RECOVERY OF OPEN-SEA RESULTS FROM NARROW TANK TESTS

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1. Introduction

During physical model tests of offshore structures in a narrow wave tank, wave reflections from the tank walls may change substantially the hydrodynamic pressures and forces on the structure from the values that would be measured in open water. In principle, integral-equation methods may be used to obtain numerical solutions for the scattering and radiation by a structure of *any* geometry when placed in a tank (for example, this has been done for a structure consisting of a group of four vertical circular cylinders [1]). Such numerical calculations, together with similar calculations for the open sea, might be used to aid interpretation of experimental results in a narrow tank. The present work considers the possibility of an alternative procedure in which open-sea hydrodynamic characteristics are deduced solely from measurements made in a narrow tank, *without* resort to detailed numerical calculations for the specific structure under test.

This abstract will describe a 'correction' procedure for the mean horizontal drift force on a structure, related procedures can be formulated for other hydrodynamic quantities. It is well known that drift forces may be significantly modified from their open-sea values by the presence of tank walls. This is not difficult to understand when it is recalled that the drift force may be expressed in terms of the far-field scattered waves which are fundamentally different for the two cases. In the open sea the scattered field consists of radially spreading waves, while in a tank the waves propagate to infinity along the tank with, in the absence of viscous effects, no decrease in amplitude. Here, an approximate method is described for constructing the far-field scattered waves in the open sea from measurements of the free surface disturbance in a narrow tank. With this information the open-sea value of the mean drift force may be calculated. For simplicity of exposition it is assumed here that the structure is vertically axisymmetric; this assumption may be relaxed at the expense of additional complexity.

2. Formulation

A long wave tank of uniform depth h has vertical, parallel walls a distance $2b$ apart. Cartesian coordinates (x, y, z) are chosen with the origin in the mean free surface and midway between the channel walls so that the x-axis is directed along the channel and the z-axis vertically upwards. A fixed structure with a vertical axis of symmetry is placed midway between the tank walls. Horizontal plane polar coordinates r and θ are defined by

$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta. \tag{1}$$

A plane wave of amplitude A and frequency ω is incident from large negative x. The wave will be scattered by the body and, in general, the wave field can be decomposed into propagating modes and evanescent modes that decay exponentially with distance from the structure. Under the usual assumptions of linear water-wave theory the time-harmonic flow may be described by a velocity potential Φ. Far enough from the body for the evanescent modes to be negligible,

$$\Phi(x, y, z, t) = \Re\left\{-\frac{igA\cosh k(z+h)}{\omega\cosh kh}\phi_T(x, y)e^{-i\omega t}\right\}, \tag{2}$$

where k is the wavenumber satisfying the usual dispersion relation, g is the acceleration due to gravity and $\Re$ indicates that the real part is to be taken. This form of the potential satisfies the linearised free-surface condition, on $z = 0$, and the condition of no flow through the bed on $z = -h$.

The potential Φ satisfies the three-dimensional Laplace equation so that, on substituting the form (2), the complex-valued function $\phi_T(x, y)$ may be seen to satisfy the Helmholtz equation. The incident wave is described by

$$\phi_I = e^{ikx} = e^{ikr\cos\theta} \tag{3}$$

and the total potential ϕ_T in (2) is decomposed as

$$\phi_T = \phi_I + \phi. \tag{4}$$

The potential ϕ describing the far-field scattered wave will then satisfy the Helmholtz equation, the condition of no flow through the channel walls, and also a radiation condition specifying that the scattered waves must be outgoing as $|x| \to \infty$. The full determination of the potential Φ requires the application of a suitable body boundary condition but, in general, this requires the inclusion of the evanescent modes.

3. The solution in the open sea

In the open-sea situation, the formulation is as described in the previous section except that the walls are 'moved out' to infinity so that the radiation condition requires waves propagating radially outwards. Far enough from the body for evanescent modes to be negligibly small, the scattered wave field may be expressed as a multipole expansion

$$\phi^{(o)} = \sum_{n=0}^{\infty} A_n^{(o)} H_n(kr) \cos n\theta \tag{5}$$

where H_n denotes the Hankel function of the first kind and order n. The scattering coefficients $\{A_n^{(o)}; n = 0, 1, \ldots\}$ can, in principle, be determined from the body boundary condition.

