The Numerical Simulation of Long Unstable Wave Trains Leading to Group Formation, Wave Deformation and Breaking

Marshall P. Tulin, Pei Wang & Yitao Yao
Ocean Engineering Lab, University of California, Santa Barbara

In connection with energetic waves in the ocean, there is now great interest in planar wave groups, and wave deformation and breaking within such groups (Tulin & Li, 1992). The non-linear processes at work, leading to breakdown, are largely not susceptible to analysis. For the understanding of these processes, numerical simulation is therefore required.

The progress of real waves under unstable conditions leads to strong spatial variations. The downtank distance (x/λ) over which these non-linear processes occur, increases with decreasing initial wave steepness ($a_o k_o$), roughly as $(a_o k_o)^{-2}$. Experiments show, Su & Green (1984), that distances, (x/λ), of order 10^2 are required. This is roughly one order of magnitude larger than numerical tank lengths achieved to date (Cointe,1990; She, *et al*, 1992). In these prior investigations, the waves were created by a wavemaker at one end, and absorbed at a beach on the other. Approximately 400 nodes, over 10-15 wavelengths, were distributed on the free surface; the cubic increase in computing time with tank length prevents longer computations.

Here we have developed a two-dimensional numerical wave tank, called *LONGTANK*; tank lengths to (x/λ) of 120 have been achieved, utilizing 5000 nodes on the free surface. The waves are generated by a wavemaker at one end; a moving beach is placed beyond the front of the complete wave group which is generated in time, and this beach damps the smaller fast waves beyond the front.

The computations utilize a multi-subdomain approach, Figure 1, the coefficient matrix of the simultaneous equations becomes block banded, or diagonalized, and an efficient banded-matrix solver can be applied, with a great savings in computer time. The price of a larger system of equations may accompany the advantage of diagonalization, but is trivial when the number of grids on the free surface is much larger than the number (N) in the vertical direction, as in the present case, where $N=O(10)$. The size of the subdomain may be optimized, and when this is done, we have found that the CPU per time step in the wave calculation increases less than linearly with the number of nodes on the free surface, Figure 2. Projection of the wave in time is carried out using the mathematics first introduced by Longuet-Higgins & Cokelet (1976) on a periodic domain, although somewhat different forward stepping is used.

LONGTANK has been used to calculate the progress of unstable wave systems (a central wave and two resonant side bands) toward deformation and breaking. The following phenomena have been observed:

1.) Sufficiently steep waves will first break when passing through the front of the entire wave group. Therefore, we have usually suppressed breaking there in order to obtain deformation and breaking well away from the influence of the wave front.

2.) With growth in the sidebands, very strong modulations leading to group formation are observed, Figure 3.

3.) For sufficiently large ($a_o k_o$), wave group formation leads eventually to a very rapid (a few wave periods) front face steepening, and then to rapid increase in horizontal velocity at the wave crest, to the formation of a jet, and thus to breaking, Figure 4 ($a_o k_o$=0.191). Wave breaking typically occurs close to the center of the wave group, as observed in the ocean.

4.) In our computations to date, the condition which sensitively differentiates waves which quickly go on to breaking, from those which do not, is that the horizontal velocity at the wave crest, U(0), reaches a value equal to half the phase velocity ($C_p/2$): $(U(0)/C_p)_{critical}=0.5$. It may be significant that this corresponds, in the cases treated, to $U(0)=C_{group}$.

5.) It is a remarkable feature of the breaking waves that very large gradients in horizontal particle speed occur in the vicinity of the crest, Figure 5.

6.) The time to breaking & the shape of the wave during its final stage is compared to measurements of Bonmarin, *et al* (1985), with excellent agreement, Figure 6. The precise shapes obtained vary somewhat from case to case, depending on the exact conditions within the group.

7.) The steepness of the simulated waves at breaking compares very favorably with the measurements of Su & Green (1984), Figure 7. More comparisons are desirable, and we are continuing our simulations.

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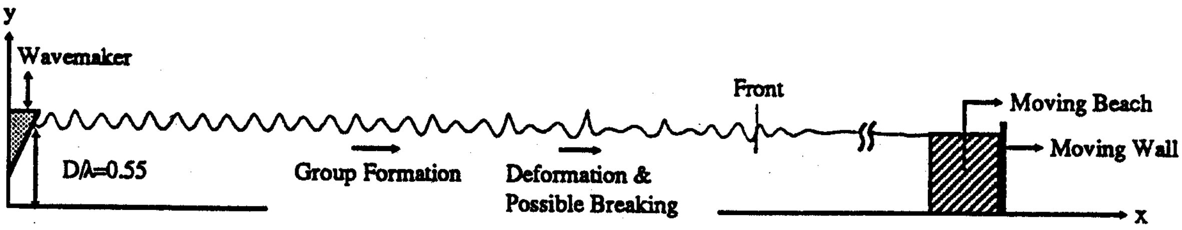

Figure 1(a): Wave Train Generated by a Wavemaker in a Numerical Wave Tank - *LONGTANK* (Schematic).

