WAVE FORCES ON CYLINDERS ADVANCING WITH SMALL FORWARD SPEED IN WATER OF MODERATE DEPTH
(Wave drift damping)

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In order to verify the validity of a simple formula for the calculation of wave drift damping [2][7], in the case of cylinders free to move at the encounter frequency of the waves, and inspired by the work of Emmerhoff & Sclavounos [3] and Matsui & al. [6], we develop here a quasi-analytic solution for vertical cylinders free in surge and sway. On the other hand, we would like to obtain an exact and complete solution for the first and constant second order forces exerted on the body freely advancing in the waves, which can be used as a reference solution for verification of numerical methods.
If we proceed in the usual manner [8] of linearisation, with respect to wave steepness $\epsilon = k_0 A$, and supposing the total periodic potential as a sum of the incident potential ϕ_0, scattering potential ϕ_7 and radiation potentials ϕ_1-surge and ϕ_2-sway, $\Phi = \Re\{\phi e^{i\omega_e t}\} = \Re\{[A(\phi_0+\phi_7) + i\omega_e \sum_{j=1}^{2} \xi_j \phi_j] e^{i\omega_e t}\}$, we obtain the following boundary value problem for a body advancing with small forward speed on waves in water of restricted depth (Laplace equation and radiation condition are understood):

$$-\omega_e^2 \phi + 2i\omega_e U \nabla\phi_s . \nabla\phi + g\frac{\partial\phi}{\partial z} - i\omega_e U \phi \frac{\partial^2\phi_s}{\partial z^2} = 0 \qquad \text{on the free surface}$$

$$\frac{\partial\phi}{\partial n} = \xi\left(i\omega_e \mathbf{n} - U(\mathbf{n}\cdot\nabla)\nabla\phi_s\right) \qquad \text{on the body}$$

$$\frac{\partial\phi}{\partial n} = 0 \qquad \text{on the bottom}$$

where ϕ_s is a potential due to the current and the steady flow around the body, for one cylinder case $\phi_s = (\bar{\phi} - x) = -(\frac{a^2}{r}\cos\theta + x)$.
If we introduce a perturbation series :

$$\phi = \varphi + \tau\psi \ , \ \ \phi_j = \varphi_j + \tau\psi_j \ , \ \ \xi_j = \xi_j^0 + \tau\xi_j^1 \ , \ \ \tau = \frac{U\omega_0}{g}$$

in the above mentioned boundary value problem, and knowing that encounter frequency ω_e can be written as a function of the fundamental wave frequency ω_0, $\omega_e = \omega_0 - k_0 U\cos\beta$, (β-incidence angle of the incoming waves), we can express the potentials φ and ψ as :

$$\varphi = A(\varphi_0 + \varphi_7) + i\omega_0 \sum_{j=1}^{2} \xi_j^0 \varphi_j$$

$$\psi = A\psi_7 + i\omega_0 \sum_{j=1}^{2} \left[\xi_j^0 \left(\psi_j - \frac{k_0}{\nu}\cos\beta\varphi_j\right) + \xi_j^1 \varphi_j\right]$$

and obtain the following equations :
On the free surface :

$$-\nu\varphi_j + \frac{\partial\varphi_j}{\partial z} = 0 \qquad j = 0,1,2,7$$

$$-\nu\psi_7 + \frac{\partial\psi_7}{\partial z} = 2i\frac{\partial\varphi_7}{\partial x} - 2k\cos\beta\ \varphi_7 - 2i\nabla\bar{\phi}\cdot\nabla(\varphi_0 + \varphi_7)$$

$$-\nu\psi_j + \frac{\partial\psi_j}{\partial z} = 2i\frac{\partial\varphi_j}{\partial x} - 2k\cos\beta\ \varphi_j - 2i\nabla\bar{\phi}\cdot\nabla\varphi_j \qquad j = 1,2$$

