A Perturbation Formulation of The Ship Wave Resistance Problem

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Introduction

A general method of formulating non-linear boundary value problems of ship hydrodynamics was presented in (Pawlowski 1992). The method uses a domain mapping technique to transform the governing equations which define a boundary value problem (b.v.p.) on an instantaneous fluid domain of unknown, and in general time dependent, geometry into the governing equations of a transformed b.v.p. on a reference domain of known geometry. The solution to the original problem on the instantaneous domain is obtained from the solution to the transformed problem on the reference domain by means of the mapping that also transforms the reference fluid domain onto the instantaneous domain.

An approximation to the solution to the transformed problem can be determined by means of a perturbation series. The perturbation solution results from solving the corresponding series of b.v.p.s obtained from the transformed b.v.p.. It might be expected that the b.v.p.s of the perturbation series are linear, and that in particular the first problem in the series would be linear since perturbation techniques are most often used to linearise boundary value problems.

The b.v.p. that defines the flow induced by a steady forward motion of a ship in calm water, in short the ship wave resistance b.v.p., is one of the most interesting problems since until now it has eluded satisfactory solutions. Below, using the technique of fluid domain mapping, it is shown that the problem can be formulated as a series of perturbation b.v.p.s which are non-linear including the first problem in the series. In order to indicate this the formulation is referred to as a non-linear perturbation formulation.

The theoretical considerations are illustrated by some results of computations performed by Mr Shaowen Song who partly implemented the formulation as a modification of another algorithm. Those results are shown here by courtesy of Mr. Song.

The Formulation of The Ship Wave Resistance Problem

The instantaneous fluid domain which is bounded by an instantaneous free surface and wetted surface of the hull is denoted by **D** . The reference fluid domain, $\mathbf{D_0}$, is bounded by the undisturbed free surface (plane $\mathbf{x_3 = 0}$) and the part of the hull that is below it. A domain displacement field is denoted by $\eta(\mathbf{x,t})$, with:

$$y = x + \eta \qquad (1a)$$

where **y** is the radius vector in **D** and **x** is a radius vector in $\mathbf{D_0}$. This relation can be rewritten as:

$$y=\exp(\eta\cdot\nabla)\,x \qquad (1b)$$

The exponential operator is defined by:

$$\exp(\eta\cdot\nabla)=\Sigma_\alpha \frac{1}{\alpha!}\eta^\alpha\partial^\alpha \qquad (2)$$

with α denoting the multi-index of differentiation. As explained above the transformation (1b) applies also to tensor fields which describe the problem:

$$u(y,t)=\exp(\eta\cdot\nabla)\,v(x,t) \qquad (3)$$

where **u(y,t)** denotes the tensor field on **D** obtained by the transformation of tensor field **v(x,t)** on $\mathbf{D_0}$.

It is shown in (Pawlowski, 1992) that the governing equations of the non-linear ship wave resistance problem on $\mathbf{D_0}$ are:

$$\Delta\Phi=0 \qquad (4a)$$

$$\exp(\eta\cdot\nabla)\,(g\partial_3\Phi+\nabla\Phi\cdot\nabla\otimes\nabla\Phi\cdot\nabla\Phi)=0 \qquad (4b)$$

$$\exp(\eta\cdot\nabla)\,\nabla\Phi\cdot(\bar{\bar{I}}+\nabla\otimes\eta)^{-1}\cdot N=0 \qquad (4c)$$

Equation (4a) is the field equation satisfied in $\mathbf{D_0}$ by velocity potential Φ. Equation (4b) represents the free surface condition on $\mathbf{x_3=0}$, and equation (4c) is the impermeability condition on the reference wetted surface, with **I** and **N** denoting respectively unit tensor and unit normal vector. A causality condition must added to the above equations. Perturbation formulations of the boundary value problem are obtained by assuming solutions in the form of perturbation series:

$$\eta=\eta^{(1)}+\eta^{(2)}+\ldots \qquad (5a)$$

$$\Phi=\Phi^{(0)}+\Phi^{(1)}+\Phi^{(2)}+\ldots \qquad (5b)$$

Series **(5a)** cannot include an **O(1)** term ($\eta^{(0)}$) and therefore the perturbation solutions imply a small wave amplitude.

