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THE USE OF KOCHIN FUNCTIONS FOR THE SECOND-ORDER WAVE-BODY INTERACTION

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1. Introduction

There is a remarkable theorem introduced to ocean engineers by Haskind (1957) and Newman (1960), which relates wave exciting forces to corresponding far-field wave amplitude of the forced oscillation (radiation) problem. Therefore, once radiation problem is solved, wave exciting forces can be obtained from the Haskind relation without explicitly solving for the diffraction potential. The far-field asymptotic behaviors of the first-order diffraction and radiation potentials are explicitly given by the so-called Kochin Function, $\mathcal{H}$:

$$\begin{pmatrix} \phi_D^{(1)} \\ \phi_R^{(1)} \end{pmatrix} \sim \sqrt{\frac{k}{2\pi r}} f_o(z) e^{i(kr+\frac{\pi}{4})} \begin{pmatrix} \mathcal{H}_{\mathcal{D}}(\pi+\theta) \\ \mathcal{H}_{\mathcal{R}}(\pi+\theta) \end{pmatrix} \quad \text{for } r >> 1, \tag{1}$$

where (r, θ, z) is the cylindrical coordinate, k the wave number, and $f_o(z) = \cosh k(z+h)/\cosh kh$ with h being the water depth. The diffraction and radiation Kochin functions $\mathcal{H}_{D,R}$ are defined as

$$\mathcal{H}_{D,R}(\theta) = \int\int_{S_B} \left(\frac{\partial \phi_{D,R}^{(1)}}{\partial n} - \phi_{D,R}^{(1)} \frac{\partial}{\partial n} \right) f_o(\tilde{z}) e^{ik(\tilde{x}\cos\theta + \tilde{y}\sin\theta)} \, d\tilde{S}, \tag{2}$$

in which, $(\tilde{x}, \tilde{y}, \tilde{z})$ denotes the source point and S_B the mean body surface.

It is well known that the second-order mean drift force can directly be obtained form the above Kochin function (e.g. Newman, 1967). It is also recently found that the Kochin function plays a key role in the asymptotic analysis of the second-harmonic potential at large depths (Newman, 1990). In this paper, we will address these two problems. Particularly, the existing theory is extended to the case of multi-directional waves in order to observe the sensitivity of the second-order mean and double-frequency forces to the change of wave directional spreading.

2. Mean Drift Forces in Multi-directional Waves

Using momentum conservation for the fluid volume surrounded by an infinite-radius vertical cylinder, the mean force on a three-dimensional body can be expressed in terms of the far-field integral (Newman, 1967), where (1) and (2) can be used. In the presence of monochromatic bi-directional waves of frequency ω, wave headings, β_k, β_l, and amplitudes, A_k, A_l, the far-field integral yields

$$\begin{pmatrix} \bar{F}_x^{(2)} \\ \bar{F}_y^{(2)} \end{pmatrix} = \sum_{k=1}^{2}\sum_{l=1}^{2} -\rho g A_j A_l^* G(kh) \times$$

$$\left(\frac{k}{8\pi}\int_0^{2\pi} \tilde{\mathcal{H}}_k(\pi+\theta)\tilde{\mathcal{H}}_l^*(\pi+\theta)\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} d\theta + \frac{i}{4}[\tilde{\mathcal{H}}_k(\pi+\beta_l)\begin{pmatrix}\cos\beta_l \\ \sin\beta_l\end{pmatrix} - \tilde{\mathcal{H}}_l^*(\pi+\beta_k)\begin{pmatrix}\cos\beta_k \\ \sin\beta_k\end{pmatrix}]\right); \quad (3)$$

where ρ and g are the fluid density and gravitational acceleration, respectively, $\tilde{\mathcal{H}} = \mathcal{H}/(-igA/\omega)$, and $G(kh) = \tanh kh + kh/\cosh^2 kh$.

