A Slender Body Model of Ringing

Rick Jefferys
Conoco EPT, Houston/London
116 Park Street, London W1Y 4NN

Introduction

Rainey's second order consistent Slender Body theory introduces additional terms to Morison's equation. This paper derives the forces which act on a vertical, moving, non-rotating cylinder such as a TLP or GBS column and reformulates the 'axial divergence correction'. A slender body model for TLP ringing is presented and shown to produce good agreement with the ringing observed in model tests in spite of a number of approximations.

1. Slender Body Theory

The Slender Body equations for a constant section vertical column with zero angular accelerations, lying in the x-z plane, subject to plane waves are derived to illustrate the features important for TLPs. The notation conforms to refs.1 & 2. Note that these equations only apply to a uniform cylinder; additional terms are required to describe forces on a tapering or flaring member and may be derived from the equations for a complete structure (ref.1).

1.1 The vector lateral load per unit length is, ignoring rotational terms:

$$F_l = \rho c[a-g]_T + M[a + (l.Vl)w - du/dt] \quad (1)$$

Subscript T denotes resolution in the transverse direction and $l=[0\ 0\ 1]'$, $w=[w_x\ 0\ w_z]'$. Here ρ is the water density, c is the cylinder cross sectional area, **a** is the vector fluid acceleration, **g** is the vector acceleration due to gravity. Relative velocity is w while the velocity of the structural cross section is denoted by **u**. M is the 2D added mass per unit length matrix with diagonal elements [M_x, M_y, 0]. Note that the axial added mass is taken to be zero since the body is 'slender'.

Other than the Ml.Vl term, F_{ad}, known as the Axial Divergence (AD) term, F_l is exactly the same as the inertial term of the Morison equation. Integration of the 'conventional' Morison terms in (1) to a moving free surface will produce double frequency forces (changing added mass) and moments M_M with second and third harmonic terms (changing moment arm).

The AD term can be simplified and reformulated in a more physically appealing way. V is the fluid velocity gradient matrix so $V_{ij} = \partial v_i/\partial x_j$.

Hence $Vl = [\partial v_x/\partial z\ \partial v_y/\partial z\ \partial v_z/\partial z]$ and $l.Vl = \partial v_z/\partial z$

Then $$F_{ad} = M(l.Vl)w = M.\partial v_z/\partial z.[w_x\ \ 0\ w_z] = [M_x.w_x.\partial v_z/\partial z\ \ 0\ \ 0] \quad (2)$$

Laplace's equation in 2D shows that $\partial v_z/\partial z = -\partial v_x/\partial x$. Substitute $w_x = v_x - u_x$ (u_x = cylinder vel.):

$$F_{ad} = -M_x.\partial v_x/\partial x.(v_x - u_x) = -\partial/\partial x(\tfrac{1}{2}M_x.(v_x - u_x)^2)$$

Thus the AD term is due to the work done in accelerating the fluid as the structure moves through the velocity field at constant speed. It may corrupt some measurements of drag coefficient C_d in spatially non-uniform flows. The AD term is the spatial part of the conventional added mass term.

The double frequency AD term can produce a third harmonic moment M_{ad} if integrated to a moving free surface. The value is crucially dependent on the wave kinematics near the free surface where the velocity gradient is high. Its pitch moment is largest on deep draft TLPs.

1.2 The slender body free surface 'slam' load F_{fs} is due to rate of change of added mass:

$$F_{fs} = \frac{1}{2}\tan\Theta[(t.w)Mw - (t.(l\wedge Mw))(l\wedge w)] \quad (3)$$

The second term is zero in this case since the fluid flow and the column lie in the same plane.

$$t.(l\wedge Mw) = t.([0\ 0\ 1]\wedge[M_x.w_x\ 0\ M_z.w_z]) = [1\ 0\ 0].[\ 0\ M_x.w_x\ 0] = 0$$

The first term simplifies as:

$$(t.w)Mw = ([1\ 0\ 0].[w_x\ 0\ w_z\ ])[M_x.w_x\ \ 0\ 0]' = [M_x.w_x^2\ \ 0\ \ 0]$$

