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The Slow Yaw Motion of Offshore Structures

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Introduction

Offshore structures undergo large amplitude slowly varying motions when constrained by weak restoring forces. The amplitudes of these motions are greatly influenced by the damping forces acting on the system. The potential flow force resisting the slow-drift translation in the horizontal plane (the wave drift damping) has been subject to a number of theoretical studies, (*e.g.* Emmerhoff and Sclavounos 1992). It is also of interest to analyze the slowly varying rotation of such bodies, in particular the yaw motion. Grue and Palm (1992) have studied the yaw moment on a translating marine structure. In the present abstract, we are studying the yaw drift damping moment on a slowly yawing body, i.e. the mean momemt resisting the yaw motion.

Formulation

Consider a body undergoing a constant rotation about the Z-axis of an inertial coordinate system with the X- and Y-axis in the undisturbed free surface and the Z-axis pointing upwards. A body-fixed coordinate system xyz is defined to coincide with XYZ when $\alpha(t) = 0$ where $\alpha(t) = \Omega t$ is the angle between the X-axis and x-axis and Ω the angular velocity of the rotation. The coordinate transformation is then given by: $x = X\cos\alpha + Y\sin\alpha$, $y = -X\sin\alpha + Y\cos\alpha$ and $z = Z$. A long-crested incident wave with amplitude A and frequency ω propagates along a direction which forms an angle β with the X-axis.

Assuming potential flow, the fluid motion can be described by a potential $\Phi(X,Y,Z,t) = \phi(x,y,z,t)$. In order to derive the correct boundary conditions for ϕ in the body-fixed frame, we must use the following relation between time derivatives of Φ in the inertial frame and ϕ in the rotating frame:

$$\frac{d}{dt}\Phi(X,Y,Z,t) = \left(\frac{\partial}{\partial t} - \Omega\frac{\partial}{\partial \theta}\right)\phi(r,\theta,z,t), \tag{1}$$

where (r,θ) are the polar coordinates in the xy-plane and $\dot{\alpha}(t) = \Omega$.

Introduce the parameters $\delta = kA$ and $\bar{\Omega} = \Omega/\omega$ where k is the wave number. Assuming that δ and $\bar{\Omega}$ are small, we expand the potential $\phi(x,y,z,t)$ as follows:

$$\phi = \underbrace{\phi_{01}}_{O(\bar{\Omega})} + \underbrace{\phi_{10}}_{O(\delta)} + \underbrace{\phi_{11}}_{O(\delta\bar{\Omega})} + O(\delta^2, \bar{\Omega}^2), \tag{2}$$

where the first sub-index refers to the order of δ and the second to the order of $\bar{\Omega}$. ϕ_{01} represents the steady double body flow, while ϕ_{10} is the linear diffraction potential, with $\alpha(t) \equiv 0$. ϕ_{11} is the leading order correction to ϕ_{10} in $\bar{\Omega}$. We can neglect ϕ_{20}, which does not contribute to the mean yaw moment. Note, however, that the steady part will contribute to the mean horizontal forces. In the case of a translating body, it contributes to the yaw moment, but not the horizontal forces (Grue and Palm 1992).

The boundary conditions for the potentials ϕ_{ij}, which satisfy the Laplace equation, are stated below. $\phi_{01} = \Omega\bar{\phi}$ satisfies a rigid wall condition both on $z = 0$ and on the body. ϕ_{10} and ϕ_{11} are both linear in δ, and can be written as: $\phi_{10} = Re\{\varphi e^{i\omega t}\}$ and $\phi_{11} = Re\{\psi e^{i\omega t}\}$. Note that φ and ψ both depend on the angle of rotation $\alpha(t)$, such that the partial time derivative of ϕ_{10} and ϕ_{11} takes the form: $\partial/\partial t = i\omega + \Omega\partial/\partial\alpha$. The boundary conditions on $z = 0$ can therefore be written as:

$$-\omega^2\varphi + g\varphi_z = 0 \tag{3}$$

$$-\omega^2\psi + g\psi_z = 2i\omega\Omega\left(\varphi_\theta - \varphi_\alpha - \nabla\bar{\phi}\cdot\nabla\varphi,\right) \tag{4}$$

where the subscripts z, α and θ denote differentiation with respect to the respective variables. Both φ and ψ satisfy the Neumann condition on the body boundary.

