A 3D Panel Method for Free-Surface Flows around Ships at Small Angles of Yaw

Z. J. Zou

Institut für Schiffbau
Universität Hamburg

Introduction

During the last decade, three-dimensional free surface flow computations using panel methods with either Rankine singularities or Kelvin singularities have been developed rapidly in the field of marine hydrodynamics. Great success has been achieved mainly for the wave resistance problem. There are, however, only a few successful efforts reported with respect to seakeeping and ship manoeuvrability problems.

This paper describes a numerical approach for computing the free surface flow and the hydrodynamic force on yawed ships in deep and/or shallow water. Based on potential theory, a three-dimensional panel method using Rankine sources and a semi-infinite dipole sheet was developed. The nonlinear free surface conditions and the nonlinear Kutta condition at the trailing edge were introduced and linearized. The flow around the ship with small angles of yaw was divided into a symmetric one due to the longitudinal motion of the ship and an antisymmetric one due to the lateral motion; the symmetric flow and the antisymmetric flow were determined separately. Preliminary numerical results obtained for a Wigley model are presented.

Formulation

The flow is assumed to be irrotational and steady. The water depth is assumed to be constant. The absolute velocity of the fluid is represented by the gradient of a perturbation velocity potential $\phi(x,y,z)$ which satisfies Laplace's equation in the fluid domain and the following boundary conditions:

$$(\nabla\phi - \vec{V}_s)\cdot\nabla\zeta = \phi_z \qquad \text{on the free surface,} \qquad (1)$$

$$\zeta = \frac{1}{g}(-\vec{V}_s\cdot\nabla\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi) \qquad \text{on the free surface,} \qquad (2)$$

$$\nabla\phi\cdot\vec{n} = \vec{V}_s\cdot\vec{n} \qquad \text{on the ship's hull,} \qquad (3)$$

$$\phi_z = 0 \qquad \text{on the water bottom,} \qquad (4)$$

$$\nabla\phi = (0,0,0) \qquad \text{at infinity,} \qquad (5)$$

where $\vec{V}_s = (u, v, 0)$ is the speed of the ship, ζ the elevation of the free surface, g the acceleration of gravity and $\vec{n}$ the normal vector on the ship's surface. Moreover, the velocity potential should satisfy a

- radiation condition which states that no waves appear ahead of the ship at a great distance;
- Kutta condition which demands a finite velocity at the trailing edge of the ship.

Eliminating the unknown ζ from (1) and (2) gives the combined condition on the free surface

$$(\nabla\phi - \vec{V}_s)\cdot\nabla(-\vec{V}_s\cdot\nabla\phi + \frac{1}{2}\nabla\phi\cdot\nabla\phi) = g\phi_z. \qquad (6)$$

Following a procedure proposed by Jensen [1], the nonlinear conditions (2) and (6) are linearized about the approximate free surface $z = Z$. The corresponding approximation to the velocity potential ϕ is Φ. It is assumed that the yaw angle is small, so that the velocity potential ϕ can be divided into a symmetric part Φ due to the longitudinal motion and an antisymmetric part φ due to the lateral motion, and that $\nabla\varphi \ll \nabla\phi$ on the free surface.

The longitudinal symmetric flow is first determined by an iterative scheme starting from $Z = 0$ and $\Phi = 0$ like in Jensen's method. Symmetric Rankine sources are distributed on the ship's hull and on a horizontal plane above the undisturbed free surface. The strengths of the singularities are determined so that the corresponding conditions are satisfied at the collocation points on the ship's hull and on the free surface. The radiation condition is satisfied by Jensen's technique of "staggered grids". The shallow water effect is taken into account by the method of images. During the iteration, the equilibrium position of the ship is adjusted, the free-surface elevation and the source strengths are updated. The iterative procedure is continued until the nonlinear free-surface condition is satisfied.

In this paper, a raised point-source method proposed by Jensen is applied for the singularities above the free surface. On the other hand, unlike Jensen's sphere method which is based on the assumption of a closed body, panels with constant source distribution on the ship's hull are used. The velocity induced by the source distribution can be calculated analytically (see Bai & Yeung [2], appendix B). However, since the second- and third-order derivatives of the velocity potential are required for collocation points on the free surface, and since analytic formulae seem to be too complicated, for a collocation point on the free surface the source distribution over a panel on the ship's hull is approximated by one or several point sources, depending on the distance between the panel and the collocation point.

In regard to the antisymmetric flow, we have a three-dimensional lifting problem. In order to determine this flow, an antisymmetric Rankine source distribution on the ship's hull and on a horizontal plane above the undisturbed free surface, and a dipole sheet on the ship's lateral plane and the symmetry plane downstream of the ship are used.

Taking the solution for the longitudinal symmetric flow as an approximation to the total flow, the linearized condition on the free surface is obtained for the antisymmtric velocity potential φ. This condition is used to determine the strengths of the singularities. No nonlinear effects of the antisymmetric velocity potential on the free-surface condition are taken into account.