When $\beta_k = \beta_l = 0$, (3) is reduced to the expression given in Newman (1967). If the input directional wave spectrum is narrow banded in frequency, the above monochromatic bi-directional drift force results may be used in the so-called "diagonal (or Newman's) approximation" for the efficient computation of slowly-varying drift forces in short-crested irregular seas (Kim & Yue, 1989).

3. Second-harmonic Diffraction Potential at Large Depths

In this section, we consider an asymptotic form of the second-harmonic diffraction potential at large depths in the presence of dual waves of frequency ω and wave headings β_k and β_l (i.e. $\phi^{(1)} = \phi_k^{(1)} + \phi_l^{(1)}$). The analysis basically follows the procedure used in Newman (1990) where the asymptotic behavior of the second-harmonic potential is obtained for the regular wave ($\beta = 0$), but the theory is now extended to multi-directional waves. Here, only the final results are presented as follows:

$$\phi^{(2)}(r,\theta,z) \sim \frac{e^{i\frac{\pi}{4}}}{4\sqrt{2\pi}K^2}\frac{z}{\sqrt{r^2+z^2}}\sum_{k=1}^{2}\sum_{l=1}^{2}\frac{\mathcal{F}(\pi+\beta_k;\beta_k,\beta_l)}{\sqrt{r^2+z^2}+r\cos(\theta-\beta_k)}, \quad (4)$$

where $K = \omega^2/g$, and the function $\mathcal{F}$ is related to the Kochin function $\mathcal{H}$ as follows:

$$\mathcal{F}(\theta;\beta_k,\beta_l) = i\sqrt{\frac{2}{\pi}}A_k K^3 e^{i\frac{\pi}{4}}(1-\cos(\theta-\beta_k))\mathcal{H}_l(\theta+\pi). \quad (5)$$

In case of monochromatic mono-directional wave ($\beta_k = \beta_l = 0$), we recover the equation (19) of Newman (1990). From (4), we see that the leading term of $\phi^{(2)}$ decays algebraically with depths, as $O(1/z)$, in contrast to the exponential decay of $\phi^{(1)}$. It is also seen that the amplitude of $\phi^{(2)}$ at large depths is determined by the the Kochin function $\mathcal{H}$.

4. Application to Arrays of Deep Vertical Cylinders

The diffraction and radiation Kochin functions can be obtained analytically for arrays of bottom-mounted vertical circular cylinders. The diffraction potential due to N cylinders can be written in the form of Fourier-Hankel series (Linton & Evans, 1990):

$$\phi_D = \sum_{j=1}^{N}\phi_D^j = -\frac{igA}{\omega}f_o(z)\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j Z_n^j H_n(kr_j)e^{in\theta_j}, \quad (6)$$

where H_n is the first-kind Hankel function of order n, $Z_n^j = \frac{J_n'(ka_j)}{H_n'(ka_j)}$ with a_j being the radius of the j-th cylinder, and (r_j, θ_j) is the local polar coordinate system of the j-th cylinder. The unknown coefficients A_n^j in (6) can be determined from the body boundary condition on each cylinder.

The total radiation potential ϕ_R can be written in terms of the normalized radiation potentials φ_i for modes i=1~6; $\phi_R = \sum_{i=1}^{6} -i\omega\xi_i\varphi_i$, where ξ_i designates the amplitudes of six-degree-of-freedom body motions, and φ_i is given by a sum of propagating and local waves:

$$\varphi_i = \sum_{j=1}^{N} \sum_{n=-\infty}^{\infty} \left(B_n^j f_o(z) \frac{H_n(kr_j)}{kH_n'(ka_j)} + \sum_{l=1}^{\infty} L_{nl}^j f_l(z) \frac{K_n(\kappa_l r_j)}{\kappa_l K_n'(\kappa_l a_j)} \right) e^{in\theta_j}, \tag{7}$$

where K is the second-kind modified Bessel function and the depth function for local waves $f_l(z) = \cos\kappa_l(z+h)/\cos\kappa_l h$. The unknowns B_n^j and L_{nl}^j can be determined from the body boundary condition of each cylinder.

