: The method works equally well for modes symmetric with respect to the midplane.

McIVER: Do you think that there are any trapped modes for bodies in finite depth which satisfy the John condition?
SIMON: For bodies that satisfy the John condition it is hard to see what would trap the waves under the body.