Shallow-water entry problem

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The plane problem of a blunt-body entry into a shallow water is considered. The body is rigid, undeformable and touches at the initial instant of time $t = 0$ the undisturbed horizontal free surface of the liquid at a single point taken as the origin of the Cartesian coordinate system xOy. The liquid is ideal, incompressible and occupies at the initial moment the strip $-h < y < 0$. Initially both the body and the liquid are at rest. Then the body starts to penetrate the liquid layer, the initial impact velocity being V_0. We shall determine the body motion and the characteristics of the spray jets initiated under the impact.

In general, we have not any specific practical problems in mind, but we can think about several cases. An example is when a huge solid mass falls into a lake from a surrounded mountain. We are then also interested the waves generated by the impact. The present analysis will then provide the initial conditions for a study of the generated waves.

The main assumptions of the paper are:

1. the rigid contour is very shallow in comparison with the thickness of the liquid layer;
2. the liquid is ideal, incompressible, and surface tension is absent.

In this case the flow region can be divided onto the following four parts (see figure 1):

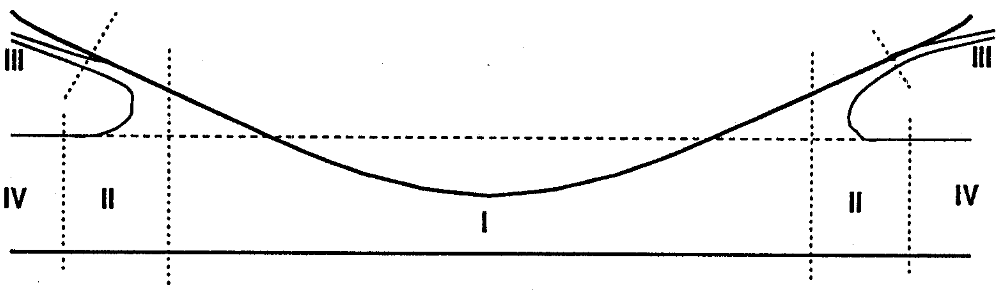

Figure 1: Scheme of the flow

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I. the region beneath the entering body;
II. the jet root;
III. the spray jet;
IV. the outer region.

This problem has some peculiarities which distinguished that from other water-impact problems. For example, the presence of the rigid bottom, $y = -h$, is of major importance and cannot be neglected.

In region I the horizontal characteristic linear size of the flow variation is essentially greater than the vertical one. This makes it possible to consider the flow to the leading order as y-independent one. It is worth to note that the dimension of the region, $2c(t)$, is unknown in advance and should be found together with the flow characteristics. However, it can be proved that the velocity of this contact region expansion is essentially greater than the entry one.

We have to indicate also the critical velocity for the liquid layer, it is equal to $\sqrt{gh}$, where $g = 9.81 m/s^2$. The stage of the process when $dc/dt >> \sqrt{gh}$ is considered only. At this stage the liquid outside of the contact region, i.e. in region IV, is at rest. When the velocity of the contact region expansion approaches the critical one a soliton is formed. Further, the soliton escapes and propagates at the velocity near the critical value.

The scheme of the flow in region II is shown in figure 2. The dimension of the region is of order h. In the moving coordinate system which translates rights at the velocity dc/dt, the flow can be considered as quasi-stationary one, and the entering body velocity can be neglected. If we are not interested in a detailed analysis of the flow near the contact point, we can use the integral laws of mass, momentum and energy conservation to find relations between the flow characteristics away this region.

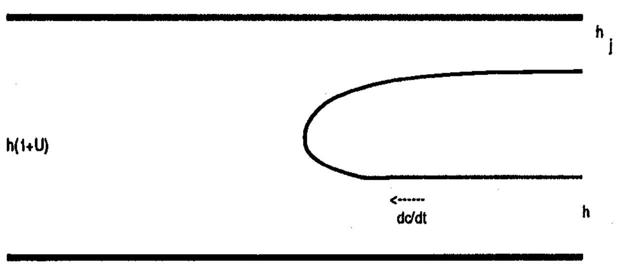

Figure 2: Flow geometry near the contact point

Matching the solutions in regions I, II, IV, we obtain one ordinary differential equation for the height hU of the liquid pilled up at the contact point with respect to the penetration depth hs. All other quantities are given in quadratures when the dependence $U(s)$ has been determined.

The flow inside the jet (in region III) should be analysed separately, because the pressure here is near atmospheric value and, hence, liquid particles in the jet move inertially.

