(9th International Workshop on Water Waves and Floating Bodies)
(Kuju, Oita, Japan, 17-20 April, 1994)

Nonlinear Computation of Wave Loads and Motions of Floating Bodies in Incident Waves

Yusong Cao, Robert F. Beck and William W. Schultz
The University of Michigan
Ann Arbor, Michigan 48109, USA

1 Introduction

At several previous WWWFB workshops, we have presented results of nonlinear computations of waves generated by moving disturbances and shown the effectiveness of semi-Lagrangian time stepping combined with the desingularized boundary integral technique in solving nonlinear wave problems. The disturbances considered included moving pressure on the free surface, moving bodies under or on the free surface, and the wavemaker in wave tanks. In these previous studies, the strength or motion of a disturbance was assumed known. In the present study, the desingularized method is extended to problems of floating bodies in incident waves.

In the wave-induced body motion problem, there are some additional difficulties. The first is the force calculation. Unlike the body-induced wave problems where the body motion is given and the calculation of the forces acting on the body is not coupled with the time-stepping of the free surface and therefore can be post-processed, the body motion in the wave-induced body motion must be solved together with the nonlinear computation of the free surface. The forces computed from integrating the pressure over the wetted body surface require the evaluation of $\frac{\partial \phi}{\partial t}$ on the body surface, typically using backward differencing because the potential is known only up to the present time. The backward differencing appears, however, to be inadequate for the time-stepping of the wave-induced body motion problem (Cointe *et al*, 1990). Another difficulty is that the acceleration of the body which is essential for the time-stepping of the body motion can not be expressed explicitly in terms of the position and the velocity of the body.

In the present study, like Cointe *et al.* (1990), we obtain $\frac{\partial \phi}{\partial t}$ on the body directly by solving a similar boundary integral equation since $\frac{\partial \phi}{\partial t}$ is also harmonic in the fluid domain. While the body acceleration at a given time instant is obtained using an iterative procedure. The formulation of the wave-induced body problem and the solution method for a 2-D example are discussed in the following.

2 Formulation and Solution Procedure

We solve the two-dimensional problem of the body motions induced by incident waves generated by a wavemaker in a tank. Potential flow is assumed. The non-penetration condition on the body surface and nonlinear kinematic and dynamic boundary conditions on the free surface are satisfied.

The problem is solved by a time-stepping procedure. At a given time instant, the free surface and body surface positions are known. The potential on the free surface and the normal velocity on the body surface are also known. Thus, a boundary value problem (BVP) for ϕ can be solved to determine the velocity on the free surface and the potential on the body so that the free surface position and its potential can be updated by time integration of the free surface kinematic and dynamic boundary conditions.

Once the flow is solved, the free surface dynamic condition gives $\frac{\partial \phi}{\partial t}$ on the free surface. The normal derivative of $\frac{\partial \phi}{\partial t}$ on the body can be shown to be

$$\frac{\partial}{\partial n}(\frac{\partial \phi}{\partial t}) = \frac{\partial \vec{n}}{\partial t}(\vec{V}_b - \nabla \phi) + \vec{n}\frac{\partial \vec{V}_b}{\partial t} \tag{1}$$

where $\vec{n}$ is the body surface unit normal vector and $\vec{V}_b$ is the velocity of the body surface. Given $\frac{\partial}{\partial n}(\frac{\partial \phi}{\partial t})$, we then have the same type of BVP for $\frac{\partial \phi}{\partial t}$ as for ϕ. At a given instant, the quantities in the right hand side of Eq. (1) are already known except $\frac{\partial \vec{V}_b}{\partial t}$ which is a function of the acceleration of the body and has to be determined in conjunction with the body's equations of motion, given in matrix notation:

$$M\frac{dV_i}{dt} = F_i \tag{2}$$

where M is the generalized mass matrix, V_i is the body motion (sway, heave and roll velocities), and F_i is the generalized force on the body. Let $V_i = \frac{dX_i}{dt}$, and we obtain a state variable representation of the dynamic system including free surface dynamics and body dynamics,

$$\frac{d\vec{X}_f}{dt} = \vec{F}_k(\vec{X}_f, \phi_f, X_i, V_i) \tag{3}$$

$$\frac{d\phi_f}{dt} = F_d(\vec{X}_f, \phi_f, X_i, V_i) \tag{4}$$

$$\frac{dX_i}{dt} = V_i \tag{5}$$

$$\frac{dV_i}{dt} = M^{-1}F_i(\vec{X}_f, \phi_f, X_i, V_i, \frac{dV_i}{dt}) \tag{6}$$

This is a system of first-order ordinary equations with respect to time about the state variables $(\vec{X}_f, \phi_f, X_i, V_i)$, where $\vec{X}_f$ is the location of the free surface nodes and ϕ_f the potential at the nodes. Eqs. (3) and (4) are the kinematic and dynamic free surface conditions.

