# TIME-MARCHING SCHEMES FOR SHIP MOTION SIMULATIONS

Torgeir Vada
Det Norske Veritas Research AS
Høvik, Norway

Dimitris E. Nakos
MIT
Cambridge, MA, USA

This study is part of a project aiming at the development of a computer program for the simulation of linear ship motions in waves, later to be extended to solve the corresponding non-linear problem. Because of this the decision has been taken to employ a numerical solution based on a Rankine Panel Method.

The design of an appropriate time-marching scheme is addressed for the enforcement of the linearized free surface conditions within the framework of a Rankine Panel Method for transient free surface flows. For a discussion on the numerical enforcement of the radiation condition the reader is referred to Nakos /1/. The performance of various algorithms is examined by means of their application on the model problem of a submerged transient source moving at a steady forward speed $U$, formulated as follows,

$$\phi(\vec{x},t) - \iint_{\overline{F}} \frac{\partial\phi(\vec{\xi},t)}{\partial z} G(\vec{x};\vec{\xi})d\vec{\xi} = \sigma(t)R(\vec{x}) \quad \text{, with } \vec{x}\in\overline{F} \ , \tag{1}$$

$$\frac{\partial\phi}{\partial t} + U\frac{\partial\phi}{\partial x} = -g\eta \quad , \text{ on } \overline{F} \ , \tag{2}$$

$$\frac{\partial\eta}{\partial t} + U\frac{\partial\eta}{\partial x} = \frac{\partial\phi}{\partial z} \quad , \text{ on } \overline{F} \ , \tag{3}$$

where $\overline{F}$ is the plane $z=0$, $g$ the acceleration of gravity, $\sigma(t)$ is the source strength, and $\phi$ and $\eta$ the velocity potential and free surface elevation of the resulting wave flow.

The discrete formulation follows from the space-approximation of the unknowns in terms of some polynomial basis functions, collocation at an appropriately selected set of points and, finally, choice of a marching scheme for the time evolution of the solution. A systematic methodology for the analysis of stability and accuracy characteristics of transient Rankine Panel Methods is discussed in Nakos /1/. The discrete dispersion relation depends upon the detailed design of the discretization scheme as well as on the following normalized discretization parameters,

$$\beta = \frac{\sqrt{h_x}}{\Delta t\sqrt{g}} \quad , \quad F_h = \frac{U}{\sqrt{gh_x}} \quad , \quad \alpha = \frac{h_x}{h_y} \ . \tag{4}$$

where $(h_x, h_y)$ are the grid spacings in $x-$ and $y-$directions and $\Delta t$ is the time step.

The study of the discrete dispersion relation is carried out by examining its roots with respect to the frequency $\omega$ or, equivalently, $z=e^{i\omega\Delta t}$. A scheme is stable if and only if all the $z-$roots lie within the unit circle of the complex $z-$plane. Of particular interest are the neutrally stable schemes, with roots on the unit circle, for which the discrete dispersion relation may be compared with its continuous counterpart. The difference between the two is identified as numerical dispersion. Some of the conclusions of this stability analysis are summarized in the following. A bi-quadratic B-spline scheme with collocation at the midpoints of the elements serves as the spatial discretization for all of the following algorithms.

*Euler Schemes*

As expected, neither the fully explicit Euler scheme (which is always unstable) nor the fully implicit Euler scheme (which is strongly stable) fulfill our requirements. However, a combination the two, based on implicit treatment of (2) and explicit treatment of (3), results in a neutrally stable time evolution, under the stability condition that $\beta > \beta_{cr}(\alpha, F_h)$. The critical value of the free surface grid number is a weak function of the panel aspect ratio, but it increases dramatically with the grid Froude number, as shown in figure 1. Consequently, the "emplicit" Euler scheme, hereafter referred to as the EE scheme, is expected not to be suitable for simulations of high speed vessels.

*Second-order schemes*

Second order accuracy may be obtained by employing the Trapezoidal scheme on both (2) and (3). The resulting scheme may be shown to be unconditionally neutrally stable. While this is certainly a very desirable property, the full-implicitness of this scheme makes it less attractive. Second order accuracy may also be obtained by an explicit three-step scheme (see e.g. Lambert /2/). Such schemes for (3) combined with the Trapezoidal scheme on (2) were analyzed. A systematic search led to the conclusion that only the Leap-Frog scheme gave neutral stability. This combination of the Trapezoidal and Leap-Frog schemes will hereafter be referred to as the TLF scheme. The neutral stability of the TLF scheme is conditional upon a stability condition similar to the one of the EE scheme, but the critical value of the free surface grid number $\beta_{cr}$ increases much less rapidly with increasing grid Froude number, as shown in figure 1. The comparison of the numerical and analytical dispersion relations is shown in figure 2, where the dispersion relation corresponding to the EE scheme has also been included. The higher order accuracy of the TLF scheme is mainly reflected on the branch of the dispersion relation corresponding to the long wave system. The numerical dispersion of the short waves are mainly governed by errors due to the spatial discretization, and therefore remain relatively insensitive to the order of the time-marching scheme.

