A REALIZABLE FORCE FEEDBACK-FEEDFORWARD CONTROL LOOP FOR A PISTON WAVE ABSORBER

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We consider the problem of the absorption of 2D water waves by the horizontal motions of a vertical plane in response to the hydrodynamical force it experiences. The solution of this problem is straightforward in the frequency domain. Unfortunately, the transposition of the frequency domain solution to the time domain by inverse Fourier transforming leads to a non-causal impulse response function. Thus it cannot be used just as it is, neither as a control loop for physical absorbing devices, nor as a non radiating boundary condition (NRBC) in numerical modelling. In this paper, we propose a method to derive causal approximations of this ideal controller. Two time-domain absorbing relations are proposed, differing in whether or not one knows a dominant frequency of the incident wave train to be absorbed. Their performances are compared with the absorption efficiency of the low frequency asymptotic Sommerfeld relation which simply reads, for such a piston device: $U(t)=F(t)$

TRANSFER FUNCTION AND IMPULSE RESPONSE OF THE IDEAL WAVE ABSORBER

Let $\Phi_I e^{i\Omega t}$ be the complex Airy velocity potential of the incident left-going wave train (see Fig.1), $\Phi_I = \frac{\pi\gamma}{\Omega}\frac{\cosh M_0(Y+1)}{M_0 \cosh M_0} e^{iM_0 X}$, and $\Phi_D e^{i\Omega t}$ the potential of the corresponding (right-going) reflected wave: $\Phi_D = \frac{\pi\gamma}{\Omega}\frac{\cosh M_0(Y+1)}{M_0 \cosh M_0} e^{-iM_0 X}$ with: Ω the frequency, γ the steepness, M_0 the solution of $M_0 \tanh M_0 = \Omega^2$,M_k the solution of $M_k \tan M_k = -\Omega^2$, all the physical quantities being nondimensionalized using the constant water depth h and the gravity acceleration g as usual.

When a time harmonic motion is imposed to the paddle with a given velocity law: $u(t) = \Re\{U.e^{i\Omega t}\}$, a right-going wave train deriving from a the radiated velocity potential $\Phi_R e^{i\Omega t}$ is generated in the basin. The induced linearized hydrodynamic force acting upon the the paddle surface is simply derived by integrating the dynamic pressure $p = -\Phi_t$ from the bottom $(Y=-1)$ to the free surface $(Y=0)$. The radiation force then reads: $F_R e^{i\Omega t} = [N(\Omega) + i\Omega M(\Omega)]U e^{i\Omega t}$, where:

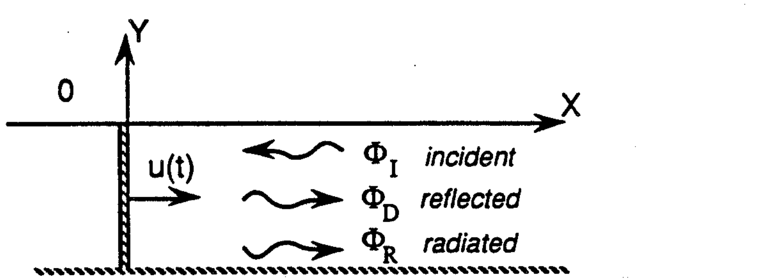

Figure 1: *principle of dynamic absorption*

$$N(\Omega) = 2\frac{\Omega^3}{M_0^3}\frac{1-e^{-4M_0}}{1-e^{-4M_0}+4e^{-2M_0}} = \frac{\Omega^3}{M_0^3}\frac{2\sinh 2M_0}{2M_0+\sinh 2M_0} \quad , \quad M(\Omega) = 2\sum_{k=1}^{\infty}\frac{\Omega^4}{M_k^3(\Omega^4-\Omega^2+M_k^2)} = \sum_{k=1}^{\infty}\frac{\Omega^2}{M_k^3}\frac{2\sin 2M_k}{2M_k+\sin 2M_k}$$

In the linearized approach we use, the total potentiel is the algebraic sum of the three above mentionned components: $\Phi_T = \Phi_I + \Phi_D + \Phi_R$ and the total force upon the paddle: $F_T e^{i\Omega t} = \{F_I + F_D + [N(\Omega) + i\Omega M(\Omega)]U\} e^{i\Omega t}$

The complete absorption of the incident wave train requires the velocity U to be such that, at least at a certain distance d from the paddle, the reflected waves and the propagating part of the radiated wave cancel each other. Let us denotes by $\tilde{U}$ the optimal complex value (amplitude and time phase) of the piston velocity U leading to this ideal result. Thus, for every given frequency Ω, we can determine the complex transfer function $H(i\Omega)$ of the ideal wave-absorber controller which would give access to the optimal velocity from the knowledge of the measured hydrodynamic force:

$$H(i\Omega)=\frac{\tilde{U}(i\Omega)}{F(i\Omega)}=\frac{1}{N(\Omega)-i\Omega M(\Omega)}$$