Hence $F_{fs} = [\frac{1}{2}\tan\Theta.M_x.w_x^2\ \ \ 0\ \ \ 0]$

This is third order in wave elevation since tan Θ is the wave slope $\approx$ ka. However, in the very steep waves under consideration, it is not necessarily third order in kinematics and is retained. This 'slam' term can produce third harmonic lateral forces and hence a fourth harmonic moment M_{fs} about the centre of inertia z_c. It may not be accurate when a sharp crest passes a large diameter cylinder and surface curvature is important.

$$M_{fs} = \frac{1}{2}\tan\Theta.M_x\,.w_x^2.(h - z_c)$$

1.3 The load at the immersed end is:

$$F_i = (\frac{1}{2}\,w.Mw - cp)l - (l.w)Mw$$

$$= (\frac{1}{2}\,M_x.w_x^2 - cp)l - w_z[M_x.w_x\ \ \ 0\ \ 0] = [-M_x.w_z.w_x\ \ 0\ \ \frac{1}{2}.M_x.w_x^2 - cp]$$

where p is the total pressure in the incident wave and c is the cylinder cross sectional area. The lateral force is a simple rate of change of added mass term and appears only at immersed ends where fluid is being accelerated, not so at the free surface. It is not clear why the axial force is included while axial added mass forces are not. In this application, the end force is negligible.

2. Pitch moments due to the slender body forces

Pitch/roll is the major contribution to ringing on a deep draft TLP. The pitch (along-wave tilt) response is much more non-linear than cross wave or heave motions. Pitch moments are calculated about the centre of inertia which decouples surge and pitch (the rotation centre for ringing), at

$$z_c = (M.V_{CG} + M_{15}(\omega))/(M + M_{11}\,(\omega))$$

The pitch exciting moment signal can be constructed approximately from experimental wave data, using Airy wave theory to 'Wheeler destretch' the measured free surface motion into the fluid vertically below the measurement point. More accurate methods are desirable but the results presented here are a major improvement on those previously available.

Fluid kinematics are derived in the frequency domain, but for an iterative estimate of slope and v_z. A cutoff frequency of 0.4Hz is imposed to ensure convergence.

$$v_x(t,z) = IFFT(e^{kz}.\omega.FFT(h)) \quad a_x(t,z) \Rightarrow IFFT(e^{kz}.j\omega^2.FFT(h))$$

Loop: $\Theta(t)=a_x(t,0)/(g - a_z(t,0))$ $\quad v_z(t,0) = dh/dt + \Theta.v_x(t,0)$

$$a_z(t,0) = IFFT(j\omega.FFT(v_z(t,0))\ \ \text{end}$$

Then $v_z(t,z) = IFFT(e^{kz}.FFT(v_z(t,0)))$ $a_z(t,z) = IFFT(e^{kz}.j\omega.FFT(v_z(t,0))$

$$\partial v_z(t,z)/\partial z = IFFT(k.e^{kz}.FFT(v_z(t,0)))$$

For numerical stability, the first kinematic point is evaluated 0.1m below the surface. All kinematics/forces are evaluated at fixed distances below the instantaneous free surface and forces are integrated over the instantaneous z coordinates. Platform motion contributions appear relatively insignificant but are included using measured first order velocity and acceleration.

The response of the structure is evaluated by FFT of the moment, multiplied by the transfer function of the pitch mode $G(j\omega) = 1/I_{pp}/(-\omega^2+2c\omega_n j\omega+\omega_n^2)$. Hence $P(t)= IFFT(G(j\omega).FFT(M(t))$. All this analysis is implemented in PC-386 Matlab.

3. Results

Figures 1a,b shows the 'Morison to free surface' (filtered $T<5s$), axial divergence and free surface moments M_M, M_{ad}, M_{fs}, with the driving wave, h, shifted for clarity. All values relate to a single column. Figure 2 shows the corresponding responses in the pitch mode. The AD and free surface terms are significant so shallow draft TLPs ring less as z_c is close to zero. It is also consistent with the experimental observation that raising the CG and hence the centre of inertia z_c, decreases ringing.

Figure 3 shows the measured pitch angles (upper curve) and model response with the moments time shifted, according to the relative displacement of the columns, to represent those acting in 45° waves; the wave is assumed non-dispersive which is reasonable given its high speed ($T_p = 18.4$).

$$M_T = M(t-\tau) + 2\ M(t) + M(t+\tau)$$

The results are sensitive to the lag τ since 1.6s, a typical lag, is half a pitch natural period.