*Filtering in space and time*

For all time-marching schemes the discrete dispersion relation deviates violently from its continuous counterpart within the outer portion of the principal wavenumber domain. More importantly, it may be seen in figure 2 that the discrete solution will contain a resonant wave mode which does not exist in the continuous problem. An efficient low-pass filtering technique is employed in order to eliminate the energy associated with length scales smaller than 4-5 panel sizes (see Nakos /1/). Filtering in time is also necessary in connection with the TLF scheme, whose dispersion relation supports a "spurious" neutrally stable branch. This spurious branch contains wave modes of all wavelengths but very high frequency, comparable to the Nyquist frequency $\pi/\Delta t$. This is a phenomenon frequently encountered in connection with the use of the Leap-Frog scheme and for its remedy Asselin /3/ has proposed the following filtering algorithm,

$$\overline{f}^{n} = f^{n} + \gamma\left(f^{n+1} - 2f^{n} + \overline{f}^{n-1}\right) \tag{5}$$

where the overbar signifies filtered quantities. Numerical experimentation showed that effective filtering may be obtained even for very low values of the filter parameter $\gamma$. A surprising result is that the filter (5) may improve the numerical dispersion of the discretization algorithm (figure 2).

*Higher order schemes*

The time-marching scheme based on fourth order Adams-Bashforth for (3) and fourth order Adams-Moulton for (2) was analyzed as a representative of higher-order schemes. This gives a dispersion relation with 5 spurious roots. All of these are, however, damped. The main problem with the scheme is that that the critical value of $\beta$ is significantly higher than for the TLF scheme (2.3 at $F_h = 0$ and 7.5 at $F_h = 2$) and that there exist significant numerical dissipation for $\beta$'s close to the stability limit. For practical applications a beta in the range 10-15 is needed. It is thus concluded that this scheme is not suited for our purpose.

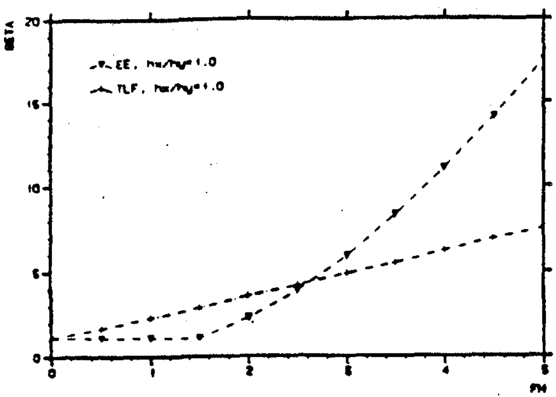

**Figure 1:** Stability Diagram

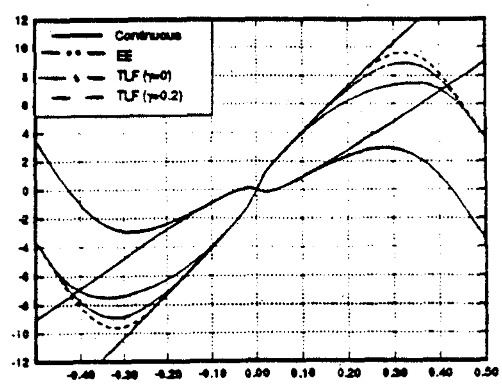


**Figure 2:** Dispersion Relations

*Numerical results*

Figure 3 shows the numerical solution of the model problem at reduced frequency $\tau = \omega U/g = 1$ and compares it with the corresponding continuous solution, obtained by algorithms discussed in Newman /4/. The two schemes give virtually indistinguishable results, thus only the ones due to the EE scheme are shown. It is evident that the wavelength and group velocity of the shorter wave system are smaller than in the continuous solution, as predicted by the comparison of the corresponding dispersion relations. Development of the capability to produce numerical simulations of the linear motion of realistic vessels in waves is currently under way and progress will be reported in the workshop.

