The Drift Force on a Rectangular Body Close to the Free Surface

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1. INTRODUCTION

The hydrodynamic pressure force acting on an oscillating submerged rectangular body has a sharp peak at a certain frequency because resonant standing waves will occur in the shallow region on top of the rectangle. Newman [1] developed a linear theory to analyze this phenomena with the fundamental approximation that the submergence of the top face is small compared with the width and wave length.

On the other hand, the same phenomena can be expected in the case of the restrained body. When the amplitude of incident waves is large, the nonlinear effect is expected in the shallow region on the top of rectangle and higher harmonics will be induced. Longuet-Higgins [2] reported that if the energy of the first harmonic completely changes to the energy of the second harmonic, the amplitude of the second harmonic would be larger than that of the first harmonic because of the energy conservation, and the negative drift force would be expected.

A nonlinear theory is developed in the present work to analyze the drift force on a submerged rectangular body by extending Newman's linear theory and the results show that the negative drift force will occur at a certain frequency.

2.THEORY

The geometrical configuration of the restrained rectangular body close to the free surface is shown in Fig.1, with a two dimensional fluid domain of a finite depth. The flow is assumed to be incompressible and irrotational It is also assumed that the problem is harmonic in time. The orders of basic properties are assumed as follows.

$$\frac{h_{IO}}{\lambda_{IO}} = \varepsilon \ll 1, \ \frac{\zeta_{IO}}{\lambda_{IO}} = O(\varepsilon), \ \frac{h_{IO}}{B} = O(\varepsilon) \tag{1}$$

The overall problem can be divided into three parts which are an external outer region, an internal outer region and an inner region as shown in Fig.1.

2.1 External Outer Region

In this region, the velocity potential can be introduced and the governing equation for the first order problem is the two dimensional Laplace's equation with linearized free surface condition. The first order solution of the velocity potential in the right domain has the following form.

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$$\Phi_{EO} = \sum_{n=1}^{\infty} Re[\phi_{EO}^{(n)} e^{in\omega t}] + i\frac{g}{\omega}\frac{\cosh m_0^{(1)}(y+h)}{\cosh m_0^{(1)}h}\left\{a_0 e^{im_0^{(1)}x} + a_R^{(1)} e^{-im_0^{(1)}x}\right\}$$
$$+ \sum_{k=1}^{\infty} C_k^{(1)} e^{-m_k^{(1)}x} \frac{\cos m_k^{(1)}(y+h)}{\cosh m_k^{(1)}h} \quad (2)$$

where ,

$$\phi_{EO}^{(n)} = M_R^{(n)} G^{(n)}(x, y, \frac{B}{2}, 0)$$
$$+ i\frac{g}{\omega}\{M_R^{(n)} a_{SR}^{(n)} + M_L^{(n)} a_{ST}^{(n)}\} e^{-im_0^{(n)}x} \frac{\cosh m_0^{(n)}(y+h)}{\cosh m_0^{(n)}h}$$
$$+ \sum_{k=1}^{\infty}\left\{M_R^{(n)} C_{SR}^{(n,k)} + M_L^{(n)} C_{ST}^{(n,k)}\right\} e^{-m_k^{(n)}x} \frac{\cos m_k^{(n)}(y+h)}{\cosh m_k^{(n)}h}$$
$$G^{(n)}(x, y, x', y') = 2\pi i \frac{1}{m_0} \frac{m_0^{(n)2} - \nu^{(n)2}}{hm_0^{(n)2} - h\nu^{(n)2} + \nu^{(n)}} \cosh m_0^{(n)}(y - y' + h) e^{-im_k^{(n)}(x-x')}$$
$$- 2\pi \sum_{k=1}^{\infty} \frac{1}{m_k^{(n)}} \frac{m_k^{(n)2} + \nu^{(n)2}}{hm_k^{(n)2} + h\nu^{(n)2} - \nu^{(n)}} \cos m_k^{(n)}(y - y' + h) e^{-m_k^{(n)}(x-x')}$$
$$\nu^{(n)} = \frac{\omega^2 n^2}{g} = m_0^{(n)} \tanh m_0^{(n)}, -\frac{\omega^2 n^2}{g} = m_k^{(n)} \tanh m_k^{(n)}$$