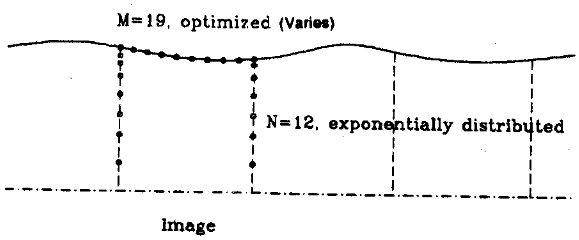

Figure 1(b): Division of Multi-Subdomain.

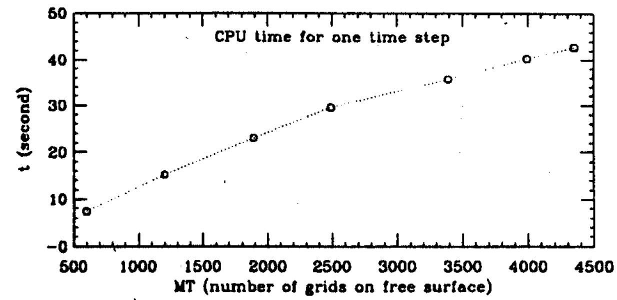

Figure 2: CPU Per Time Step in the Wave Calculation verse Number of Nodes on the Free Surface

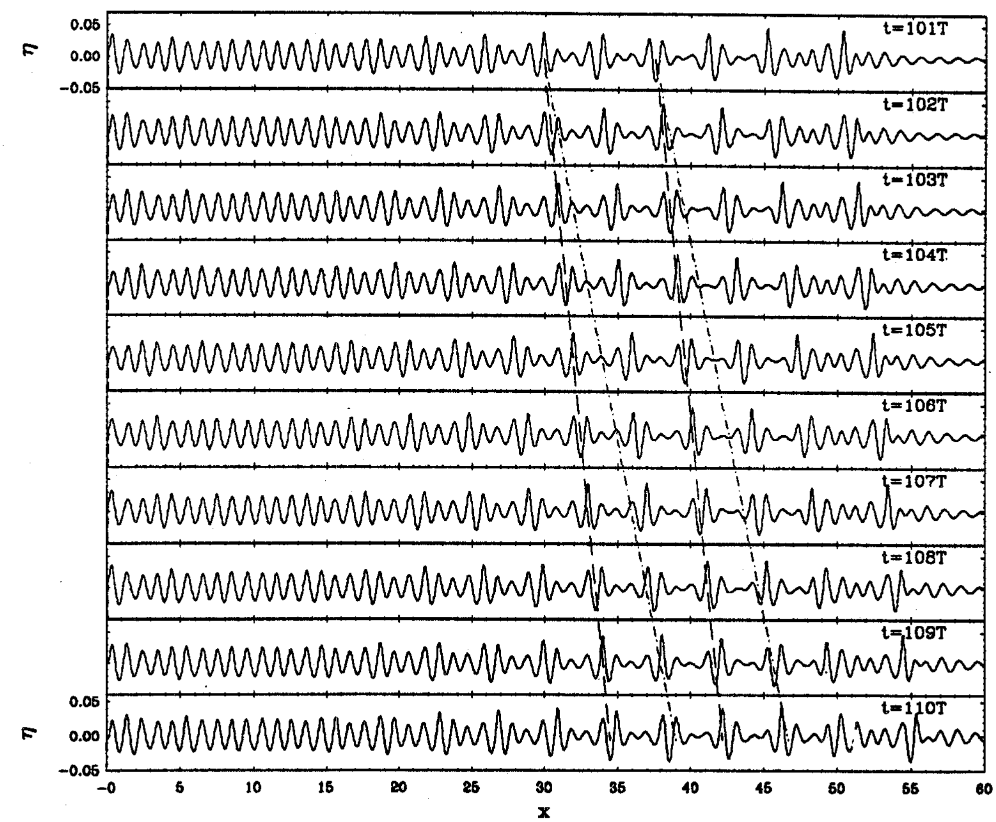

Figure 3: Simulated Wave Train (left to right). Central wave, $a_o k_o$=0.14 $\delta k/k_o$=0.14 and $\varepsilon_o^{\pm}/a_o$=0.16. Dashed lines indicate propagation at the group velocity (steeper) and phase velocity. Breaking did not occur.