In the present work, it will be assumed that the scattered field can be well represented by taking the first few terms in the series in (5); in the results presented below only the first three terms are retained. Although this assumes that the waves are much longer than the body dimension a, so that $ka \ll 1$, there is numerical evidence to suggest that a good approximation to the wave field is obtained even when $ka = O(1)$. The mean horizontal drift force may be calculated once the scattering coefficients in (5) are known [**2**, §7.10].

4. The solution in the tank

Far enough from the body for evanescent modes to be negligible, the scattered field when the body is in the tank may be expressed in terms of so-called 'channel multipoles' as

$$\phi = \sum_{n=0}^{\infty} A_n \phi_n(kr, \theta). \tag{6}$$

This form of solution is directly analogous to the more familiar open-sea solution given in equation (5). By construction, each multipole ϕ_n satisfies the Helmholtz equation and the tank-wall conditions [**3**, **4**]. The multipoles may be written

$$\phi_n(kr, \theta) = H_n(kr) \cos n\theta + \bar{\phi}_n, \tag{7}$$

where $\bar{\phi}_n$ is the correction to the open-sea multipole potential due to the channel walls.

As with the open-sea solution, for moderately long waves relative to the body dimension the scattering coefficients are expected to decay with increasing n. In an experimental situation the coefficients may be estimated by making measurements of the amplitude and phase of the free-surface elevation as this is directly related to ϕ through the linearised free-surface condition. Truncating the series (6) to N terms, say, and making N measurements of the free-surface motion gives enough information to determine the scattering coefficients. In the example of §6 below, $N = 3$ and the measurements of the free-surface motion are simulated numerically by calculating the reflection and transmission coefficients and the potential at a point on the tank wall.

5. Relation between tank and open-sea solutions

When the structure is in the tank, the scattering problem may be interpreted in terms of images. The image structures are centred about $|y| = 2mb$, $m = 1,2,\ldots$, and each scatters the incident wave as well as the waves scattered by the remaining image structures. For very wide tanks, the disturbance in the vicinity of the original structure, due to the waves scattered by neighbouring images, is very small so that first approximations to the tank scattering coefficients are just the corresponding open-sea coefficients. A systematic correction to the tank scattering coefficients may be developed using a 'wide-spacing' approximation to the channel multipoles [4]. The wide-spacing approximation assumes the images are widely separated compared to the wavelength which is equivalent to the condition $kb \gg 1$ although numerical evidence [4] suggests the approximation works well even when $kb = O(1)$.

From (6) and (7), the effect of the image system for the complete wave field is

$$\sum_{n=0}^{\infty} A_n \bar{\phi}_n \sim C\left(e^{-iky} + e^{iky}\right) \quad \text{where} \quad C = \frac{e^{-i\pi/4} I_0}{2\pi (kb)^{1/2}} \sum_{n=0}^{\infty} A_{2n} \tag{8}$$

and I_0 is an $O(1)$ function of kb. Equation (8) shows that, near the original structure, the primary effect of scattering by the image system is to generate a pair of plane waves, with complex amplitude C, propagating along the line of images. Thus, a structure in a tank may be regarded as being in open water and irradiated by the incident wave (3) and the plane waves (8). In this equivalent open-sea representation, the scattered field due to the specified incident wave propagating along the tank is given by (5) and, with the assumed axisymmetry, the scattered wave field for the incident waves $e^{\pm iky}$ is

$$\sum_{n=0}^{\infty} A_n^{(o)} H_n(kr) \cos n(\theta \pm \pi/2). \tag{9}$$

Equating the waves propagating *away* from the structure in the tank and in the equivalent open-sea representation yields