To solve the lifting problem by means of an inviscid-flow model, it is essential to impose the Kutta condition. Here the Kutta condition is applied indirectly by requiring equal pressure on both sides of the ship along the trailing edge (equal-pressure condition, see Hess [3]). This condition is nonlinear and requires an iterative procedure for exact satisfaction (see e.g. Nakatake et al.[4]). However, by making use of the decomposition of the total flow into a symmetric one and an antisymmetric one, this Kutta condition can be linearized in the form

$$\nabla\Phi \cdot \nabla\varphi - u\,\varphi_x = v\,\Phi_y. \tag{7}$$

(7) was satisfied at the collocation points of the ship's hull panels adjacent to the trailing edge. This condition supplies additional equations for determining the unknown dipole distributions.

In order to ensure that there are enough conditions to determine the unknown singularity strengths, the semi-infinite dipole sheet is divided horizontally into strips. It is assumed that the dipole strength on each strip changes proportionally from zero at the leading edge to the maximal value at the trailing edge and remains constant downstream of the trailing edge, so that there is only one unknown for each strip. This dipole sheet is equivalent to a system of horseshoe vortices, with the vertical bound vortices on the same strip having the same strength.

Numerical results

A numerical investigation was performed for a Wigley model in deep and shallow water. The free-surface elevation, the wave resistance, the steady sinkage and trim, the lateral hydrodynamic force and the yaw moment were calculated. As an example, figure 1 shows the calculated free-surface elevation at the collocation points on the free surface along the Wigley hull in deep water in symmetric flow to allow a comparison with Jensen's results. Figures 2 and 3 show the numerical wave pattern for the Wigley model at 5° yaw in deep and shallow water, respectively. More numerical results will be presented at the workshop.

Acknowledgment

I am very grateful to Prof. H. Söding for his continual guidance during this work. The financial support by DFG (Deutsche Forschungsgemeinschaft) is also acknowledged.

References

[1] Jensen,G., Bertram,V. and Söding,H.(1989): *Ship Wave-Resistance Computations.* 5th International Conf. Numerical Ship Hydrodynamics, Hiroshima

[2] Bai,K.J. and Yeung,R.W.(1974): *Numerical solutions to free-surface flow problems.* Proc. 10th Symposium on Naval Hydrodynamics, Cambridge, Mass.

[3] Hess,J.L.(1974): *The problem of the three-dimensional lifting potential flow and its solution by means of surface singularity distribution.* Comput. Methods Appl. Mech. Eng. 4, pp. 283-319

[4] Nakatake,K., Ando,J., Komura,A. and Kataoka,K.(1990): *On the Flow Field and the Hydrodynamic Forces of an Obliquing Ship.* Trans. West-Japan Society of Naval Architects, No. 80, pp. 1-12 (in Japanese)

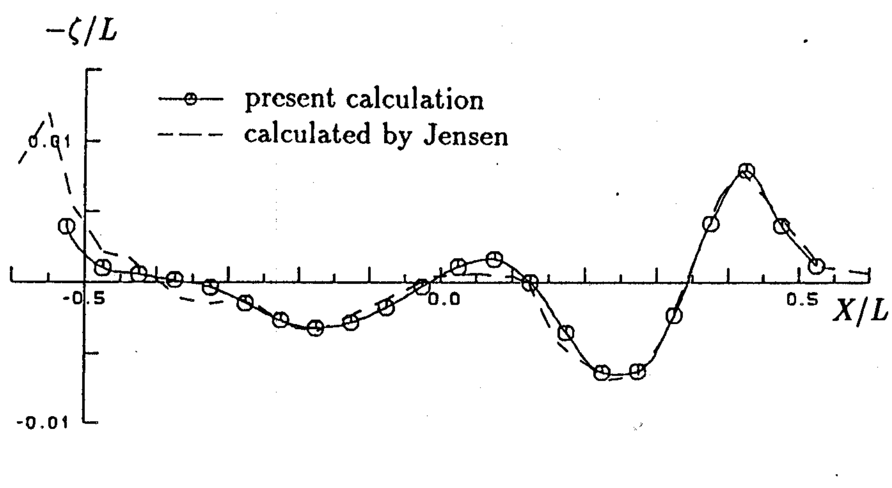

Fig. 1 Wave profile along the Wigley hull
$F_n = 0.267$, $F_{nh} = 0$ ($\frac{h}{T} = \infty$), $\beta = 0°$