4. Conclusions

In spite of major approximations (no wave diffraction or dispersion, linear destretching), the model performs well and gives significant insight into the ringing problem for the first time. The axial divergence force appears to be a major contributor to ringing and the pitch moment depends on the depth of the centre of inertia. Response is very dependent on the wave speed and direction which determines the sum of excitation from individual columns. A better model of large wave kinematics is desirable and ultimately time domain fully nonlinear diffraction codes will be required. The model also gives a relatively simply diagnostic ($\Theta.v_x^2$ or $h^2.a_x$) for waves likely to cause ringing and hence a means to compare sea and model tank waves for their realism.

5. Acknowledgements

Conoco Inc. and the Heidrun project gave permission to publish and Rod Rainey explained Slender Body Theory.

6. References

1. Rainey, R.C.T., 1989, A new equation for calculating wave loads on offshore structures, J. Fluid Mech., **204** pp 295-324.

2. Eatock Taylor, R., R.C.T. Rainey & D.N. Dai, 'Non-linear hydrodynamic analysis of TLPs in waves: slender body and diffraction theories compared', BOSS, London 1992, pp.569-583.

Discussion:

Newman: If you had done a similar statistical analysis based on a different third order hydrodynamic model, might you have found similar results?

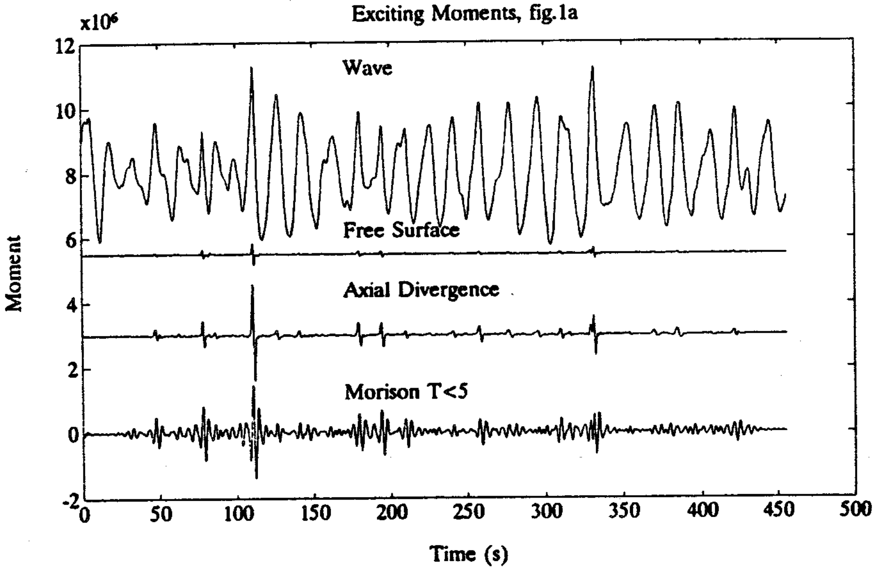
Jefferys: A variety of third order parameters predict the occurrence of ringing events but not their size. Hilbert envelope x wave height has been used by C.T. Stansberg for this purpose. Fig.1a shows that the Morison to free surface (leading term $h^2.a_x$) is in phase with the slam term $\Theta.w_x^2$ (necessarily so in an Airy wave) and also the axial divergence term. However, wave height cubed is not a good predictor of ringing as large waves do not always cause ringing events. It is also worth noting the fair quantitative agreement between the predictions of the Slender Body model and the measured responses.

Grilli: Although the waves are 'long', the video shown indicates that waves are steep and highly non-linear. Hence linear 'low order' wave theory may not be adequate to predict the kinematics, particularly in the crest region far above MWL, so the ringing loads may be poorly predicted. Of course I would favour the development of a fully nonlinear 3D diffraction model but this is a long term goal. In the meantime, a correction of the wave kinematics could be used to account for nonlinear effects, particularly in the crest. The method suggested by Sobey (1991), who analysed high wave kinematics in shallow water by higher order theory and experiment could be used here.