This may be considered as the transmittance of the feedback branch of the open loop system described on fig. 2. The real and imaginary parts of $H(i\Omega)$ are plotted on fig. 3. This feedback controller being linear and time invariant, the classical theory of LTI systems results in that its impulse response function $h(t)$ is the inverse Fourier transform of its transfer function in the frequency domain. Then, its output in the time domain $\tilde{u}(t)$ is given by the following convolution integral:

$\tilde{u}(t)=\int_{-\infty}^{+\infty} f(\tau)h(t-\tau)d\tau$, where $f(t)$ is the time varying hydrodynamic force. On the figure 4, we have plotted $h(t)$ and $h^*(t)$ which are the inverse Fourier transform of $H(i\Omega)$and its complex conjugate:$H^*(i\Omega)$. As one can see, $h(t)$ is "anticausal" (*i.e* h(t)=0; t≥0), while $h^*(t)$ which is by construction its symetric with respect to the time variable is causal.
Refering to the convolution integral above, that means that the calculation of the optimal velocity $\tilde{u}(t)$ to be given to the paddle to perform the total absorption involves all the future values of the input $f(t)$, and none from the past ! In other words, for a given geometrical deformation mode of the vertical cut, we allways may derive the transfer function of an ideally wave-absorbing controller, but we cannot practically realize it.

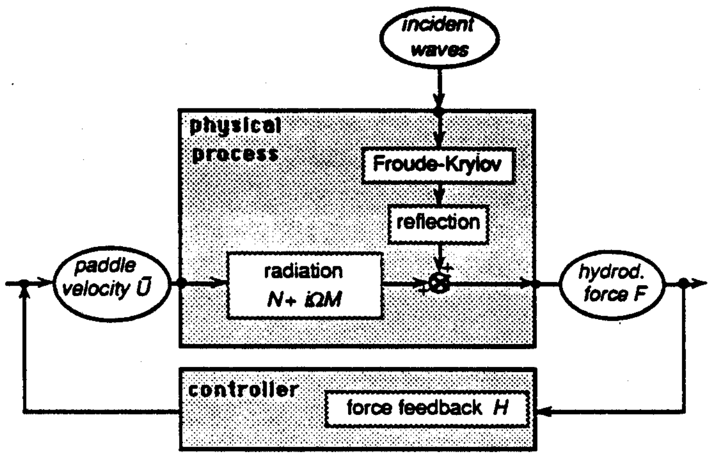

Figure 2: *non causal force feedback control*

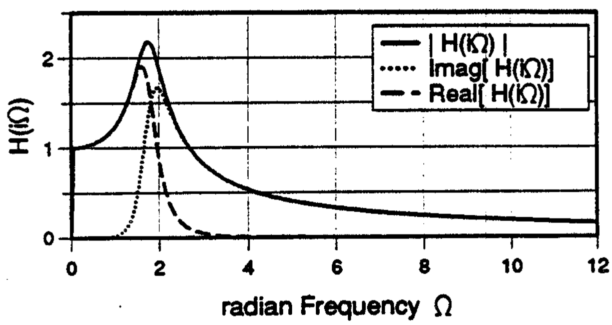

Figure 3: *Force-to-Velocity transfer function of the ideal piston wave-absorber*

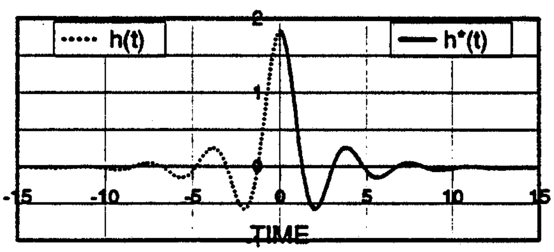

Figure 4: *direct and retrograde impulse response functions of the ideal wave-absorber.*

Low Frequency Limit *(LFL)* mode. On figure 3, we can see that in the low frequency range (i.e $0 \le \Omega \le 1.$), the phase lag between force and optimal velocity is negligible while the gain remains in the range: $1. \le |H(i\Omega)| \le 1.2$. Thus, a very first rough approximation of a time domain absorption relation between measured force and velocity stands in the limit relation: $\tilde{u}(t) = f(t)$. This is nothing but the well known Sommerfeld relation between local pressure and normal velocity [see Orlanski (1976)], integrated from Y=-1 to Y=0; it is exact in the limit $\Omega \to 0$, because of the cancelation of the radiated near field and of the transformation of the 2D flow into a monodimensional one. As a logical consequence, the performances of this absorption relation are very good in this frequency range, as we shall see later (Fig. 7).

