Extended Abstract for the 8th International Workshop on Water Waves and Floating Bodies, St. John's, Canada, May 1993.

THE SHELL FUNCTIONS: A GLOBAL METHOD FOR COMPUTING TIME-DEPENDENT FREE-SURFACE FLOWS

Ronald W. Yeung and Christian A. Cermelli
Department of Naval Architecture & Offshore Engineering
University of California at Berkeley, CA 94720, USA.

Introduction. A standard procedure to solve two-dimensional unsteady potential flows around moving bodies is by use of a time-dependent Green function. Most of the necessary computational time is then spent on the evaluation of the Green-function coefficients. Our present work describes a numerical method that significantly decreases the computational efforts required to solve these problems when a generalized problem is once solved. Its implementation is limited here to the study of submerged bodies, but there is no restriction in principle to its application to surface-piercing bodies.

The shell function method is based on a concept presented by Yeung [1] and initially tested for the special case of axisymetric flows by Lee [4]. A control surface (or shell) surrounding the body is used. Green's theorem and Green's functions are used to derive a relation on the shell which includes all external free-surface effects. This relation is then coupled with a description of the flow inside the shell using the elementary source solution. An accurate solution can then be quickly derived for the whole problem for a new geometry or new instantaneous configuration once the shell functions are developed and stored. The solution for a cylinder heaving near the free surface is given here.

Numerical Procedure. The shell can be any arbitrary surface enclosing the body. The potential flow outside the shell satisfies Laplace's equation. The unsteady Green function given by Finkelstein [5] is used with Green's theorem on a contour defined by the shell S and the free surface. After some manipulations (see e.g. Yeung [2]), the following equation can be obtained:

$$\begin{aligned}\pi\Phi(P,t) &= \int_S \Phi(Q,t)G_\nu(P,Q,0)ds(Q) - \int_0^t \int_S \Phi(Q,\tau)H_{\tau\nu}(P,Q,t-\tau)ds \\ &\quad - \int_S \Phi_\nu(Q,t)G(P,Q,0)ds + \int_0^t d\tau \int_S \Phi_\nu(Q,\tau)H_\tau(P,Q,t-\tau)ds \quad \forall\ P\in S\,, \end{aligned} \tag{1}$$

where

$$G(P,Q,t) = \Re\left[\log(z-\zeta) - \log(z-\bar{\zeta}) - 2\int_0^\infty \frac{dk}{k}(1-\cos\sqrt{k}t)e^{-ik(z-\bar{\zeta})}\right] \tag{2}$$

$$H_\tau(P,Q,t-\tau) = \Re\left[2\int_0^\infty \frac{dk}{\sqrt{k}}\sin\sqrt{k}(t-\tau)e^{-ik(z-\bar{\zeta})}\right] \tag{3}$$

Equation (1) can be considered as a special kind of boundary condition on the shell; it is a combined Volterra and Fredholm equation that relates the velocity potential and the normal velocity on the shell through the unsteady Green function. Following [2], after time and spatial discretization of (1), the following numerical relation can be derived:

$$\left(\Phi_S^{(k)}\right) = [C]\left(u_{ns}^{(k)}\right) + (E^{(k)})\,, \tag{4}$$

where $\left(\Phi_S^{(k)}\right)$ and $\left(u_{ns}^{(k)}\right)$ are vectors representing the potential and normal velocity on the shell at the k-th time step. $[C]$ is a matrix made up of integrals of G and H_τ and $(E^{(k)})$ is a vector that depends on integrals of H_τ and on $(\Phi_S^{(m)})$ and $(u_{ns}^{(m)})$ for $m<k$.

In the second step, Green's theorem with simple source and dipole distribution is applied on the shell and the body of surface B to describe the interior flow. It is assumed that the flow is also potential inside the shell. A linear relation among $(\Phi_S^{(k)})$, $(u_{ns}^{(k)})$, $(\Phi_B^{(k)})$ and $(u_{nB}^{(k)})$ is obtained at every time step. The subscript B

denotes quantities taken on the body. The coefficients of this linear system are simple integrals of $\log r$. Since the motion of the body is prescribed, $(u_{n_B}^{(k)})$ is known. Continuity of the potential and the normal velocity across the shell is applied in a manner similar to Nestegard and Sclavounos [3]. By use of the shell relation (4), $(\Phi_S^{(k)})$ is replaced by $[C](u_{n_S}^{(k)})+(E^{(k)})$. $(\Phi_B^{(k)})$ and $(u_{n_S}^{(k)})$ are the only unknowns left in the linear system. The solution at time step k can easily be obtained by an inversion of the system and then be used to build the new system which will give the solution for the following time step.

