(Abstract for the 9th Intl. Workshop on Water Waves and Floating Bodies, April 1994)

A New Green-Function Method for the 3-D Unsteady Problem of a Ship with Forward Speed

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Introduction

Recently several numerical methods have been developed for the 3-D unsteady problem of a ship advancing in waves, e.g. the Green-function method [1], the free-surface Rankine panel method [2], and the combined boundary-integral equation method [3]. Notwithstanding fairly good numerical results, a number of moot questions remain in those methods, such as the satisfaction of the radiation condition and the treatment of the so-called line-integral term.

These are investigated theoretically in the present paper, which is based on Green's second identity with the Green function satisfying the classical linearized free-surface condition. Since the effects of the double-body flow are taken into account in the formulation, the resultant integral equation for the velocity potential includes integrals over the free surface. However, the use of Gauss' theorem proves that there exists no water-line integral and the integrals over the free surface can be confined to a relatively smaller region near the ship.

Also shown with the proposed free-surface condition is that the Timman-Newman relation must be satisfied without ambiguity concerning the line-integral term, and the same is true of the Haskind-Newman relation, in which the Kochin function is newly defined to contain the free-surface integral.

Investigations proceed to the energy and momentum conservation principles. The outcome is that the energy flux through the free surface is exactly zero, while the momentum flux is not zero but negligible. This can be regarded as a proof for the appropriateness of the proposed free surface condition and the solution method.

The free-surface condition

We consider the free-surface condition first, because the final result is slightly different from existing ones in published papers so far.

A ship is advancing at constant forward speed U along the positive x-axis and oscillating in waves with encounter frequency ω. The z-axis is positive downward, and the fluid is assumed ideal with irrotational motion.

When the total velocity potential is decomposed into the double-body flow Φ, the steady wave flow $\overline{\phi}$, and the unsteady wave flow ϕ, the unsteady pressure (P_u) and wave elevation (ζ_u) linear in ϕ can be given from Bernoulli's equation, in the form

$$-\frac{P_u}{\rho} = \frac{\partial \phi}{\partial t} + \nabla\Phi\nabla\phi - gz + O(\overline{\phi}\phi,\ \phi^2) \tag{1}$$

$$\zeta_u = \frac{1}{g}\Big(\frac{\partial \phi}{\partial t} + \nabla\Phi\nabla\phi\Big) + O(\overline{\phi}\phi,\ \phi^2) \tag{2}$$

Here ρ is the fluid density and g is the acceleration of gravity.

The unsteady free surface condition must be given by the zero substantial derivative of the unsteady pressure on the unsteady wave surface. Therefore we write

$$\Big\{\frac{\partial}{\partial t} + \nabla(\Phi + \overline{\phi} + \phi)\nabla\Big\}\ P_u = 0 \qquad \text{on } z = \zeta_u \tag{3}$$

Using Taylor-series expansion of the velocity potential and its derivatives about the undisturbed free surface $z = 0$ and neglecting higher-order terms, we have

$$\frac{\partial^2\phi}{\partial t^2} + 2\nabla\Phi\nabla\frac{\partial\phi}{\partial t} + \nabla\Phi\nabla(\nabla\Phi\nabla\phi) - g\frac{\partial\phi}{\partial z} - \frac{\partial^2\Phi}{\partial z^2}\Big(\frac{\partial\phi}{\partial t} + \nabla\Phi\nabla\phi\Big) = 0 \qquad \text{on } z = 0 \tag{4}$$

Since Φ is the double-body flow, Φ satisfies the rigid-wall boundary condition on $z = 0$, and thus ∇ means only the horizontal gradient.

With expressions $\phi = \Re[\varphi(x,y,z)\exp(i\omega t)]$, $\Phi = U\phi_S = U(-x+\chi_S)$, the above free-surface condition can be expressed as

$$\begin{aligned}\frac{\partial\varphi}{\partial z} + K\varphi \;&+\; 2i\tau\frac{\partial\varphi}{\partial x} - \frac{1}{K_0}\frac{\partial^2\varphi}{\partial x^2}\\ &-\; 2i\tau\left(\nabla\chi_S\nabla\varphi + \frac{1}{2}\varphi\nabla^2\chi_S\right)\\ &-\; \frac{1}{K_0}\left\{\nabla\Big(\nabla\phi_S(\nabla\chi_S\nabla\varphi)\Big) - \nabla\left(\nabla\chi_S\frac{\partial\varphi}{\partial x}\right)\right\} = 0 \quad \text{on } z = 0\end{aligned} \tag{5}$$

where $K = \omega^2/g$, $\tau = U\omega/g$, $K_0 = g/U^2$.