$$\sum_{n=0}^{\infty} A_n H_n(kr) \cos n\theta = \sum_{n=0}^{\infty} H_n(kr) \left\{ A_n^{(o)} \cos n\theta + C A_n^{(o)} \cos n(\theta + \pi/2) + C A_n^{(o)} \cos n(\theta - \pi/2) \right\}. \tag{10}$$

and so

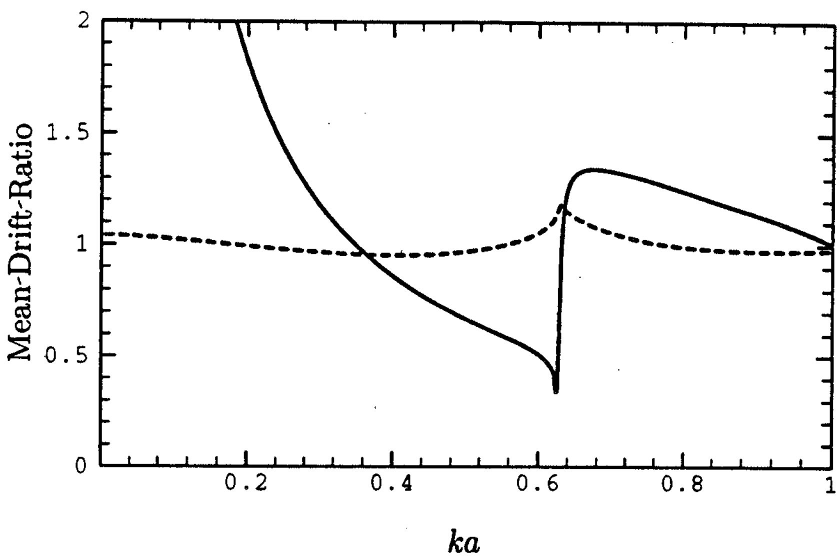
$$A_n = A_n^{(o)} + 2C A_n^{(o)} (-1)^{n/2}, \quad \text{for} \quad n \text{ even}, \quad \text{and} \quad A_n = A_n^{(o)}, \quad \text{for} \quad n \text{ odd}. \tag{11}$$

6. Results

To test the ideas described above, the drift force on a vertical, circular cylinder is considered. The drift force in the open sea may be found from the open-sea scattering coefficients [2, §7.10], which are given in terms of the tank coefficients by (11), and the latter are estimated by the experimental procedure outlined at the end of §4. Numerical results from an 'exact' tank formulation [3, 4] are used to simulate the experimental procedure.

The results presented here are for a tank half-width b equal to five times the cylinder radius a and are scaled by the open-sea drift force [3, eq. (4.24)]. The solid curve is the ratio of the drift force on a cylinder in a tank [3, eq. (4.23)] to the open sea drift force; if the tank walls had no effect then this ratio would be unity for all wavelengths of incident wave. The dashed curve is the estimate of the open-sea drift force obtained using the procedure described here; if the procedure were fully accurate then this ratio would be unity. Numerical tests have also been made with two coefficients and the results were significantly worse than those for three coefficients given here.

The example given here is a relatively severe test of the method. Two approximations were used in the formulation, namely that the wavelength is much greater than the body radius, $ka \ll 1$, and that the tank width is much greater than the wavelength, $kb \gg 1$. With the range of ka used each of the fundamental assumptions is violated over some part of the range. The only significant departure from unity of the estimated open-sea result is near the tank resonance at $kb = \pi$ where the method breaks down. (For structures symmetric about the tank centre-plane, $x = 0$, resonances corresponding to cross-tank standing waves occur for kb equal to integer multiples of π.)



While the results presented here offer encouragement that the method is worth pursuing, there is much further investigation required. The only tests so far made are for a vertical circular cylinder. Clearly, more complex geometries must be considered where evanescent modes will be present. In addition, it is likely that with non-axisymmetric geometries taking only three coefficients in the multipole expansion will not give the same degree of accuracy. Furthermore, the method requires measurements of the amplitude and phase of the free-surface motion in a tank and it must be established that this can be done with sufficient accuracy for the method to yield meaningful results. It will also be necessary to assess the sensitivity of the method to the choice of measurement points.

7. References

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