Results. The case of a cylinder moving near the free surface has been studied to test the accuracy of the shell method. Comparison has been made with a direct method that uses distribution of unsteady source and dipole on the body. First, a vertical velocity consisting of a step-function in time is given to the cylinder. The location of the body is assumed to be constant; we shall call this case: "linearized body boundary condition (LBBC)".

Figure 1 shows the resulting force on the body. Figure 2 is a plot of the relative error in computing the force using the two methods with the time-step value as a parameter. It demonstrates good convergence characteristics as the time step gets smaller. It also confirms that the shell method gives results comparably accurate to those of the direct method. Similar convergence is observed for both methods as the number of segments on the body and the shell increases. Figure 3 shows the difference in computation time. The intercept at initial time step on the shell curve corresponds to the time required to read the shell coefficients (integrals of H_r) assuming that they have been computed beforehand since the idea of the shell here is to enable one to study any general body moving inside.

Second, a flow about the same oscillating cylinder has been studied using a non-linear body boundary condition (NLBBC): i.e. the *exact* location of the cylinder is taken into account at any time step. Figure 4 shows the force in the linear and non-linear case for oscillation amplitude equal to 20% of the cylinder diameter and a non-dimensional period equal to 6.5. Figure 5 illustrates the convergence in the non-linear case as the time step gets smaller. Again, very favorable convergent characteristics are observed. Figure 6 shows the magnitude of the computational savings when the shell method is used.

Conclusions. The shell method is an extremely useful formulation to solve two-dimensional time-dependent potential flows. Computational savings are substantial because:

- The Green-function coefficients need to be computed only once. Any problem taking place inside the shell can be solved.
- Relatively few segments on the shell are required to model free-surface effects even when a larger number of segments is needed for complex body geometry.
- When a non-linear boundary condition is applied on the body, the total computational time grows as k^2, where k is the time-step index, when using the direct method as compared to k when using the shell method.

The shell method can also be easily extended for free-surface piercing bodies. It may also be possible to implement the shell method in conjunction with a finite-difference scheme. Since the boundary condition on the shell is exact, the radiation condition can be satisfied without having to consider a large domain. Three-dimensional applications could also be implemented.

References

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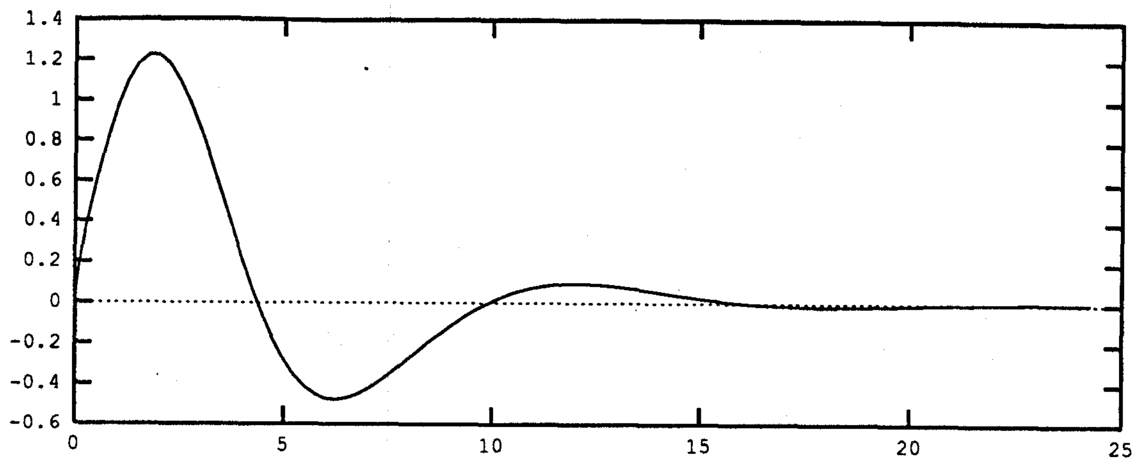