On the body surface :

$$\begin{aligned}
\frac{\partial \varphi_7}{\partial n} &= -\frac{\partial \varphi_0}{\partial n} \\
\frac{\partial \varphi_j}{\partial n} &= n_j \qquad\qquad j=1,2 \\
\frac{\partial \psi_7}{\partial n} &= 0 \\
\frac{\partial \psi_j}{\partial n} &= \left[\frac{i}{\nu}(\mathbf{n}\cdot\nabla)\nabla\vec{\phi}\right]_j \qquad\qquad j=1,2
\end{aligned}$$

Analytic solution for potential φ in the case of one cylinder free in surge and sway, is well known and can be written as follow:

$$\begin{aligned}
\varphi_0 &= \frac{igA}{\omega_0} f_0(z) \sum_{m=-\infty}^{\infty} e^{-im(\frac{\pi}{2}+\beta)} J_m(k_0 r) e^{im\theta} \\
\varphi_7 &= -\frac{igA}{\omega_0} f_0(z) \sum_{m=-\infty}^{\infty} e^{-im(\frac{\pi}{2}+\beta)} \frac{J_m'(k_0 a)}{H_m^{*\prime}(k_0 a)} H_m^*(k_0 r) e^{im\theta} \\
\varphi_1 &= \left[f_0(z) B_0 \frac{H_1^*(k_0 r)}{k_0 H_1^{*\prime}(k_0 a)} + \sum_{n=1}^{\infty} f_n(z) B_n \frac{K_1(k_n r)}{k_n K_1'(k_n a)}\right] \cos\theta \\
\varphi_2 &= \left[f_0(z) B_0 \frac{H_1^*(k_0 r)}{k_0 H_1^{*\prime}(k_0 a)} + \sum_{n=1}^{\infty} f_n(z) B_n \frac{K_1(k_n r)}{k_n K_1'(k_n a)}\right] \sin\theta
\end{aligned}$$

with $\nu = \frac{\omega_0^2}{g} = k_0 \tanh k_0 H = -k_n \tan k_n H$ and:

$$f_0(z) = \frac{\cosh k_0(z+H)}{\cosh k_0 H}, \quad f_n(z) = \frac{\cos k_n(z+H)}{\cos k_n H}, \quad B_0 = \frac{2\sinh 2k_0 H}{2k_0 H + \sinh 2k_0 H}, \quad B_n = \frac{2\sin 2k_n H}{2k_n H + \sin 2k_n H}$$

We remark that all the potentials φ_j can be written in the forme $\varphi_j = \sum_{n=0}^{\infty} f_n(z) g_{jn}(r,\theta)$.
The solutions for the potentials ψ_j is less evident because of inhomogeneous free surface condition. Special attention is given to research of particular solution which satisfy a first part of the free surface condition $2i\frac{\partial \varphi_j}{\partial x} - 2k_0 \cos\beta \varphi_j$. The solution can be found in the following form, (valid for all ψ_j $\;j = 1, 2, 7$):

$$\psi_{jP} = 2(i\frac{\partial}{\partial x} - k_0 \cos\beta)\chi_j$$

Emmerhoff & Sclavounos proposed χ_j as a derivative of φ_j with respect to the wavenumber ν, $\chi_j = \frac{\partial \varphi_j}{\partial \nu}$. Here we present another solution depending only on space derivatives of φ_j. In the case of infinite water depth expression for χ_j is found in the compact form $\chi_j = \frac{1}{\nu}\mathbf{r}\cdot\nabla\varphi_j$. For finite water depth problem, and in the case when the potential φ_j can be written in the form $\varphi_j = \sum_{n=0}^{\infty} f_n(z) g_{jn}(r,\theta) = \sum_{n=0}^{\infty} \varphi_{jn}$, the solution is more complicated because of the fact that bottom condition is not automatically satisfied :

$$\chi_j = \sum_{n=0}^{\infty} \frac{1}{k_n}\frac{\partial k_n}{\partial \nu}\left[\mathbf{r}\cdot\nabla\varphi_{jn} + H(-\nu\varphi_{jn} + \frac{\partial \varphi_{jn}}{\partial z})\right]$$