From the far-field asymptotic behaviors of (6) and (7), we finally obtain the Kochin functions of the N columns in the following explicit forms:

$$\mathcal{H}_D(\pi+\theta) = -\frac{igA}{\omega}\frac{2}{k}\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} A_n^j Z_n^j e^{-i(kR_j\cos(\theta-\Theta_j)-n\theta_j+\pi(n+1)/2)}, \tag{8}$$

$$\mathcal{H}_R(\pi+\theta) = \sum_{i=1}^{6} -i\omega\xi_i \frac{2}{k^2}\sum_{j=1}^{N}\sum_{n=-\infty}^{\infty} \frac{B_{ni}^j}{H_n'(ka_j)} e^{-i(kR_j\cos(\theta-\Theta_j)-n\theta_j+\pi(n+1)/2)}, \tag{9}$$

where (R_j, Θ_j) is the location of the center of each cylinder. These analytic Kochin functions can be used in (3) and (4) to calculate the second-order mean and double-frequency forces on multiple columns.

For illustration, we first consider the surge drift force in monochromatic bi-directional waves, which is given by (3) in terms of the Kochin functions. The result for the four columns of the stationary ISSC TLP is presented in Figure 1 for four different combinations of incident wave angles, where the incident angle of one wave is fixed at 0° and that of the other wave is increased from 0° to 45° with 15° increment. It is noted that the magnitude of the second-order surge mean drift force gradually decreases with increasing the directional spreading in low frequency region but shows somewhat complicated pattern at high frequencies.

We next investigate the effect of wave directional spreading on second-harmonic vertical forces. For this, we compute the double-frequency vertical forces on the stationary four legs of the ISSC TLP for four different combinations of dual waves as in Figure 1. It is seen that the second-harmonic vertical forces on the four columns are very sensitive to the wave directions and do not simply tend to decrease as directional spreading increases, which implies that the assumption of wave uni-directionality does not necessarily lead to a conservative result.

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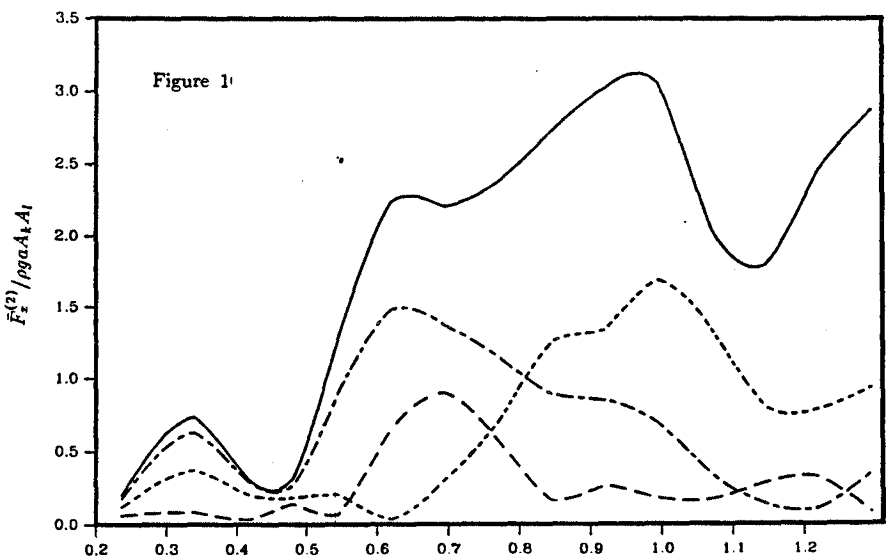

(——) (0°, 0°), (- - -) (0°, 15°), (- - - - -) (0°, 30°), and (- - - - -) (0°, 45°).