Figure 1: Force on a heaving cylinder with step velocity versus time

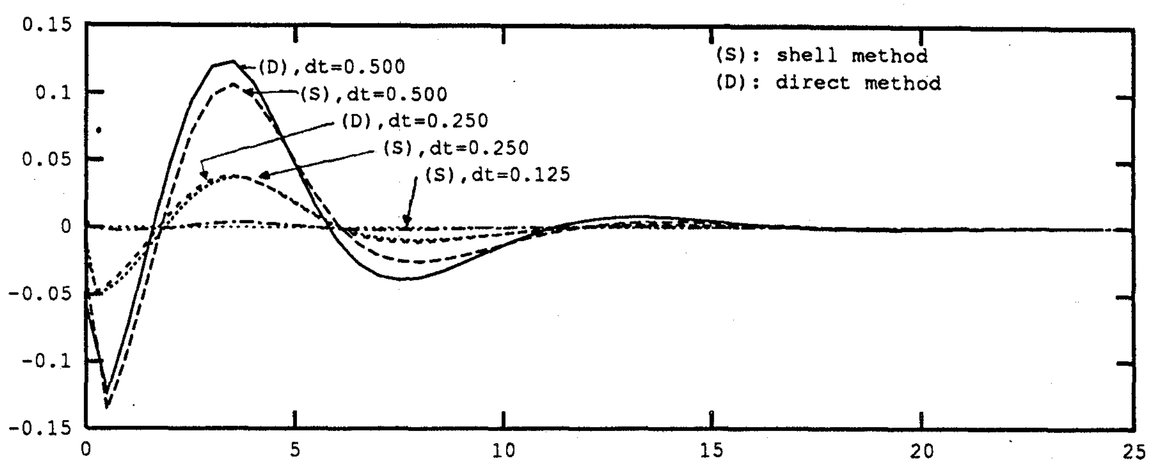

Figure 2: Relative error on the force for different size of time step and a linear body boundary condition (LBBC) versus time

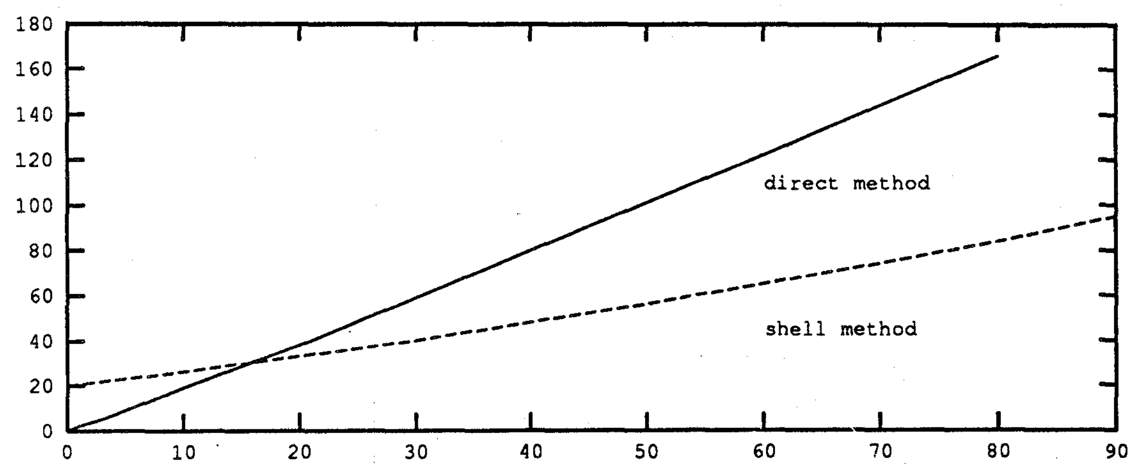

Figure 3: CPU time on IBM workstation using linear body boundary condition versus number of time steps