When a standard explicit method (*e.g.* Runge-Kutta method) is used for the time-stepping, the time derivatives of the state variables are required at a given time instant and can be determined by Eqs. (3)-(6) since the state variables are known. One difficulty in the wave-induced body motion problem is, as mentioned earlier, that the body acceleration $\frac{dV_i}{dt}$ is not explicitly expressed in terms of the state variables as seen in Eq.(6). An iterative procedure is required to obtain $\frac{dV_i}{dt}$. In our calculation, at a given time instant, the following iterative procedure is employed,

1. solve the BVP for ϕ, and calculate $\frac{\partial \phi_f}{\partial t}$, $\frac{d\phi_f}{dt}$ and $\frac{d\vec{X}_f}{dt}$ using the free surface dynamic and kinematic conditions;

2. guess $\frac{dV_i}{dt}$ (its value at the last time step);

3. calculate $\frac{\partial \vec{V}_b}{\partial t}$ using its relation to $\frac{dV_i}{dt}$, and then $\frac{\partial}{\partial n}(\frac{\partial \phi}{\partial t})$ using Eq.(1);

4. solve the BVP for $\frac{\partial \phi}{\partial t}$, and calculate $\frac{\partial \phi}{\partial t}$ on the body surface;

5. compute the pressure on the body surface using the Bernoulli equation, and integrate the pressure to get the force F_i;

6. correct $\frac{dV_i}{dt}$ using Eq. (6);

7. if the difference between the new $\frac{dV_i}{dt}$ and the old $\frac{dV_i}{dt}$ is larger than the error tolerance, replace the old one with the new one and go to 3; otherwise, stop the iteration.

Since the influence matrix for $\frac{\partial \phi}{\partial t}$ is the same as that for ϕ, no additional matrix setup and inversion is necessary in solving the BVP for $\frac{\partial \phi}{\partial t}$. Therefore, the above iteration procedure does not result in a significant increase in computational effort. The effective desingularized boundary integral method is used to solve the BVP.

1 Preliminary Numerical Results

The preliminary results here are for a floating rectangular box placed at the middle of a computational wave tank. The simulation starts with hydrostatic equilibrium. An incident wave is generated by a pneumatic wavemaker at the left end of the tank. To reduce wave reflection from the other end, an artificial wave absorber is placed in a region near the end, as shown in Fig. 1.

The problem is made non-dimensional based on the draft of the box in the static equilibrium, gravity, and the water density. In this example, the length of the tank is 40, the beam of the box is 2 and the water depth is 2. The box center of gravity is 0.3 below the still water surface, and the moment of inertia about the center of gravity is 2. The pressure distribution of the pneumatic wavemaker is,

$$p(x,t) = P_o sin(\omega t) cos(\frac{\pi}{2}x) H(t) H(x) H(1-x),$$

Where H is the Heaviside step function, and x is measured from the left wall.

Fig. 2 shows the horizontal force, the vertical force (buoyancy subtracted), and the roll moment acting on the box as functions of time. The time history of the body motion (relative to its initial static position) is shown in Fig. 3. The nonlinear effect is clearly seen in these results. The horizontal drifting of the body can be seen in Fig. 3 as expected. One interesting observation is that the roll moment and the roll motion are quite irregular. It is not quite clear at the present moment whether this is due to nonlinear free surface effects, shallow water effects, or effects of the body geometry and inertia. Further investigation into these effects is under way. We also plan to compare our results to a case in Cointe *et al.* (1990).

2 References

Cao, Y., W.W. Schultz, and R.F. Beck, Proceedings of 4th, 5th, 6th, 7th and 8th International Workshop on Water Waves and Floating Bodies.

Cointe, R., P. Geyer, B. King, B. Molin and M. Tramoni, *Nonlinear and Linear Motions of a Rectangular Barge in a Perfect Fluid*, Proc. of 18th Symposium on Naval Hydrodynamics, Ann Arbor, Aug. 19-24, 1990.