Decomposition of the transfer function: The inverse Fourier transform $k(t)$ of the function $K(i\Omega)$ defined below can be shown to be the sum of a causal function $p(t)$, and an even function of time $q(t)$ (fig. 5). The relation between the optimal velocity $\tilde{U}$ and the force F becomes, in the frequency domain: $\tilde{U} = P(i\Omega)\tilde{U} + Q(i\Omega)\tilde{U} - H^*(i\Omega)F$. In the time domain, assuming the fluid to be at rest for $t \le 0$, we have by Fourier transforming:

$$K(i\Omega) = 1 + \frac{N(\Omega) - i\Omega M(\Omega)}{N(\Omega) + i\Omega M(\Omega)}$$

$$\tilde{u}(t) = \int_0^t p(t-\tau)\tilde{u}(\tau)d\tau + \int_0^\infty q(t-\tau)\tilde{u}(\tau)d\tau - \int_0^t h^*(t-\tau)f(\tau)d\tau$$

The first two terms calculated from $\tilde{u}(t)$ may be regarded for the whole system as a feedforward control loop with a non-causal part due to the symetry of $q(t)$ with respect to t=0; the force feedback term is now causal due to the complex conjugation in the frequency domain. The motion controller is not yet realizable, but a substancial step has been made toward this goal with the decomposition we propose. At the present stage, no approximation has been introduced, and the instant velocity defined by the above relation is allways the *optimal* one in that sense that, if we were able to compute it at time t from the knowledge of the past, it would absorb 100% of any incident wave train. We shall now propose two different causal approximations of $q(t)$ and evaluate the absorption performance they result in.

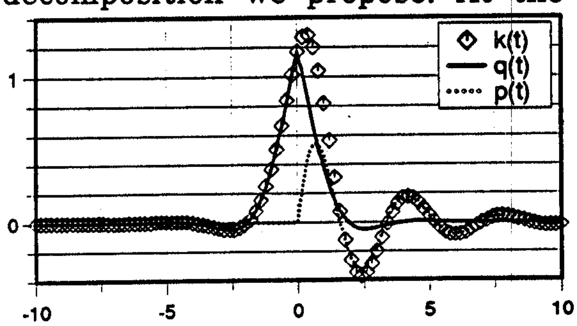

Figure 5: *impulse responses of the feedforward loop.*

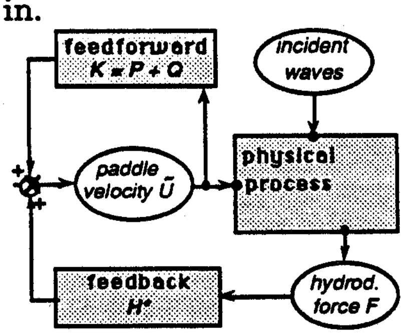

Figure 6: *Feedback-Feedforward control*

Purely Unsteady Feedback-Feedforward *(PUFF)* causal approximation. The sharp shape of the even function $q(t)$ around the origin (fig. 5) suggested us to approximate it by a Dirac δ function, and write: $q(t) \cong \alpha\delta(t)$. From energetic arguments, the weight α was set equal to the surface under $q(t)$ which is itself equal to $Q(0)$. The absorption relation is then given by:

$$\tilde{u}(t) = \frac{1}{1-\alpha}\left[\int_0^t p(t-\tau)\tilde{u}(\tau)d\tau - \int_0^t h^*(t-\tau)f(\tau)d\tau\right]$$

This absorption mode is said to be *purely unsteady* because, as the *LFL* mode, it does not require any spectral knowledge of the incident wave train.

Frequency Dependent Feedback-Feedforward (FDFF) causal approximation. In this second approach, the incident wave train spectrum is assumed to present a known dominant frequency ω. In that case, the coefficient α of the *PUFF* relation may be tuned to the exact real value $Q(i\omega)$ of the frequency domain transfer function previously defined. The *FDFF* absorption relation is the same as the *PUFF* relation above after the substitution: $\alpha \mapsto Q(i\omega)$.

Results: These absorption relations are implemented in a "2D linearized numerical wave tank". At one end, a piston wavemaker generates a short wave train consisting in a monochromatic harmonic wave modulated by a linear up and down ramp window. The motion of the opposite piston end, initially at rest like the fluid itself, is deduced from the absorption laws studied herein in response to the total hydrodynamic force. The measured wave amplitude absorption coefficients are plotted on fig. 7. They have to be squared in order to obtain the corresponding energy coefficients. It is clear from this figure that the purely unsteady feedback-feedforward method brings only a little improvement with respect to the low frequency limit; the price to pay in terms of numerical sophistication seems to high for the result.

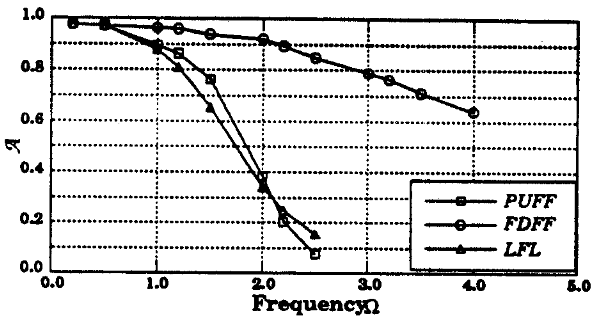

Figure 7: *absorption efficiency (amp. ratio)*

On the other hand, this kind of control scheme is very promising and give excellent results as soon as one can identify a priori a dominant frequency in the incident wave train. This conclusion agrees with the results of preceeding studies about non radiating numerical boundary conditions for unsteady water waves simulation (e.g Lee and al.). When the flow is purely unsteady, the relation we propose gives better results than Milgram's one, and is not time variable as the Orlanski's one implemented by Lee and al., Jaganathan, ... Finally, we are convinced that more a efficient relation remains to be developped in that case, for medium to high frequencies.

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