As discussed in (Pawlowski, 1992), the **O(1)** free surface condition becomes:

$$\partial_3\Phi^{(0)}=0, \qquad |\nabla\Phi^{(0)}|^2-U^2=O(\Phi^{(1)}) \qquad (6a)$$

if:

$$|\nabla\otimes\eta|=O(\Phi^{(1)}) \qquad (6b)$$

is assumed, i.e. the waves are considered to be of small steepness. Equations (6) can be satisfied exactly by the velocity potential of the uniform flow with velocity **U**, leading to a Neumann-Kelvin b.v.p. for $\Phi^{(1)}$. The same equations can also be satisfied in the perturbation sense by considering $\Phi^{(0)}$ to be determined by the double body flow. This choice results in a Dawson-like b.v.p. for $\Phi^{(1)}$. Both of those perturbation formulations give useless evaluations of wave resistance at the $\Phi^{(0)}$ level and, for ship shapes of practical interest, fall short of good predictions of wave resistance at the $\Phi^{(1)}$ level.

A different family of perturbation formulations is obtained if instead of (6b) a steep wave assumption is adopted:

$$|\nabla\otimes\eta|=O(\Phi^{(0)}) \qquad (7a)$$

The O(1) free surface condition then becomes:

$$g\partial_3\Phi^{(0)}+\frac{1}{2}\nabla\Phi^{(0)}\cdot\nabla(\nabla\Phi^{(0)}\cdot\nabla\Phi^{(0)})=0 \qquad (7b)$$

and the O($\Phi^{(1)}$) free surface condition is obtained as:

$$g\partial_3\Phi^{(1)}+\frac{1}{2}\nabla(\nabla\Phi^{(0)}\cdot\nabla\Phi^{(0)})\cdot\nabla\Phi^{(1)}+$$
$$\nabla\Phi^{(0)}\cdot\nabla(\nabla\Phi^{(0)}\cdot\nabla\Phi^{(1)})+\partial_3[g\partial_3\Phi^{(0)}+$$
$$\frac{1}{2}\nabla\Phi^{(0)}\cdot\nabla(\nabla\Phi^{(0)}\cdot\nabla\Phi^{(0)})]\,\eta_3^{(1)}=0 \qquad (7c)$$

with:

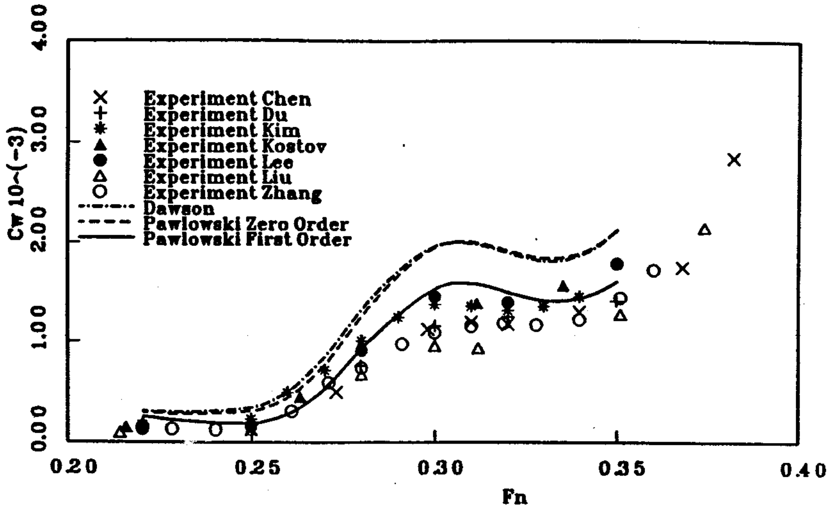
$$\eta_3^{(1)}=\frac{-1}{g+\frac{1}{2}\partial_3|\nabla\Phi^{(0)}|^2}\left[\frac{1}{2}(|\nabla\Phi^{(0)}|^2-U^2)+\nabla\Phi^{(0)}\cdot\nabla\Phi^{(1)}\right] \qquad (7d)$$

Equation (7b) is non-linear whereas equation (7c) is linear with respect to their unknown potentials.