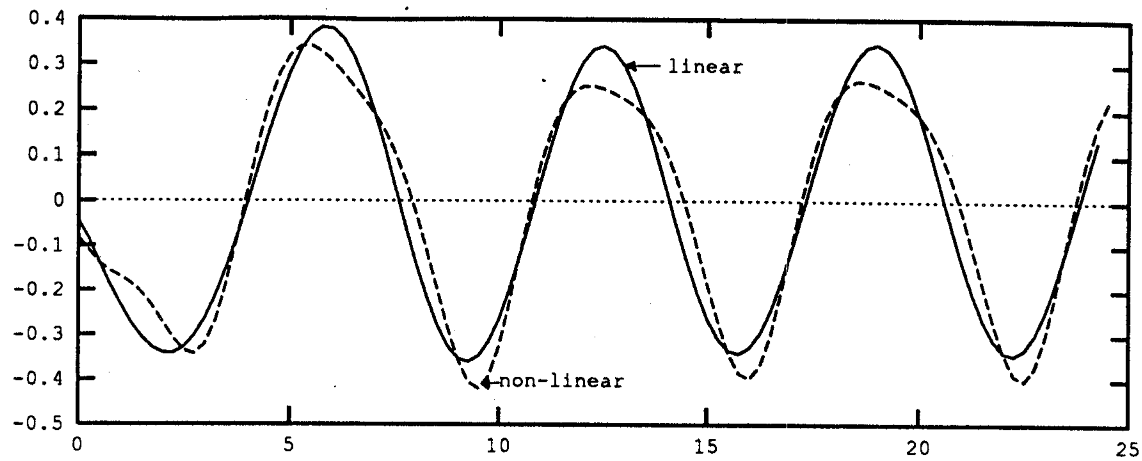

Figure 4: Force on an oscillating cylinder versus time

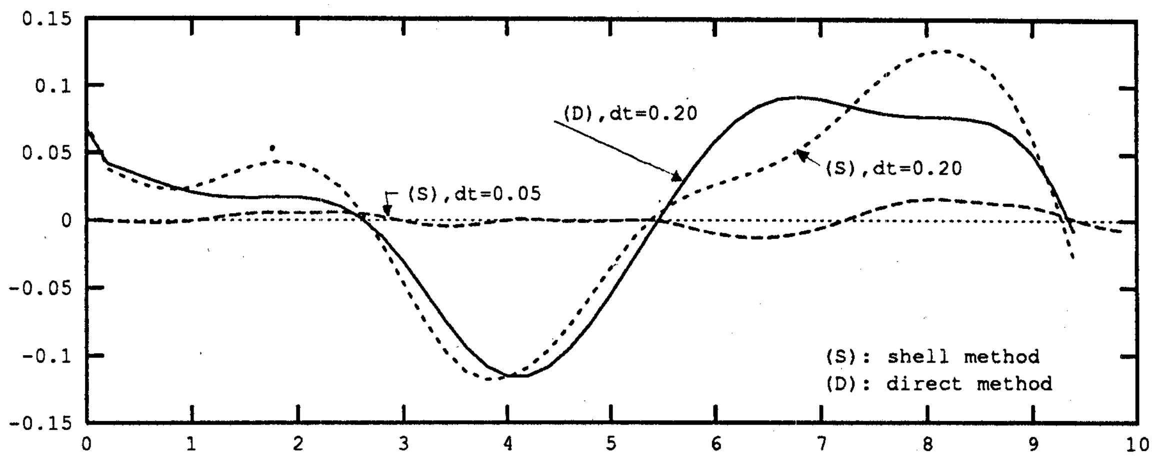

Figure 5: Relative error on the force for different size of time step with a non-linear body boundary condition (NLBBC) versus time

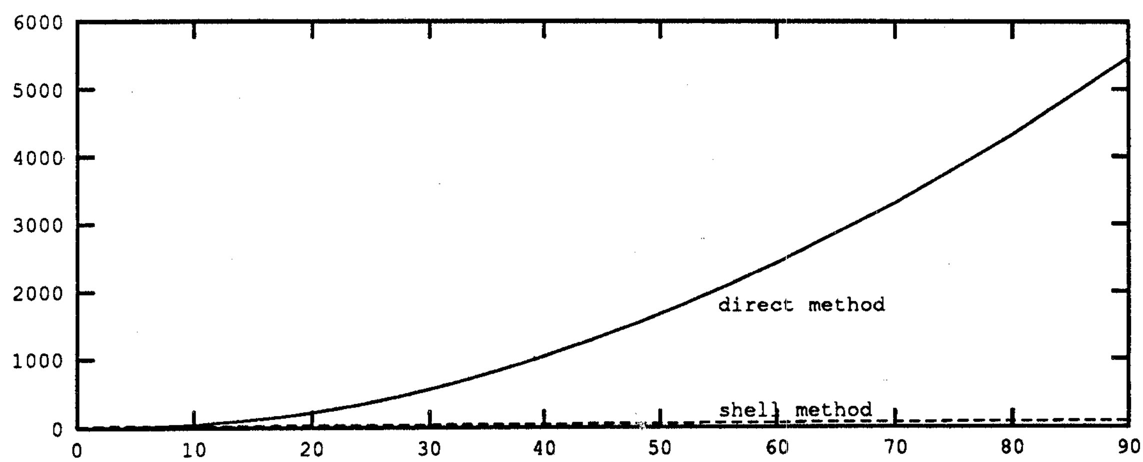

Figure 6: CPU time on IBM workstation with a non-linear body boundary condition versus number of time steps