Ship Generated Internal Waves in Highly Stratified Seas; The Inverse Problem

T. Miloh & E. Avital

Faculty of Engineering

Tel-Aviv University,

Ramat-Aviv, 69978, Israel

It is well established that ships moving in stratified oceans and lakes can generate surface signatures of internal waves, detectable by remote sensing devices (e.g. Watson et. al 1992). These signatures often appear as "bright narrow V" wake features, which can be induced by a modulation of the ambient surface wave spectrum caused by ship generated internal waves. This conjecture is supported by the recent Loch Linnhe experiment, where all images of the V-shaped waves were found to be surface signatures of ship induced internal waves. In the absence of vertical shear and ambient current the internal wave pattern is found to vary with ship speed and with the local stratification. For a ship moving along a straight course, the kinematic theory of Keller & Munk (1970) can be applied in order to predict the geometry of the wave pattern, once an appropriate dispersion relationship is used. The dispersion relationship in turn is determined numerically from the measured profile of the *Brunt – Väisälä* frequency. The Keller-Munk (KM) theory then relates the wave phase and wave number to the position of a point on the multiple wake arms. The KM method, which has been recently revisited by Yih &Zhu (1989), thus provides (within the realm of infinitesimal internal wave theory) explicit relationships for the wake coordinates in terms of the wave number, phase velocity , group velocity and the ships rectilinear speed along a straight trajectory. In situ measurements (e.g. Watson et. al 1992) seem to confirm the general validity of the kinematic KM theory for various pycnoclines.

In this paper we propose an explicit new method for solving the inverse problem, that is determining the wave dispersion relationship as well as the phase and group velocities, from direct measurements of the wake geometry. It is rather surprising that the nonlinear KM ordinary differential equations can indeed be analytically inverted. Uniqueness aspects of this inverse problem, which is governed by a particular version of a Sturm-Liouville system, are also discussed. Three different *Brunt – Väisälä* frequency profiles, typical for lakes & fjords (Loch Linnhe), seas (Gibraltar straits) and oceans (Pacific), are used in order to demonstrate the general versatility of this method. A modified Phillips-Miles dispersion formula, based on some gross parameters of the sharp thermocline, is also employed. Finally, the original KM theory for a straight course, is extended here for a ship moving along a large circlular path. These expressions reduce to Sretenski's ship wave patterns for circular trajectories, when instead of internal wave one uses the surface wave dispersion relationship. We believe that the proposed method, when used in conjunction with some appropriate remote sensing devices, can be used for the direct (non-evasive) estimation of density profiles in oceans and lakes, from the observable kinematical patterns of ship wakes.

References

1. Keller J.B. & Munk W.H. "Internal wave wakes of a body moving in a stratified fluid". Phy. Fluids 13, 1425-1431 (1970).
2. Watson G, Chapman R.D. & Apel, J.R. "Measurements of the internal wave wake of a ship in a highly stratified sea lock". J. Geophy. Res. 97, 9689-9703 (1992).
3. Yih C.S. & Zhu S. "Patterns of ship wakes", Q. App. Math 47, 17-33 (1989).