It is easy to confirm in (5) that the classical free-surface condition is recovered if the steady perturbation potential χ_S is omitted. Meticulously speaking, corresponding free-surface conditions used in the Rankine panel method include one additional term which comes from the substantial derivative of the steady hydrodynamic pressure. But it should be emphasized that, as will be made clear, the present condition (5) provides rational results in that the energy-conservation principle is exactly satisfied and the Timman-Newman and Haskind-Newman relations hold.

The Integral equation

We start with Green's theorem:

$$C\,\varphi(x,y,z) = \iint_{S_B+S_F+S_\infty}\left(\frac{\partial\varphi}{\partial n} - \varphi\frac{\partial}{\partial n}\right) G(x,y,z;\xi,\eta,\zeta)\,dS(\xi,\eta,\zeta) \tag{6}$$

Here C is a constant and S_B, S_F, S_∞ denote the body surface, the free surface, and the infinite rediation surface, respectively. $G(x,y,z;\xi,\eta,\zeta)$ is the 3-D translating and oscillating Green function satisfying the classical free-surface condition. We note that the integral in (6) must be performed with respect to the integration point (ξ,η,ζ).

Let us first consider the integral over the free surface. Although the transformation is a little complicated, if Gauss' theorem is utilized effectively after substituting (5) for $\partial\varphi/\partial n$ and the classical free-surface condition for $\partial G/\partial n$, the following relation can be obtained:

$$\begin{aligned}\mathcal{F} \;=\; &-\Big[\int_{C_\infty} + \int_{C_B}\Big]\left\{2i\tau\,\varphi G + \frac{1}{K_0}\Big(\varphi\frac{\partial G}{\partial\xi} - G\frac{\partial\varphi}{\partial\xi}\Big)\right\} d\eta\\ &+\int_{C_B}\left\{2i\tau\,\varphi G + \frac{1}{K_0}\Big(\varphi\frac{\partial G}{\partial\xi} - G\frac{\partial\varphi}{\partial\xi}\Big)\right\}\frac{\partial\chi_S}{\partial n}\,d\ell\\ &-\frac{1}{K_0}\int_{C_B}\Big(\varphi\nabla G - G\nabla\varphi\Big)\nabla\chi_S\frac{\partial\phi_S}{\partial n}\,d\ell\\ &-2i\tau\iint_{S_F}\left(\nabla\chi_S\nabla G + \frac{1}{2}G\nabla^2\chi_S\right)\varphi\,dS\\ &+\frac{1}{K_0}\iint_{S_F}\left\{\nabla\Big(\nabla\phi_S(\nabla\chi_S\nabla G)\Big) - \nabla\Big(\nabla\chi_S\frac{\partial G}{\partial\xi}\Big)\right\}\varphi\,dS\end{aligned} \tag{7}$$

Here the body boundary condition for the double-body flow is given by

$$\frac{\partial\phi_S}{\partial n} = 0\,, \quad \frac{\partial\chi_S}{\partial n} = n_x\,, \quad n_x\,d\ell = d\eta \qquad \text{on } C_B \tag{8}$$

Thus we can see that the so-called line-integral term along the periphery of the water-plane area C_B is cancelled out by the inclusion of the steady perturbation potential χ_S, but instead the integral over the free surface does not vanish. However, this integral is expected to decay rapidly with increasing distance from the body, because it contains the spatial derivatives of χ_S, and χ_S itself is $O(1/r^2)$ in the 3-D problem. Moreover in the present case, there is no ambiguity concerning the radiation condition; in fact, since the Green function satisfies the radiation condition, we can easily confirm that

$$\iint_{S_\infty}\Big[\frac{\partial\varphi}{\partial n}G-\varphi\frac{\partial G}{\partial n}\Big]dS=\int_{C_\infty}\Big\{2i\tau\,\varphi G+\frac{1}{K_0}\Big(\varphi\frac{\partial G}{\partial\xi}-G\frac{\partial\varphi}{\partial\xi}\Big)\Big\}d\eta+\begin{cases}0\\ \varphi_0\end{cases}\tag{9}$$

where the first case applies to the radiation potentials and the second to the diffraction potential $\varphi_D=\varphi_0+\varphi_7$, with φ_0 and φ_7 the incident-wave and scattering potentials respectively. Namely, (9) cancels out exactly the first term along C_∞ appearing in (7).

