A Multidomain Approach to Free Surface Viscous Flows

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INTRODUCTION

One of the classical problems in naval hydrodynamics is the computation of steady free surface viscous flows. The extremely large Reynolds numbers in the industrial applications ($\sim 10^8 \div 10^9$), and the presence of the moving boundary make such flow very difficult to simulate.

The aim of multidomain domain approach is to save both computer storage and CPU time: the full viscous model is solved only in the neighbourhood of rigid boundaries and inside the wake, while the external flow, where viscous effects are supposed to be negligible, is simulated by a linear potential model (Dawson, 1977). The linear scheme has been chosen because of its robustness and low CPU time requirements.

When using zonal approach, problems related to the matching conditions for external and internal solutions arise. Although the coupling procedure in unbounded flows is quite well established (see, e.g. Lock and Williams, 1987), the external free surface flow gives rise to new difficulties.

In the present paper, the viscous and inviscid solvers are briefly described, as well as the problem related to the interaction between the two parts in which the fluid domain is split. Finally some numerical examples are discussed and compared with experimental data (Salvesen, 1966).

SOLUTION PROCEDURE

The wave pattern generated by the motion of submerged or floating bodies is computed by a potential solver. The irrotational flow domain D is bounded by the free boundary S and by the surface Γ (closed or not) on which Neumann boundary conditions are enforced (see Fig.1-2):

$$\left.\frac{\partial \phi}{\partial n}\right|_{\Gamma} = v_n \tag{1}$$

with v_n assigned (e.g. $v_n = 0$ at rigid boundaries at rest). Following Dawson (1977), the velocity potential $\phi(x, y, z)$ is split into the double model potential $\varphi(x, y, z)$ and the free surface perturbation $\tilde{\varphi}(x, y, z)$. All the second order terms in $\tilde{\varphi}$ are neglected in the boundary condition at the free surface:

$$\varphi_l^2 \tilde{\varphi}_{ll} + 2\varphi_l \varphi_{ll} \tilde{\varphi}_l + \frac{\tilde{\varphi}_z}{Fr^2} = -\varphi_l^2 \varphi_{ll} \qquad \text{on } z = 0 \tag{2}$$

where l is a parameter defined along the streamlines of the basis flow lying on $z = 0$. Finally a condition at infinity must be imposed:

$$\lim_{x \to -\infty} |\nabla \phi| = 1 \tag{3}$$

The solution is expressed in terms of the simple layer potential; the boundaries Γ and S have been discretized with flat elements while the density is approximated with a piecewise constant function.

The solution of the Reynolds Averaged Navier–Stokes equations is required where viscous effects are dominant, namely in the region close to the body and in the wake.

For steady problems, the governing equations of the viscous flow are

$$\frac{\partial}{\partial x_j}(u_j u_i) + \frac{\partial p}{\partial x_i} = \frac{\partial}{\partial x_j}(\tau_{ij}) \tag{4}$$

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{5}$$

where the index notation has been adopted, $p = P/\rho$ and τ_{ij} is the viscous stress

$$\tau_{ij} = (\nu + \nu_T)\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{6}$$

ν_T is the turbulent viscosity, defined by the Baldwin and Lomax (1978) algebraic model.

Equations (4-5) are discretized using a finite volume scheme and solved using a marching procedure based on a pseudo-compressibility implicit scheme (Kwak et al.,1986). In this scheme, pressure and Cartesian components of velocity are located at the center of the cell, the fluxes at cell interfaces are computed by simple averaging and centered differencing and a numerical high-order dissipation is introduced to stabilize the calculation. Local time step is used to speed up the convergence rate. The coefficient matrix is split into block tridiagonal matrices by means of an approximate factorization technique, for which an efficient solution algorithm exists.

The decomposition of the flow domain requires special care in the choice of the coupling mechanism. One of the crucial points is the position of the surface where the matching must be imposed. In the present work the two domains overlap (fig.2). The solution of the potential flow is then used to enforce the boundary conditions on the viscous flow and *vice versa*, as described in the following.

The coupling procedure consists of two separate stages. In the first one, the basis flow past Γ and its image is computed by means of an iterative procedure:

(a) we solve the external flow with the boundary condition on Γ:

$$\varphi_n = v_n = \vec{u}^v \cdot \vec{n} \tag{7}$$

where $\vec{u}^v \cdot \vec{n}$ is the normal component of the viscous velocity at Γ (see fig.2), computed in the previous iteration.

(b) The potential velocity and pressure at Γ' (fig.2) are then used as boundary conditions for the viscous problem, which is iterated for a fixed number of steps.

Steps (a) and (b) are repeated until some convergence criterium is satisfied.

Once the solution of the basis flow is obtained, the influence of the free surface is taken into account. In this stage the boundary conditions for the external flow are:

$$\tilde{\varphi}_n = \vec{u}^v \cdot \vec{n} - \vec{u}^0 \cdot \vec{n} \qquad \text{on } \Gamma \tag{8}$$

$$\varphi_l^2 \tilde{\varphi}_{ll} + 2\varphi_l \varphi_{ll} \tilde{\varphi}_l + \frac{\tilde{\varphi}_z}{Fr^2} = -\varphi_l^2 \varphi_{ll} \qquad \text{on } z = 0 \tag{9}$$

where $\vec{u}^v \cdot \vec{n}$ is known from the previous viscous computation and $\vec{u}^0$ is the converged double model velocity. Next the free surface is computed from the linearized dynamic boundary condition.

The iterative procedure is identical to that used in the double model solution.