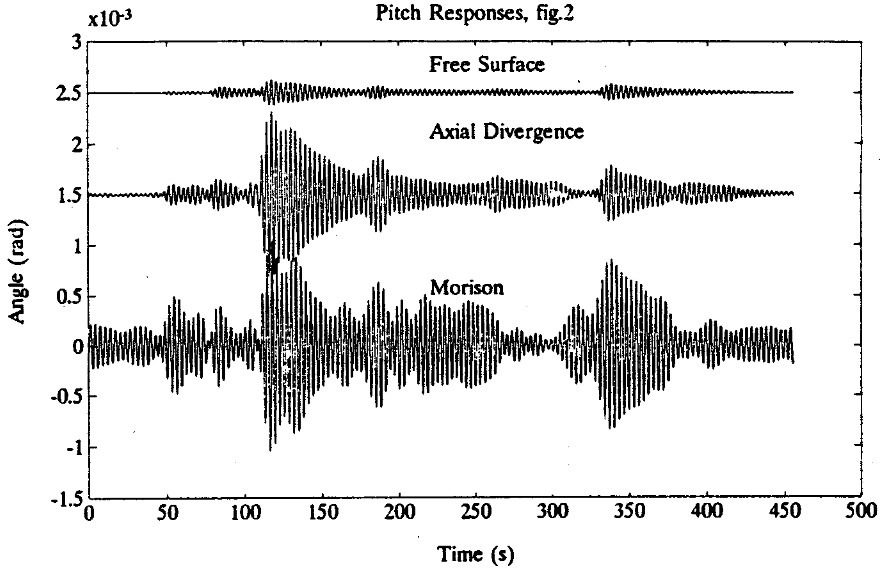
Jefferys: While the iterative method outlined in the paper produces consistent free surface slope, velocity and acceleration values which satisfy the non-linear free surface kinematic condition, the Wheeler 'destretching' method of calculating kinematic values in the fluid leaves a lot to be desired. Additionally, the 'linear' approach to calculation of x velocity and acceleration probably underestimates these values. I fully support your suggestion of a more accurate approach, since this would improve both the evaluation of the local kinematics and of the kinematic quantities up and downstream of the wave probe which is sited near the mid-columns. The active Norwegian JIP on ringing is working on a better kinematic model and we will ensure that Sobey's work is reviewed and incorporated into a recommended practice if appropriate.

Reference: Sobey, R.J., A local Fourier approximation method for irregular wave kinematics', Applied Ocean Research, **14** (1992), pp93-105.

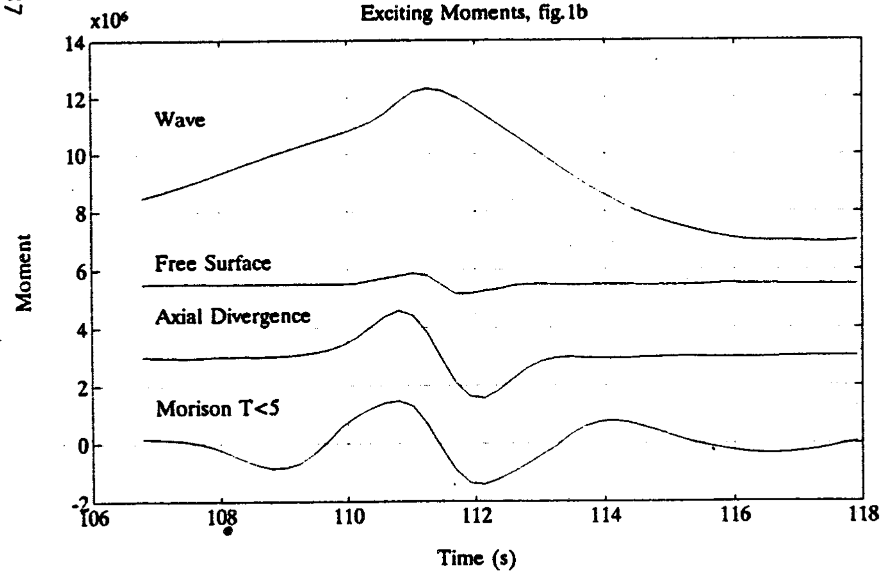
Exciting Moments, fig.1a



Pitch Responses, fig.2



Exciting Moments, fig.1b



Run 301, Predicted and Observed Pitch Response, fig.3