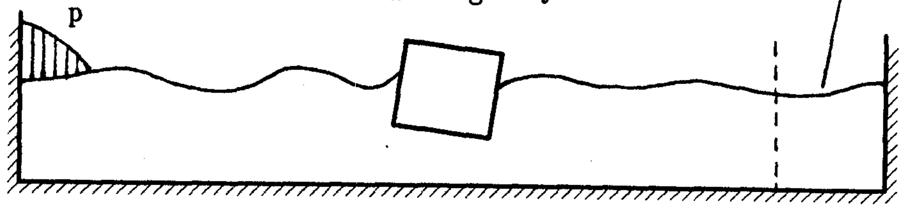

Fig.1 Sketch of the wave-induced body motion in a wave tank

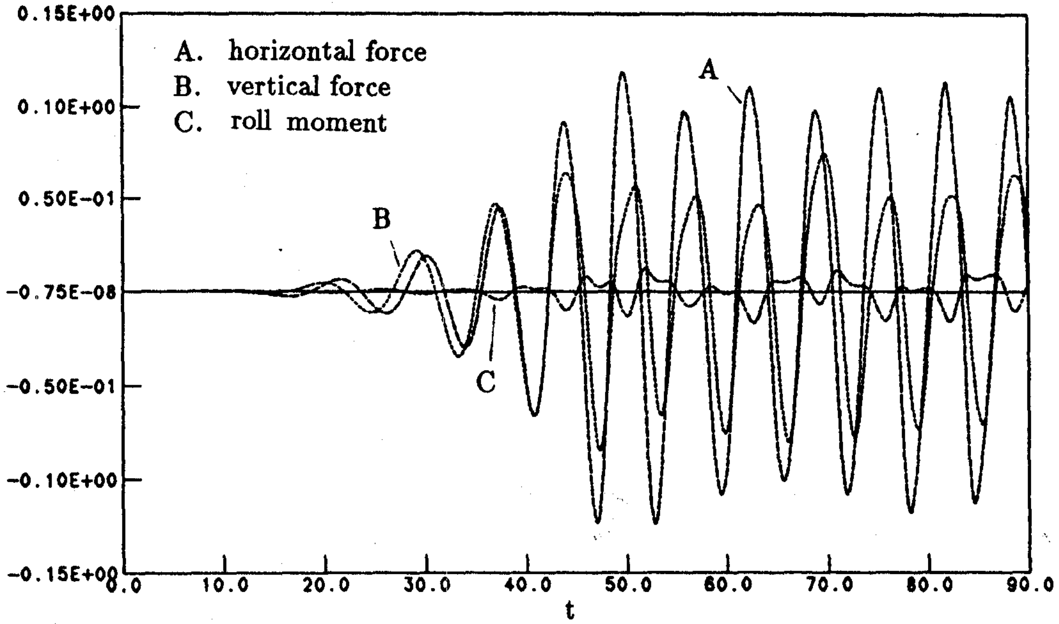

Fig.2 Forces acting on the body

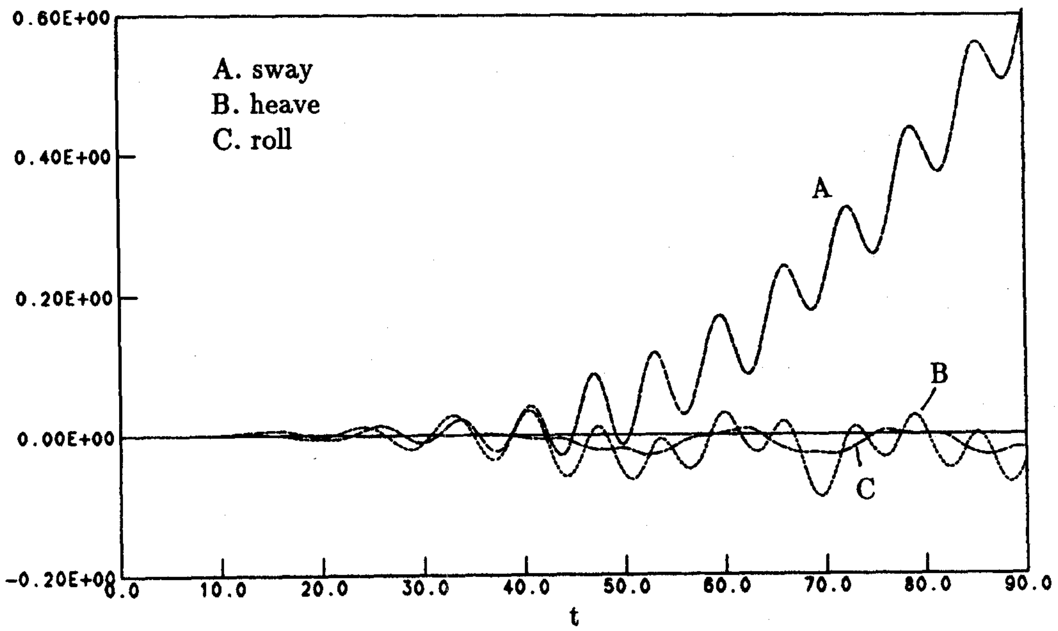

Fig.3 Motion of the body (relative to its static position)

DISCUSSION

Wang B.: There are several methods to impose the radiation condition. In the wave absorbing zone method, the zone size and absorbing efficiency depend on the characters of incoming wave. How did you choose the size of wave absorbing zone?

Cao Y.: The wave absorbing zone is one wavelength.