In summary, a new integral equation can be expressed as follows:

$$\begin{aligned}C\,\varphi_j(x,y,z)\;+\;&\iint_{S_B}\varphi_j\frac{\partial G}{\partial n}\,dS\\&+2i\tau\iint_{S_F}(\nabla\chi_S\nabla G+\frac{1}{2}G\,\nabla^2\chi_S)\,\varphi_j\,dS\\&-\frac{1}{K_0}\iint_{S_F}\Big\{\nabla\Big(\nabla\phi_S\,(\nabla\chi_S\nabla G)\Big)-\nabla\Big(\nabla\chi_S\frac{\partial G}{\partial\xi}\Big)\Big\}\,\varphi_j\,dS\\&=\begin{cases}\displaystyle\iint_{S_B}\frac{\partial\varphi_j}{\partial n}G\,dS & \text{for } j=1\sim 6\\[2ex] \varphi_0(x,y,z) & \text{for } j=D\end{cases}\end{aligned}\tag{10}$$

A key to the success in solving the above integral equation is how efficiently and accurately the Green function can be computed. The steepest-descent method developed by Iwashita & Ohkusu [1] may be the most reliable, and I have already succeeded partly in accelerating the computations by modifying their method. At present, the development of computer program is in progress, and numerical results based on (10) will be shown in the presentation at the Workshop.

Relations among hydrodynamic forces

The Timman-Newman and Haskind-Newman relations were originally proved under the classical free-surface condition, but due to the neglect of the water-line integral, their proofs are not exact. Presented here is the exact form of the Timman-Newman and Haskind-Newman relations when the double-body flow is included in the free surface condition.

First let us consider the 'transfer' function of the radiation force in the i-th direction due to the j-th mode of motion, which can be written in the form

$$T_{ij}^{+}=\rho\iint_{S_B}\Big\{\varphi_j+\frac{U}{i\omega}\nabla\phi_S\nabla\varphi_j\Big\}\,n_i\,dS=\rho\iint_{S_B}\varphi_j\Big(n_i-\frac{U}{i\omega}m_i\Big)\,dS\tag{11}$$

Here Tuck's theorem has been used, resulting in the use of the so-called m-term. It should be noted that the speed-dependent hydrodynamic restoring force, which may experimentally be analyzed as part of the added mass, is not included in (11).

After introduction of the reverse-flow radiation potential ψ_j satisfying the following body-boundary condition

$$\frac{\partial\psi_j}{\partial n}=n_j-\frac{U}{i\omega}m_j\qquad\text{on } S_B\,,\tag{12}$$

we consider the equation:

$$T_{ij}^{+}-T_{ji}^{-}=\rho\iint_{S_B}\Big(\varphi_j\frac{\partial\psi_i}{\partial n}-\psi_i\frac{\partial\varphi_j}{\partial n}\Big)\,dS\tag{13}$$

By virtue of Green's theorem, the above integral can be transformed into the integrals over the free surface and control surface at a large distance from the body. Then using the same technique as in deriving the integral equation (10), we can prove the relation:

$$T_{ij}^{+} = T_{ji}^{-} \tag{14}$$

It must be emphasized that there are no ambiguities in the proof (of course no water-line integral). We can see from (14) that no linear term in U exists in the case of $i = j$. A couple of examples of the experiments supporting this relation will be shown in the presentation.

Similar transformation can be used to show the Haskind-Newman relation. In this case, however, we should note that the incident-wave potential φ_0 satisfies the classical free-surface condition as in the Green function, and the diffraction potential $\varphi_D = \varphi_0 + \varphi_7$ (not ϕ_7 itself) satisfies the proposed free surface condition (5).

After somewhat lengthy reduction using Gauss' theorem, we can show the final result:

$$E_j = \rho g a \frac{\omega}{\omega_0} H_j^{-}(k_0, \beta + \pi) \tag{15}$$

where

$$\begin{aligned} H_j^{-}(k, u) = & \iint_{S_B} \left(\frac{\partial \psi_j}{\partial n} \mathcal{G} - \psi_j \frac{\partial \mathcal{G}}{\partial n} \right) dS \\ & + 2i\tau \iint_{S_F} (\nabla \chi_S \nabla \mathcal{G} + \frac{1}{2} \mathcal{G} \nabla^2 \chi_S) \psi_j \, dS \\ & + \frac{1}{K_0} \iint_{S_F} \left\{ \nabla \Big(\nabla \phi_S (\nabla \chi_S \nabla \mathcal{G}) \Big) - \nabla \Big(\nabla \chi_S \frac{\partial \mathcal{G}}{\partial \xi} \Big) \right\} \psi_j \, dS \end{aligned} \tag{16}$$

$$\mathcal{G}(k, u) = \exp\{-k\zeta + ik(\xi \cos u + \eta \sin u)\} \tag{17}$$

Here a, ω_0, k_0 denote the amplitude, circular frequency, and wavenumber of the incident wave respectively, and β the angle of incidence relative to the positive x-axis.