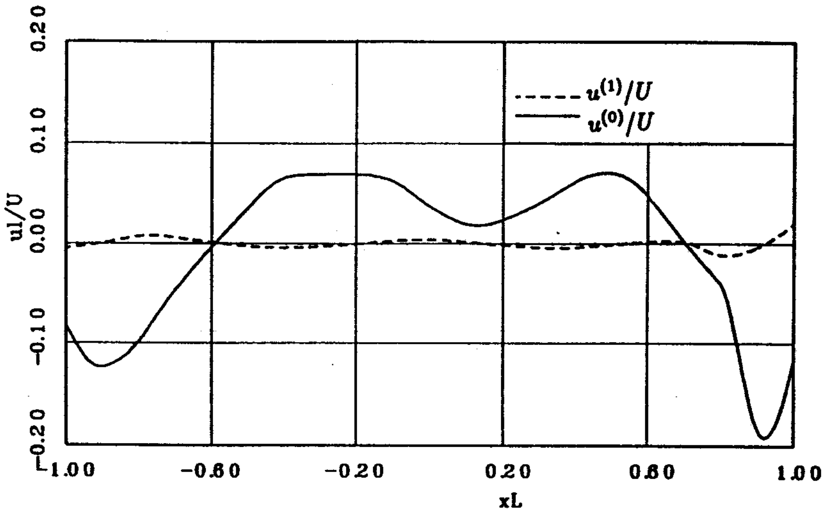
The perturbation formulation based on conditions (7b) and (7c) was implemented in a somewhat simplified form in a comparative computation of wave resistance for a Series 60, CB=0.60 vessel. The results are shown in the figures below. They indicate a convergence of the perturbation solution to a solution which matches the experimental data.

References:

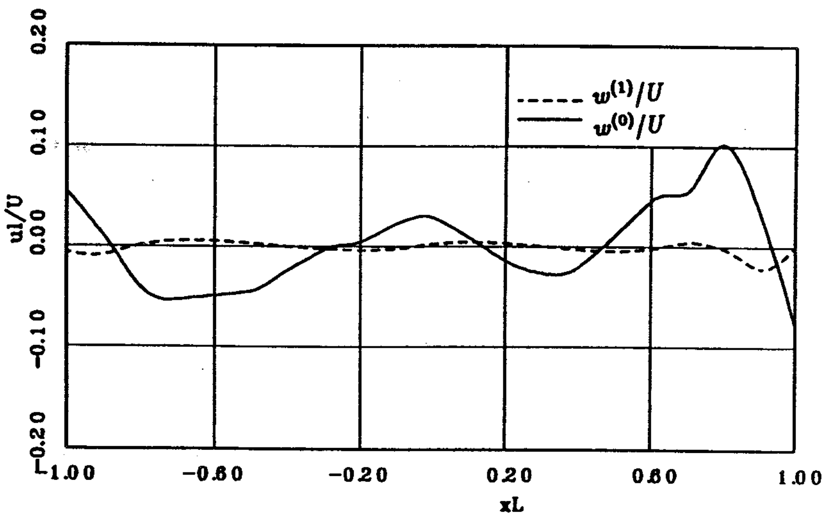
Pawlowski, J.S. "A Non-linear Theory of Ship Motion in Waves", Nineteenth Symposium on Naval Hydrodynamics, 1992.



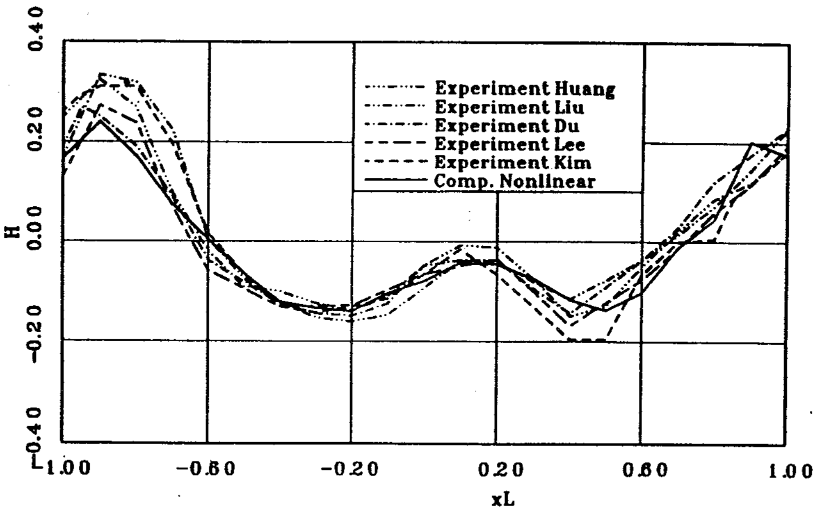
Zero and first order nonlinear C_w for Series 60 ship hull



Horizontal fluid velocity along Series 60 ship hull (Fn=0.25)



Vertical fluid velocity along Series 60 ship hull (Fn=0.25)



Comparison of nonlinear wave profiles for Series 60 hull (Fn=0.25)