Normal velocity on the cylinder induced by this potential is annulled by one standard scattering potential which satisfy homogeneous condition on the free surface and following condition on the cylinder :

$$\frac{\partial \psi_{jR}}{\partial r} = -\frac{\partial \psi_{jP}}{\partial r} = -\frac{\partial}{\partial r}\left[2(i\frac{\partial}{\partial x} - k_0\cos\beta)\chi_j\right] = -\sum_{m=-\infty}^{\infty} v_{jm}(z) e^{im\theta}$$

This potential is found, using standard procedure, in the form :

$$\psi_{jR} = \sum_{m=-\infty}^{\infty}\left[f_0(z) B_{jm0} \frac{H_m^*(k_0 r)}{k_0 H_m^{*\prime}(k_0 a)} + \sum_{n=1}^{\infty} f_n(z) B_{jmn} \frac{K_m(k_n r)}{k_n K_m'(k_n a)}\right] e^{im\theta}$$

with :

$$B_{jmn} = -\frac{\int_{-H}^{0} v_{jm}(z) f_n(z) dz}{\int_{-H}^{0} f_n(z) f_n(z) dz}$$

Finaly, the parts of the potential ψ which satisfy the last parts of the free surface condition $,-2i\nabla\bar{\phi}\cdot\nabla\varphi_j$, and homogeneous condition on the cylinder are found by employing a well known ring source method [4][6][7], which is well adapted to the probleme because the forcing term on the free surface decays rapidly when the radial distance from the cylinder goes to infinity and so we can use the standard radiation condition. The remaining parts of the potentials ψ_j can be calculated without difficulty.
Once the potential solution found it is possible to calculate the first order forces and amplitudes of motions in surge and sway, introducing the perturbation series for ξ and ϕ into following equation of motion , (M_{ij} denotes a surge-sway mass matrix) :

$$-\omega_e^2 \mathrm{M}_{ij}\xi_j = \int_S p n_i dS = -\rho \int_S \Big[i\omega_e \phi - U(\frac{\partial \phi}{\partial x} - \nabla\bar{\phi}\nabla\phi)\Big] n_i dS = F_i + \Big(\omega_e^2 A_{ij} - i\omega_e B_{ij}\Big)\xi_j$$

Accepting the usual notation $\rho\omega_0^2 \int_S \varphi_j n_i dS = \omega_0^2 A_{ij}^0 - i\omega_0 B_{ij}^0, \quad j = 1,2 \quad i = 1,2$ we obtain :

$$\begin{aligned}
\left[-\omega_0^2(M_{ij} + A_{ij}^0) + i\omega_0 B_{ij}^0\right] \xi_j^0 &= -\rho i\omega_0 \int_S (\varphi_0 + \varphi_7) n_i dS \\
\left[-\omega_0^2(M_{ij} + A_{ij}^0) + i\omega_0 B_{ij}^0\right] \xi_j^1 &= -\rho i\omega_0 \int_S \Big[\psi_7 + \frac{1}{\nu}\Big(i\frac{\partial}{\partial x} - i\nabla\bar{\phi}\nabla - k_0\cos\beta\Big)(\varphi_0 + \varphi_7)\Big] n_i dS \\
&\quad - \xi_j^0\Big\{-\rho\omega_0^2 \int_S \Big[\psi_j - 2\frac{k_0}{\nu}\cos\beta\varphi_j + \frac{i}{\nu}\Big(\frac{\partial \varphi_j}{\partial x} - \nabla\bar{\phi}\nabla\varphi_j\Big)\Big] n_i dS \\
&\quad + 2\omega_0^2 \frac{k_0}{\nu}\cos\beta M_{ij}\Big\}
\end{aligned}$$