φ may be written as the sum of the incident potential φ_I and the diffraction potential φ_D. The free surface boundary condition for ψ contains the α-derivative of φ, which may be evaluated by inspection of φ_I:

$$\varphi_I = \frac{igA}{\omega}e^{i\omega t - ikr\cos(\theta+\alpha(t)-\beta)+kz}. \tag{5}$$

It is obvious that differentiation of φ_I with respect to α and β are the same except for a minus sign. In order to see that the same is true for the diffraction potential, φ_D, and thus for φ, consider the boundary conditions for φ on the body: $\partial\varphi_D/\partial n = -\partial\varphi_I/\partial n$, where $\partial/\partial n$ denotes the normal derivative. Differentiation with respect to β gives: $\partial^2\varphi_D/\partial\beta\partial n = -\partial^2\varphi_I/\partial\beta\partial n = \partial^2\varphi_I/\partial\alpha\partial n$. By performing the same differentiation with respect to α, we see that $\partial\varphi_D/\partial\alpha$ and $-\partial\varphi_D/\partial\beta$ share the same boundary conditions, and therefore we can write:

$$\frac{\partial\varphi}{\partial\alpha} = -\frac{\partial\varphi}{\partial\beta}. \tag{6}$$

Results

The solution for ψ has been obtained for an array of vertical cylinders, following the method by Emmerhoff and Sclavounos (1992). In the computations of the mean moment acting on the body, we use the method of conservation of momentum, which gives the following expression for the yaw moment:

$$M_z = -\vec{k} \cdot \overline{\int_{S_\infty} \left(p\,\vec{x} \times \vec{n} + \rho\,\vec{x} \times \vec{V}\,\vec{V} \cdot \vec{n} \right) dS} \qquad (7)$$

where $\vec{k}$ is the unit vector pointing in the positive z-direction, p is the fluid pressure, $\vec{n}$ is the unit normal vector pointing out of the fluid, ρ is the fluid density, $\vec{V} = \nabla\phi$ is the fluid velocity and $\vec{x} = (x, y, z)$. The overbar denotes time average. By choosing S_∞ to be a cylindrical wall infinitely far away from the body, we can show that the first term in (7) vanishes. The mean moment is further expanded in the non-dimensional angular velocity, $\bar{\Omega}$, and neglecting terms of $O(\bar{\Omega}^2)$ the yaw moment is written:

$$M_z = \underbrace{M^{(0)}}_{O(\delta^2)} + \underbrace{M^{(1)}}_{O(\delta^2\bar{\Omega})} + O(\delta^2\bar{\Omega}^2). \qquad (8)$$

Therefore we can use the expansion for ϕ in (2) to derive the expression for the yaw drift-damping moment of $O(\delta^2\bar{\Omega})$:

$$M^{(1)} = -\frac{1}{2}\rho Re \int_{S_\infty} \left(\varphi_r \psi_\theta^* + \varphi_\theta \psi_r^* \right) dS, \qquad (9)$$

where the $*$ denotes complex conjugate.

Results should be presented for arrays of vertical cylinders, including the linear slow-drift yaw potential ψ and the yaw drift-damping moment over a range of wavelengths and waveheadings.

References

[1] Emmerhoff, O.J. & Sclavounos, P.D. 1992. The slow-drift motion of arrays of vertical cylinders. *J. Fluid Mech.* **242**, 31-50.

[2] Grue, J. & Palm, E. 1992. Mean yaw moment on floating bodies advancing with a forward speed in waves. *Behavior of Offshore Structures Conference (BOSS'92), London, Great Britain, 1992.* **Vol. 2**, 1238-1249.