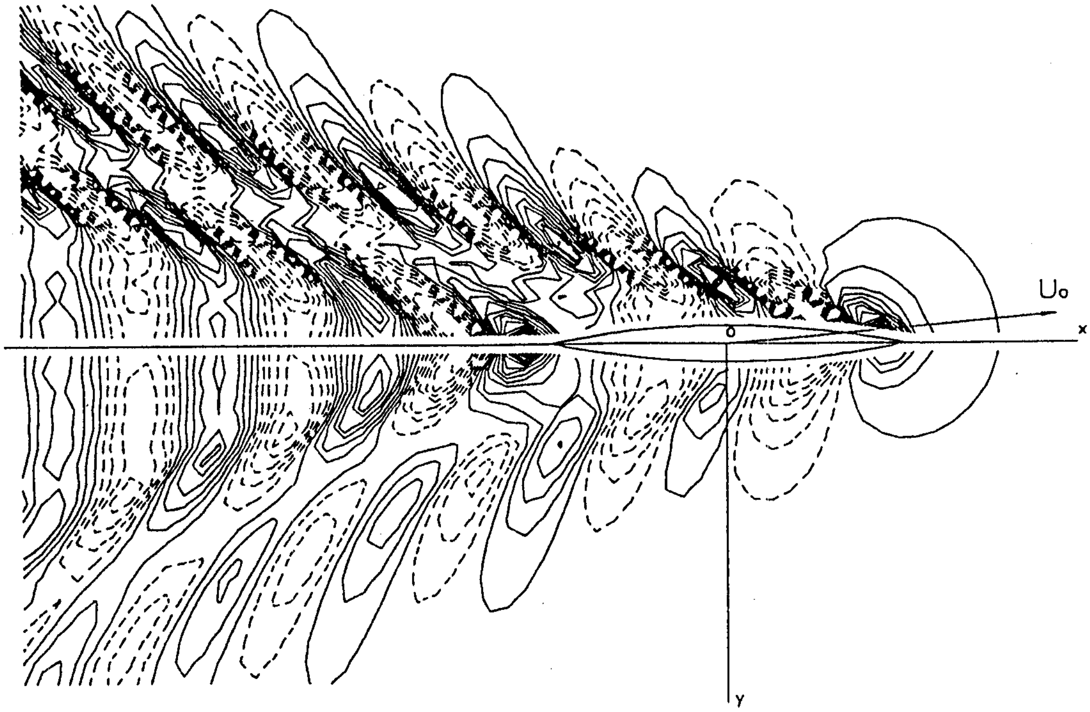

Fig. 2 Contour lines of surface elevation for the Wigley model
$F_n = 0.267$, $F_{nh} = 0$ ($\frac{h}{T} = \infty$), $\beta = 5°$
distance between the contour lines $0.6 \times 10^{-3} L$.

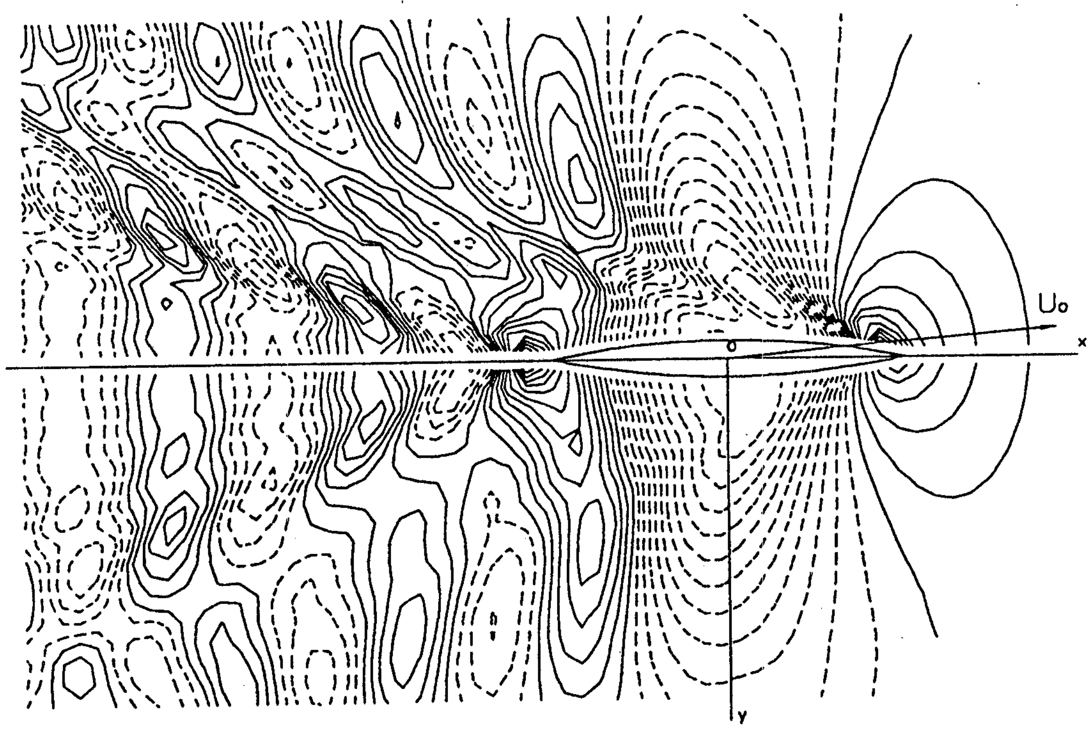

Fig. 3 Contour lines of surface elevation for the Wigley model
$F_n = 0.267$, $F_{nh} = 0.872$ ($\frac{h}{T} = 1.5$), $\beta = 5°$
distance between the contour lines $1.1 \times 10^{-3} L$.