The solutions of those two equations complete the determination of the first order potentials φ and ψ. The mean steady second order forces, mean drift force and wave drift damping, are calculated applying the momentum conservation principle in the manner suggested by Emmerhoff and Sclavounos [3]. The solution for the problem of diffraction only is presented in [7], and surprising simple formula for calculation of wave drift damping [2] is generalised for the finite water depth case :

$$B_{xx}(\omega_0) = \Big\{\Big[\Big(\frac{\partial D_x(\omega_0)}{\partial \omega_0} - \frac{1}{\alpha}\frac{\partial \alpha}{\partial \omega_0} D_x(\omega_0)\Big)\omega_0 + \frac{2}{\alpha} D_x(\omega_0)\Big]\cos\beta - \frac{1}{\alpha}\frac{\partial D_x(\omega_0)}{\partial \beta}\sin\beta\Big\}\frac{k_0}{\omega_0}$$

with convention $F_{dx} = D_x - UB_{xx}$, and with $\alpha = \frac{1}{2} + \frac{k_0 H}{\sinh 2k_0 H}$.

The complete agreement of results obtained by the simple formula and by the method briefly presented here incited us to verify the validity of the simple formula in the case of a freely floating body.
Some preliminary results are presented on figure 1,2,3 and 4. Figures 1 and 2 represent first order quantities calculated with and without the forward speed. Agreement between the results for first order exciting force, figure 1, calculated with the method presented here and results obtained by Matsui & al [6] is total. Figure 2 represents the added mass and damping, and shows that the well known Timman-Newman relations, [$A_{ij}(U) = A_{ji}(-U)$, $B_{ij}(U) = B_{ji}(-U)$] are verified. Figure 3 represents the drift force and wave drift damping for one fixed cylinder and figure 4 wave drift damping for one array of four cylinders in the cases of finite and infinite water depth, (the cylinders are centered at the corner of a square with side length equal 7 times the radius of cylinder).

Références

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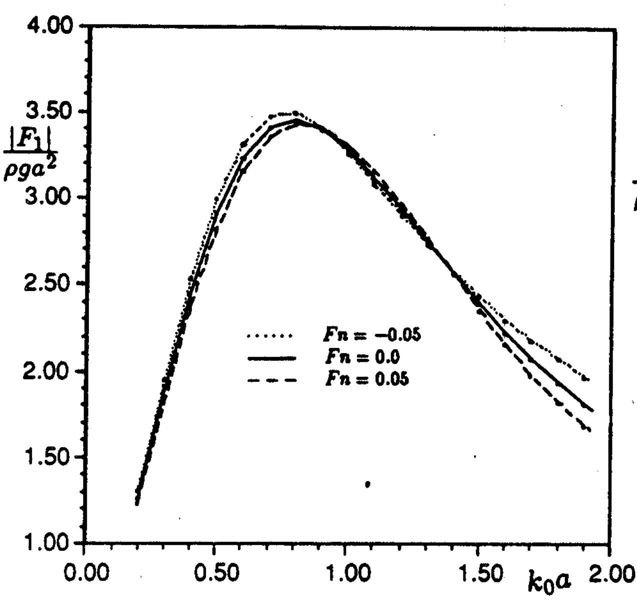

Figure 1. First order surge exciting force F_1 for one cylinder and for differents Froude numbers $Fn = U/\sqrt{ga}$. Incidence angle is $\beta = 0$ and $H = a$.