Where, a_0 is the amplitude of incident waves and the reflection coefficient a_R and C_k are obtained by solving the linearized diffraction problem of a restrained rectangular body with a draft d. Coefficients a_{SR}, a_{ST}, C_{SR} and C_{ST} are obtained by solving the radiation problem of a periodic source at the inter section between the body surface and the free surface, where the strength of the source is determined by matching it with the inner solution. It is noted that the first order solution is proportional to ε^2 and the second order solution is proportional to ε^4.

2.2 Internal Outer Region

In this region, the first order solution of velocity and the free surface deformation are governed by the nonlinear shallow water wave equations which are represented as follows.

$$\begin{cases} \dfrac{\partial u_{IO}}{\partial t} + u_{IO}\dfrac{\partial u_{IO}}{\partial x} + g\dfrac{\partial \zeta_{IO}}{\partial x} = 0 \\ \dfrac{\partial \zeta_{IO}}{\partial t} + \dfrac{\partial}{\partial x}(u_{IO}\zeta_{IO}) + h_{IO}\dfrac{\partial u_{IO}}{\partial x} = 0 \end{cases} \quad (3)$$

These equations can be solved by the finite element method with end conditions which are given by matching solutions of (3) with the inner solution. It is noted that the first order solution is correct up to the order of ε^2.

2.3 Inner Region

In order to obtain the consistent solution, the second order solution is necessary in this region. Boundary conditions for the first order problem and the second order problem are as follows.

$$\begin{cases} u_{I1}\dfrac{\partial \zeta_{I1}}{\partial x} - v_{I1} = 0 \\ \dfrac{1}{2}(u_{I1}^2 + v_{I1}^2) + g\zeta_{I1} = c_0 = constant \qquad on\ y = \zeta_{I1} \end{cases} \quad (4)$$

$$\begin{cases} \dfrac{\partial \zeta_{I1}}{\partial t} + u_{I1}\dfrac{\partial \zeta_{I2}}{\partial x} + u_{I2}\dfrac{\partial \zeta_{I1}}{\partial x} + \zeta_{I2}\dfrac{\partial u_{I1}}{\partial y}\dfrac{\partial \zeta_{I1}}{\partial x} - v_{I2} - \zeta_{I2}\dfrac{\partial v_{I1}}{\partial y} = 0 \\ \dfrac{\partial \phi_{I1}}{\partial t} + (u_{I1}u_{I2} + v_{I1}v_{I2}) + \zeta_{I2}(u_{I1}\dfrac{\partial u_{I1}}{\partial y} + v_{I1}\dfrac{\partial v_{I1}}{\partial y}) + g\zeta_{I2} = 0 \qquad on\ y = \zeta_{I1} \end{cases} \tag{5}$$

Where subscripts 1 and 2 mean the first order and the second order respectively. Introducing the complex velocity potential, the first order solution is obtained as follows.

$$w(s) = M_I \ln s + C_0 \tag{6}$$

Where the fluid domain is transformed to the upper half of s-plane by the Schwartz-Christoffel transformation.

$$\frac{dz}{ds} = -ic_1\frac{1}{s}(s-1)^{\frac{1}{2}} \exp\left[-\int_{-\infty}^{0} \frac{f(\tau)}{\tau - s} d\tau\right] \tag{7}$$

$$where, \quad z = x - \frac{B}{2} + iy$$

The complete second order solution can not be obtained. However, asymptotic values at infinity can be obtained from the first order solution, so it is possible to match the inner solution with outer solutions up to order of ε^2. The asymptotic values of the second order solution are obtained as follows.