REFERENCES

/1/ Nakos, D. E., "Stability and dispersion characteristics of gravity waves over a discrete free surface", (to be published), 1993.

/2/ Lambert, J. D., "Computational methods in ordinary differential equations", Wiley and Sons, 1973

/3/ Asselin, R., "Frequency filter for time integrations", *Monthly Weather Review*, vol. 100, no. 6, 1972.

/4/ Newman, J. N., "The evaluation of free-surface Green functions" , Num. Ship Hydrodynamics Conference, 1985.

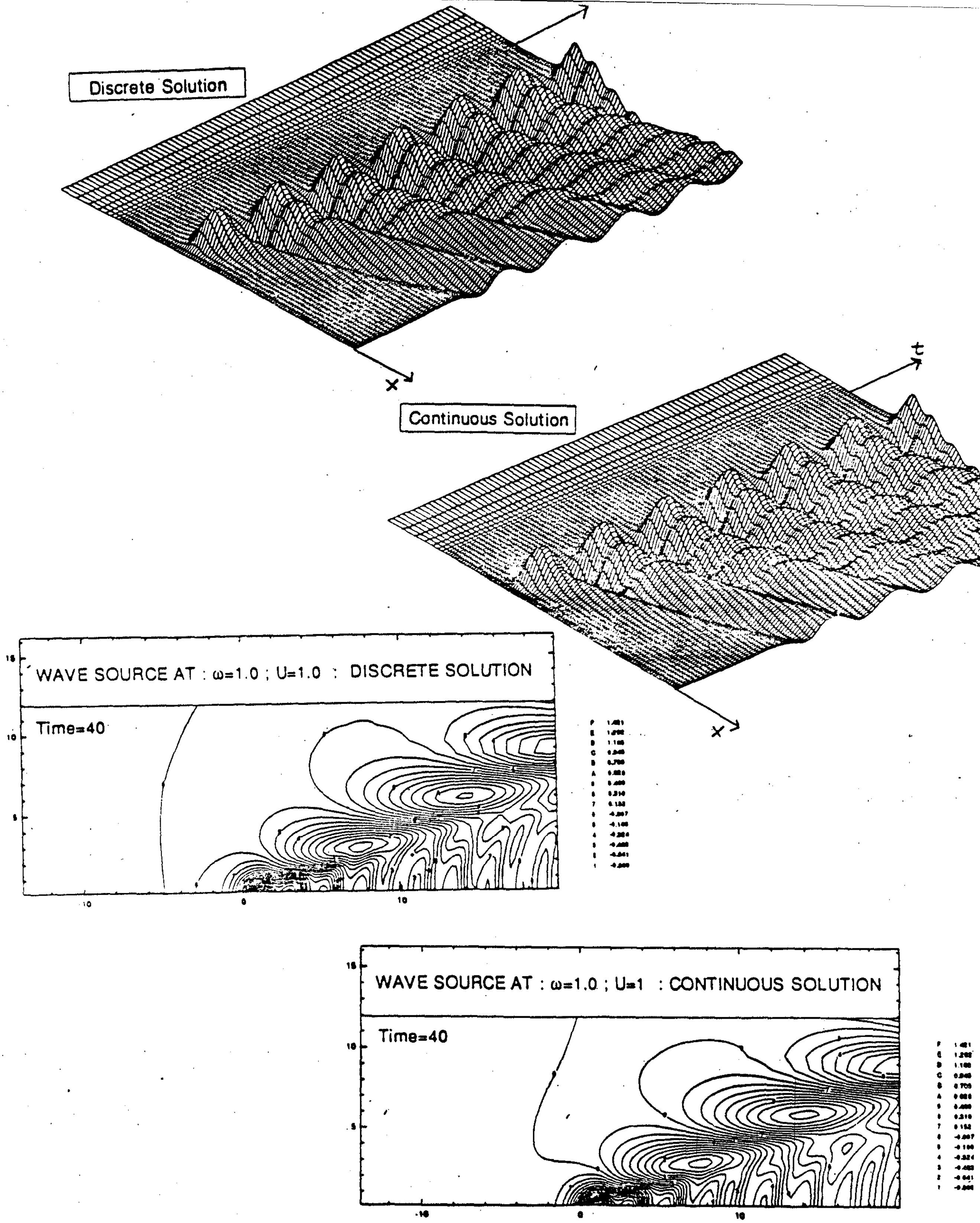


**Figure 3:** Continuous and discrete solutions for the potential over the free surface, due to a submerged point source at reduced frequency $\tau = 1$.