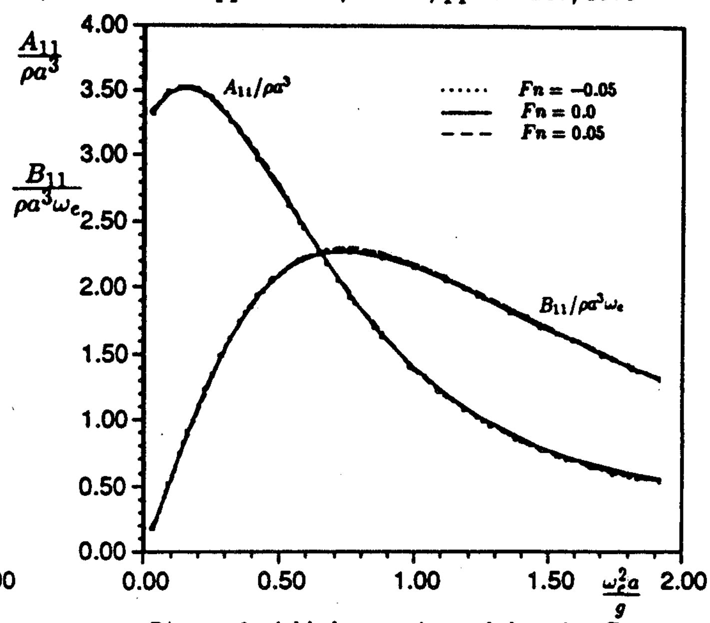

Figure 2. Added mass A_{11} and damping B_{11} in surge for one cylinder and for differents Froude numbers $Fn = U/\sqrt{ga}$ with $\beta = 0$ and $H = a$.

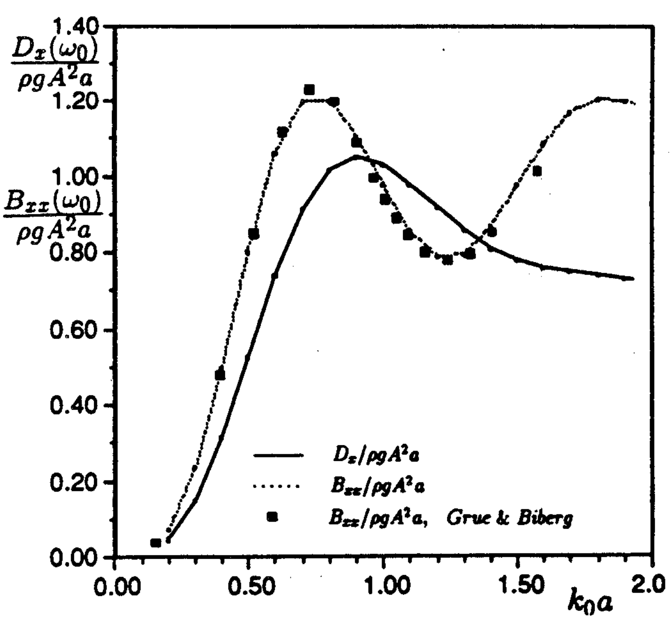

Figure 3. Mean drift force D_x and wave drift damping B_{xx} for a fixed cylinder. Incidence angle is $\beta = 0$ and depth is $H = a$.

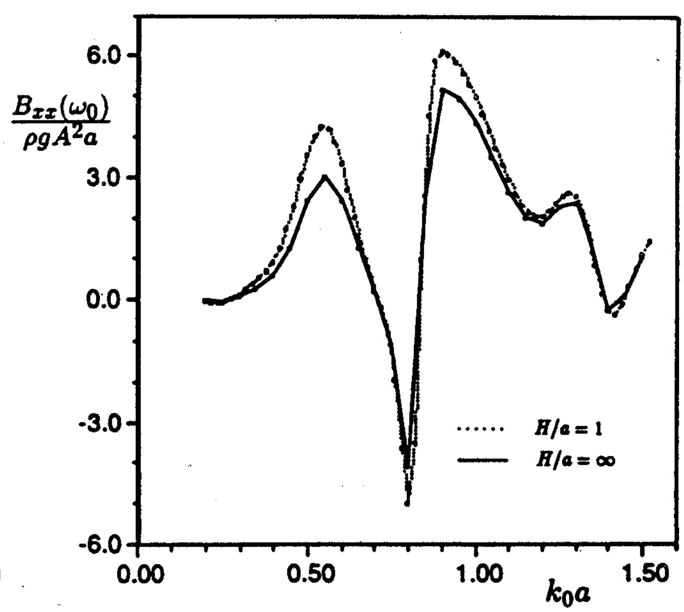

Figure 4. Wave drift damping for a fixed square configuration of four vertical cylinders with spacing $7a$ in the cases of infinite and finite water depth. Incidence angle is $\beta = \pi/4$.