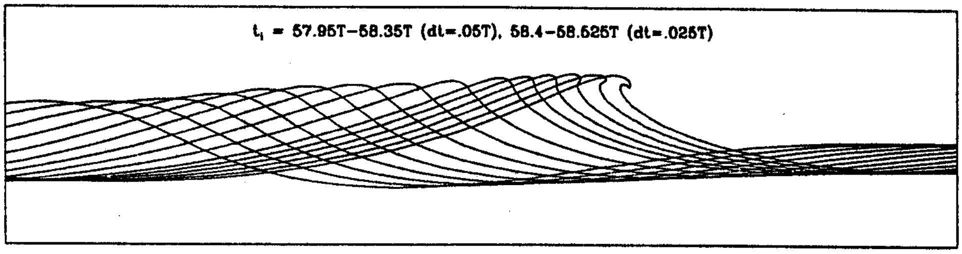

Figure 4 : Simulated Wave Shapes at Breaking. $(a_o k_o) = 0.141$, $(x/\lambda)_{breaking} = 23$.

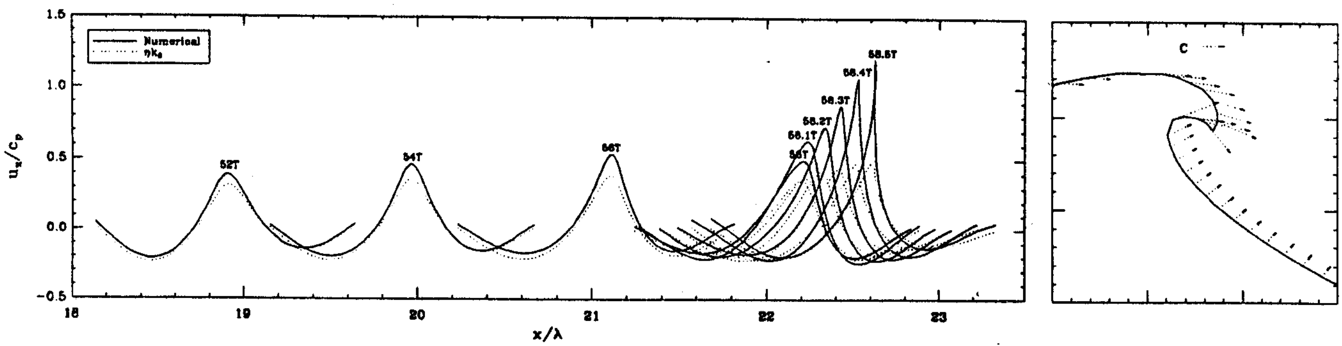

Figure 5 : Horizontal Velocities, U_x, in Simulated Wave Train Approaching and at Breaking. On the right : Surface Velocity Field at Breaking.

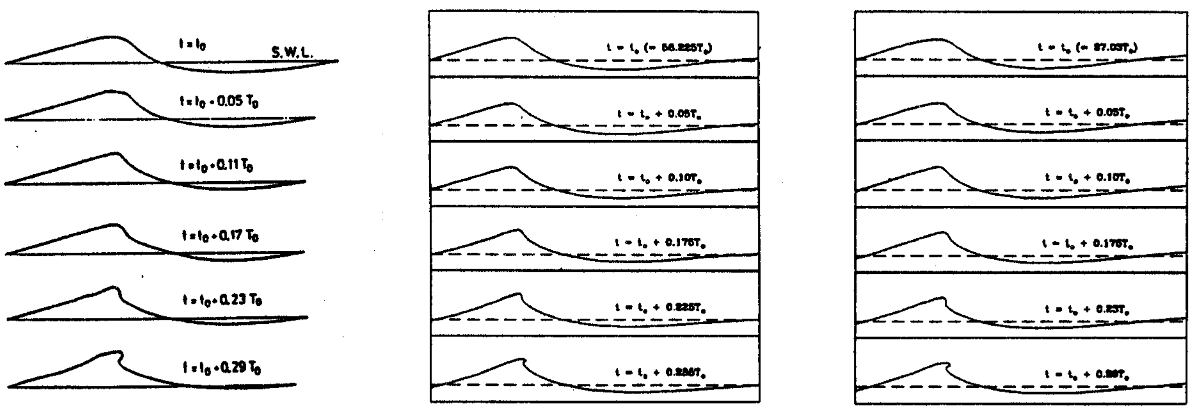

Figure 6: Waveforms Near Breaking. On the Left: From Bonmarin (1985): $a_o k_o \approx 0.25$
Center and Right: From present numerical simulations: $a_o k_o = 0.191, 0.217$.

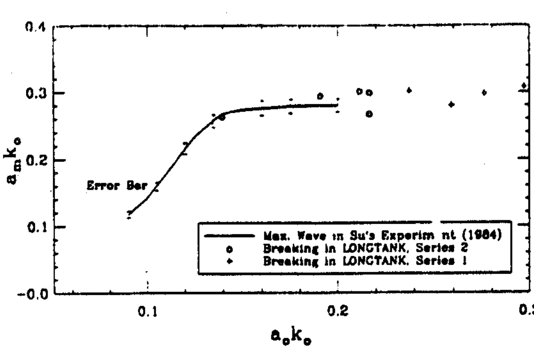

Figure 7: Maximum Wave Steepness in Wave Train verse Initial Steepness.