$$\begin{cases} \zeta_{I2} \simeq -\dfrac{1}{g}\dfrac{\partial}{\partial t}\left[2M_I(\ln r - \ln\dfrac{2}{\pi}h_{IO}) + C_0\right] \quad z \to +\infty \\ \zeta_{I2} \simeq -\dfrac{1}{g}\dfrac{\partial}{\partial t}\left[\dfrac{M_I\pi}{h_{IO} + \zeta_{I1}}(x - \dfrac{B}{2}) - 2M_I(1 - \ln 2 - c_2) + C_0\right] \quad z \to -\infty \end{cases} \tag{8}$$

$$where, \quad r^2 = (x - \frac{B}{2})^2 + y^2$$

Where coefficients c_1 and c_2 are calculated from the function f(s) which is determined so as to satisfy the first order free surface condition, however, since it is difficult to obtain the function f(s) numerically, it is assumed that f(s)=0 when c_1 and c_2 are calculated in this work.

2.4 Matching and Numerical Calculation

According to the ordinary procedure of the matched asymptotic expansion method, outer solutions are matched with the inner solution which process gives a relationship between the external outer solution and the internal outer solution.

The internal outer solution is obtained by the finite element method as stated previously. This method and the matching process give a system of nonlinear equations which can be solved by Newton-Raphson method with particular initial values.

3. RESULTS AND CONCLUSIONS

A nonlinear theory is developed to analyze the drift force on a submerged rectangular body by extending Newman's linear theory. The nonlinear theory is consistent up to the order of ε^2 and this means that the drift force induced by higher harmonics can be estimated.

Several numerical results are shown in Fig.2 and these results show that the negative drift force will occur at a certain frequency.

References

[1] Newman,J.N., Sortland,B. and Vinje,T. :Added Mass and Damping of Rectangular Bodies Close to the Free Surface. *Journal of Ship Research* , **28**, 1984, 219-225.

[2] Longuet-Higgins,M.S. :The mean forces exerted by Waves on Floating or Submerged Bodies with Applications to Sand Bars and Wave Power Machines. *Proc. R. Soc. Lond.*, A.**352**, 1977, 463-480

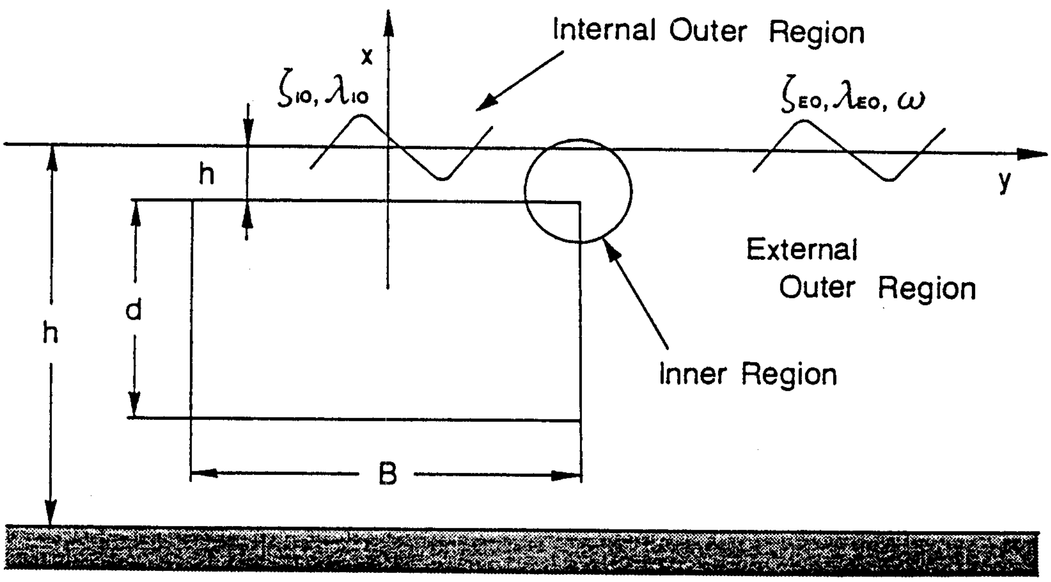

Fig.1 The geometrical configuration of the restrained rectangular body close to the free surface.