This revised algorithm brought a significant improvement with respect to (Campana et al., 1992), where a non overlapping domain decomposition was used. In the present work neither under-relaxation for the source density nor extrapolation for interface values are needed. Moreover, the convergence rate gained a relevant speed up.

NUMERICAL RESULTS

Some numerical results have been obtained for the 2-D test case of a submerged non-lifting hydrofoil, for which experimental data are available in literature (Salvesen, 1966). Residual vs. iterations is plotted in fig.3, which shows that the convergence can be ensured up to roundoff errors. In fig.4 the computed free surface profile has been compared with the experimental data. Although a linearized free surface condition was used, the agreement seems to be rather good. The whole fluid domain is depicted in fig.5; the contour lines of the pressure inside the viscous region show the effects of the interaction with the external wave pattern. No separation is observed in the velocity field reported in fig.6, in spite of the marked thickening of the boundary layer near the trailing edge. However the boundary layer is confined inside Γ, which can be arranged close enough to the body to maximize the benefit of the multidomain approach.

The algorithm will be also applied for 3-D problems of free surface piercing bodies, as the Wigley hull and the Series-60.

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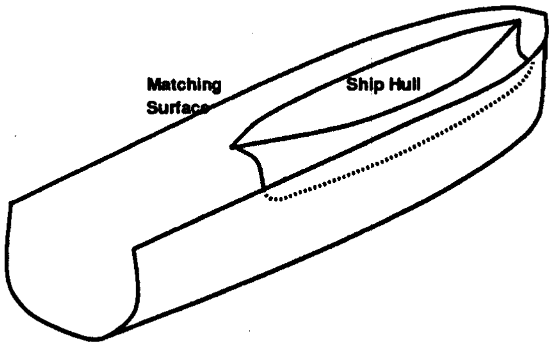

Fig.1: Domain decomposition

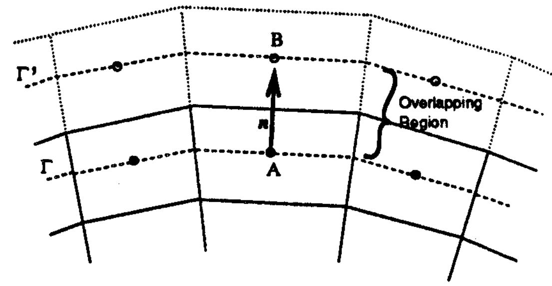

Fig.2: Matching surfaces Γ, Γ′ and variable collocation

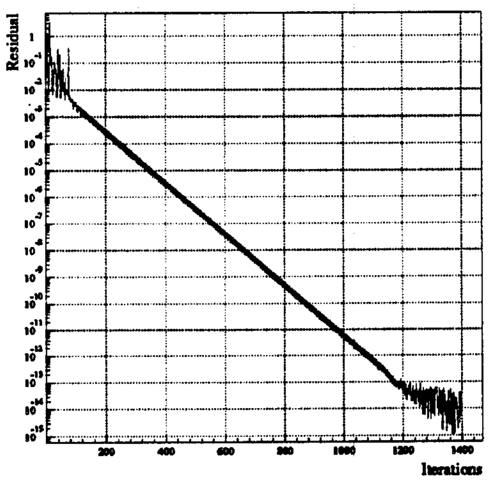

Fig.3: Salvesen profile at zero incidence. $Re = 3.5 \times 10^5$, $Fr = 0.591$. Convergence history of the $\tilde{\sigma}$ residual $\frac{|\tilde{\sigma}^{k+1} - \tilde{\sigma}^{k}|}{|\tilde{\sigma}^{k}|}$

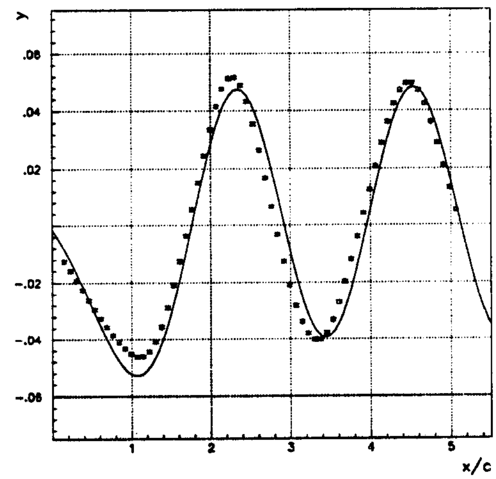

Fig.4: Free surface profiles.
*: experimental data (Salvesen 1966).
Solid line: viscous–inviscid computation.

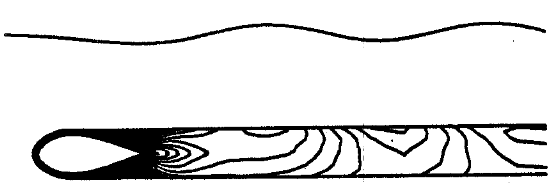

Fig.5: Computed free surface elevation and pressure field in the viscous region.

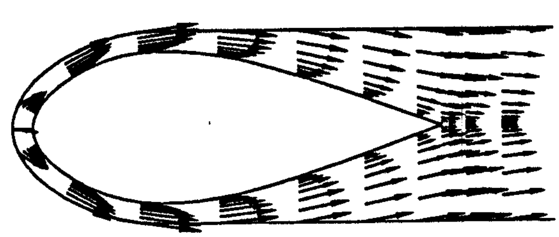

Fig.6: Computed velocity field in the viscous region.