Hydrodynamic characteristics of bodies in channels

by

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Introduction

Experiments carried out to determine the hydrodynamic characteristics of bodies usually need to be performed in wave tanks even though what is actually required is knowledge of their behaviour in the open sea. It is thus important to understand both qualitatively and quantitatively how the tank walls affect quantities such as the exciting force on a fixed body due to an incident wave or the added mass and damping coefficients for a body making small simple harmonic oscillations.

We consider the case of a channel of constant width $2d$ and constant depth h and we assume that the channel is infinitely long. We also assume that the fluid is incompressible and that the fluid motion is irrotational. Then the governing equation is Laplace's equation and it is well known that there is a countable set of discrete cut-off frequencies, with n propagating modes possible when the frequency lies between the $(n-1)$th and the nth cut-off $(n \geq 1)$. The possible modes are alternately symmetric and antisymmetric about the centreline of the channel.

There are many important non-linear aspects of the physical situation when the frequency is close to a cut-off value. In this paper however we shall restrict our attention to linear theory. There are many situations in which the linear solution provides the dominant contribution to the hydrodynamic characteristics and despite being much simpler, the nature of the linear solutions is still not fully understood.

Trapped modes

A simple geometry that can be used to shed light on the various phenomena that can occur with wave-body interactions in a channel is the vertical circular cylinder, either extending throughout the depth or truncated. Callan, Linton & Evans (1991) proved that for a vertical cylinder extending throughout the depth and placed on the centreline of the channel there exists a discrete mode below the first cut-off frequency, antisymmetric about the centreline, which satisfies the condition of zero normal velocity on all solid boundaries and which has finite energy. They term such a mode a trapped mode. Evans (1992) proved that such antisymmetric trapped modes also exist for a vertical plate on the centreline extending all the way to the bottom and Evans, Linton & Ursell (1992) proved that these modes are still possible for an off-centre plate even though in this case antisymmetry cannot be imposed and propagating modes are possible at all frequencies.

Each of these problems has an acoustical counterpart and in particular the thin plate on the centreline of an acoustic waveguide has been shown experimentally by Parker (1966) to exhibit such modes which he terms acoustic resonances. A full review of the occurrence of acoustic resonances is given in Parker & Stoneman (1989).

Evans & Linton (1991) used numerical methods based on matched eigenfunction expansions to show that such modes exist for vertical cylinders of rectangular cross-section whilst Linton & Evans (1992b) have used the numerical solution of integral equations to show that such modes exist for a wide class of cross-sections, all symmetric about the channel centreline. In all the above cases the body extends throughout the fluid depth. Numerical evidence presented in Linton & Evans (1992a) suggests that such modes exist for truncated cylinders also.

Discussion

The existence of trapped modes clearly implies the non-uniqueness of certain radiation problems since any multiple of the trapped mode can be added to the solution without affecting the boundary conditions. In this paper we will show that this non-uniqueness appears as a singularity in the added mass coefficient in a region where the damping coefficient is identically zero. This phenomenon will be explained in terms of a pole of the complex force coefficient lying on the real axis.

Now, any body symmetric about the channel centreline moving in sway will produce a fluid motion antisymmetric about the centreline so that, if $0 < 2kd < \pi$, no waves will be radiated and the damping coefficient will indeed be identically zero over this range. In all such cases considered up to now trapped modes have been found (except in the case of a vertical plate perpendicular to the channel walls where the body does not interfere with the sloshing modes of the tank) and it seems reasonable to suggest that trapped modes exist for all such bodies. By solving the problem of a swaying cylinder situated away from the centreline of the channel (using multipoles) we will show how the singularity in the added mass appears as the cylinder approaches the centreline.

It seems unlikely that trapped modes exist for non-symmetric bodies or for bodies off the channel centreline, but we will show that another problem where the damping coefficient vanishes over a range of values of kd is that of a swaying vertical plate aligned with the channel walls placed anywhere in the channel, or indeed any number of such bodies. This explains why Evans *et al.* (1992) were able to find trapped modes in this case also.

Since the damping coefficient in sway is related through the channel equivalent of the Haskind relations to the cross-channel exciting force on the body due to an incident wave from infinity, our arguments suggest that if this force is zero over a range of frequencies then a trapped mode will exist. From symmetry considerations this is always the case for a body symmetric about the channel centreline and it is also clearly true for thin vertical plates aligned with the channel walls. It seems unlikely that the cross-channel exciting force on any body which is not symmetric about the channel centreline will be zero over a range of frequencies and so in turn it seems unlikely that trapped modes will exist for such bodies.

References

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