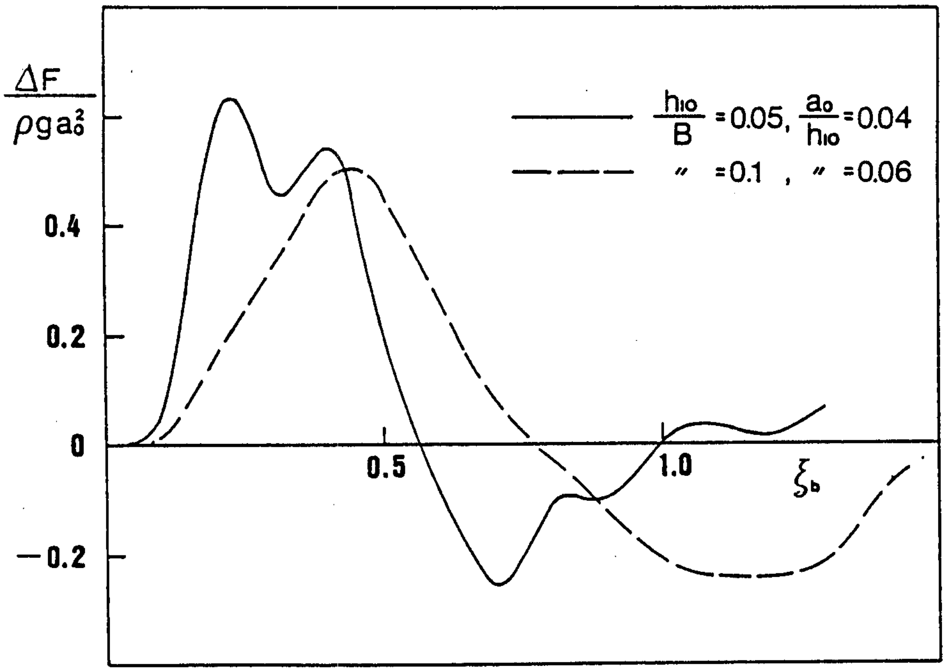

Fig.2 Numerical results of the drift force on a rectangular body.

DISCUSSION

LIU: The linear potential theory predicts the 2nd order (leading order) horizontal drift force which is positive and due to the quadratic interaction of the first order first harmonic waves. Your experimental observation for the presence of a negative drift force indicates the significance of nonlinear effect. Classical perturbation method shows the nonlinear effect to the mean drift force can be at most fourth order in wave slope. Thus, to predict the nonlinear effect using the far-field formulation, both energy and momentum should conserve to fourth order in surface wave slope. I believe the far-field formula for the mean horizontal drift force, which is accurate to the fourth-order, is messy. Therefore, in this case, it is much simpler and clearer to use direct pressure integration approach.

TAKAGI: Thank you very much for your discussion. Wave slopes of higher harmonic waves (2ω, 3ω, 4ω, ...) are as same order as wave slope of incident waves in this theory because of strong nonlinear effect of shallow water wave equation which governs the internal outer region. So, the negative drift force is obtained as a second order force (According to my definition this force is order of ε^4.), and it is easily shown that the energy and momentum are conserved up to this order.

The near field pressure integral has not been done in this paper, however I think it is easy to demonstrate that the direct pressure integral leads the same result up to order of ε^4.

GRUE: Recently I did some very similar research, both with respect to theory and measurements. For small and moderate wave slopes I found good agreement between theory and experiment. I cannot see any reason why this should also be the case for your parameters. However, a good start point is to study wave diffraction for small and moderate wave slopes.

TAKAGI: Thank you very much for your good comment. It is true that wave slopes of my experiment is too large for my theory because of the small body size in experiment. I hope that your experimental results verifies my theory.

However breaking and viscid effect on this problem should be studied as a future work.