(15) is formally idential to the relation originally proved with the classical free-surface condition. However the definition of the Kochin function is markedly different; the present Kochin function, which is of course given from the asymptotic analysis of (10) with $C = 1$, does not include the water-line integral but integrals over the free surface instead.

Principles of energy and momentum conservation

Because of the paucity of space, we can not describe the details here, but the outcome of the energy-conservation analysis with the modified free-surface condition (5) is that there is no energy flux through the free surface and thus the rate of work done by the body must coincide with the rate of energy flux across the control surface far from the ship.

However in the analysis of the rate of change of momentum flux, we have found that the momentum flux through the free surface is not zero and the remaining free-surface integral includes only the second derivations of the steady perturbation potential which decay very rapidly in the order of $O(1/r^4)$ with increasing distance from the body.

Except for this term, we can say that the pressure integral over the wetted body surface up to the unsteady free surface $z = \zeta_u$ must be equal to the rate of momentum flux across the control surface at infinity. In other words, we can apply Maruo's formula to the prediction of the added resistance in waves, with the Kochin function newly defined as in (16) taking account of the effects of steady disturbance on the free surface.

References

[1] Iwashita, H. and Ohkusu, M.: J.S.N.A.J., Vol.166, pp.187-205, 1989

[2] Nakos, D. E.: Ph.D Dissertation of MIT, USA, 1990

[3] Iwashita, H., Lin, X. and Takaki, M.: Trans. West-Japan S.N.A., No.85, pp.37-55, 1992

DISCUSSION

Eatock Taylor. R.: Can you comment on the relation between your integral equation and the one published some years ago by Palm and Grue, and the consequences such as Timmian-Newman relations?

Kashiwagi M.: You must be saying the paper published in J.F.M. Vol.227 in 1991. Their paper is based on the small speed assumption, and thus the quadratic terms in U are discarded. I believe that the present paper is an extension to the general forward-speed problem.

Yasukawa. H.: You mentioned that the term which comes from the substantial derivative of the steady hydrodynamic pressure can be neglected. But I think that this way is inconsistent. Also I would like to have a comment as your free surface condition is about the same as our condition based on double body flow.

Kashiwagi M.: A point in the present paper is that the validity of the free-surface condition should be judged by whether or not the energy flux through the free surface is zero. In that sense, the fact that your free surface condition is different from the present one implies that the energy flux across the free surface is not zero in your case.

Bingham. H.: You have panels distributed on the $z = 0$ plane, which means that the source and field points will coincide on the free-surface. I think that there will be another (in addition to $\frac{1}{r} + \frac{1}{r'}$) singularity in your Green function in this situation. How do you propose to handle this?

Kashiwagi M.: I didn't pay a special care to that difficulty, which might be a reason of some eccentricities I found in numerical results. When both of the field and source points are on the same panel, what I did is that firstly the local coordinate system was transformed into the polar coordinate system with the origin taken at the field point, and then the Gauss-Legendre quadrature was employed. I guess the analytical investigation on the singularity might do the trick.

Newman. J.N.: Expanding on Dr. Bingham's question, the Green function for $U = 0$ has the form $G = 1/R + 1/R' + 2K \log(z + \zeta + R')$ as $R' \to 0$. When collocating at points on the free surface ($z = \zeta = 0$), care is required particularly on the principal diagonal (self-influence of the panel). We use analytic integration over the panel in this case, as described in Newman and Sclavounos, BOSS '88, Trondheim. It is not obvious to me how this will be manifested when $U > 0$, but it seems likely to be worse rather than better!

Kashiwagi M.: Thank you for your information on the singularity, which is surely useful in checking the program. Regarding the numerical method used for the self-influence term of the panel, please see the reply to Dr.Bingham's question.

Yue D.K.P.: I just want to point out that in view of the second derivative of the Green function on the free surface which has a log-like singularity (in addition to the Cauchy-like Rankine singularity), special care must be needed in evaluating the free-surface kennel in your new integral equation. Can you explain how you treat that in your program?

Kashiwagi M.: As I replied to Dr.Bingham's question, I used the numerical integration with the coordinate transformation when both the field and source points are on the same panel. I will check the program with all of your indications taken into account.