For the coupled problem, when we should find not only the liquid flow but the body motion also, the position of the entering contour is assumed to be as follows:

$$y = \frac{1}{n}Rk(\frac{|x|}{R})^n - hs(t),$$

where $R = 1, k << 1$ for $n = 1$; $k = 1, h/R << 1$ for $n > 1$. In this case the initial problem for $U(s)$ is

$$\frac{dU}{ds} = \frac{n}{2}(1 + \frac{s}{U})(1 + [1 + U]^{-\frac{1}{2}}) - 1 \, (0 < s < 1)$$
$$U = 0 \, (s = 0)$$

It should be noted that $U(s)$ is dependent of the only parameter n. Then the dependence of the contact region dimension $2c$ on the penetration depth hs is given as

$$2c(s) = 2R(\frac{nh}{kR})^{\frac{1}{n}}[s + U(s)]^{\frac{1}{n}},$$

and the jet thickness h_j can be written as

$$h_j = h[\sqrt{1 + U(s)} - 1]^2.$$

Expressions for the penetration velocity ds/dt and the time t as function of s are not so simple. The behaviors of these functions can be seen in figures 3 and 4 for the wedge $y = 0.1 \mid x \mid$ of mass $7000kg/m$ entering the liquid layer of the depth $0.5m$ at the initial velocity $V_0 = 6m/s$. This case was sorted out not for simplicity of calculation but to demonstrate the typical evolution of the process.

It is worth to note that the body hits the bottom at a non-zero velocity only if $n < 3$. The calculations have shown that the body velocity is reduced very quickly due to the hydrodynamic forces.

The axisymmetrical problem can be analysed in the same way. In the three-dimensional problem some difficulties are connected with the solution inside region I only. Account for the body elasticity does not expected to meet any obstacles which are common for the classical water-entry problem due to the fact that the pressure distribution over the wetted part of the contour is finite in this approach.

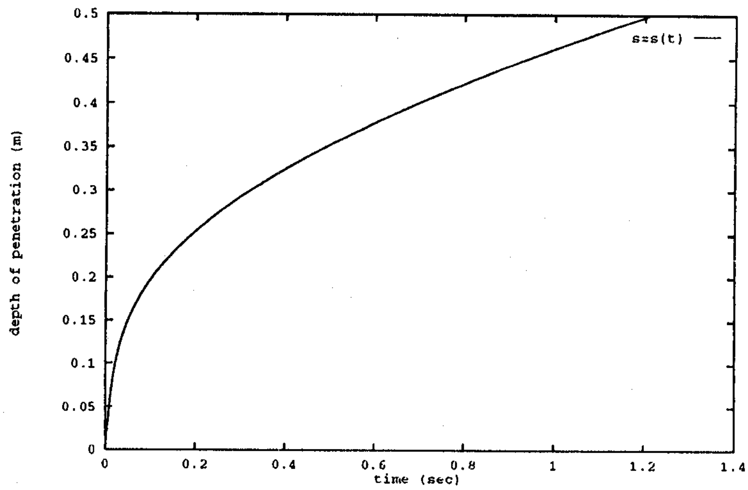

Figure 3: Depth of penetration

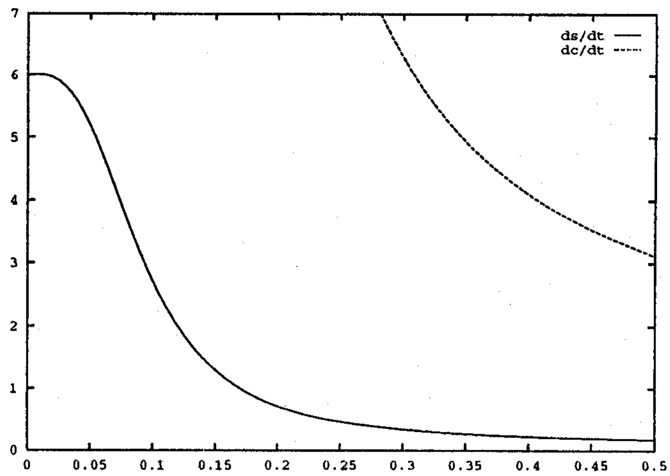

Figure 4: Penetration velocity

DISCUSSION

Tuck E.O.: Although, as the author explains, it is not necessary to solve the spray-root problem of Figure 2, it is possible to write down a simple closed-form solution if one desires it. This solution was presented in a paper by Dixon and me, in J. F. M. (Vol.205, 1989) on the "Surf-Shimmer", namely a shallow-water planing surface.

Korobkin A.: Thank you very much for this reference.

Baba E.: You are commented for your detail analysis of complicated flow around impacted bodies one of which is covered by thin-layer of a liquid. As a result of this analysis we would expect your further analysis to estimate water elevation at far distance from the point of impact.

Korobkin A.: We are going to use the KdV equation to describe the motion of the wave, generated under the entry of a body into a shallow water. The present analysis provides the initial data for the KdV equation: mass of the piled-up water, its form, the 'initial' velocity distribution.