A Combined Boundary-Integral Equation Method for Determining the Unsteady Flow around a Ship in Waves

by Xin LIN, Mikio TAKAKI and Hidetsugu IWASHITA
Hiroshima University, JAPAN

1 Introduction

Unsteady ship motions can be predicted by the well known strip theory and 3-D panel method with the classical free surface condition, but people are satisfied with this solution. Many new methods are developed to solve the boundary-value problem with more accurate free surface condition, such as Rankine source method, modified 3-D panel method and some hybrid methods. In this paper, the boundary-value problem with the modified free surface condition is solved by a combined boundary-integral equation method (CBIEM). The method is one of the hybrid methods.

2 Formulations and Numerical Method

Let us consider a ship advancing at constant forward speed U in an incompressible, inviscid and irrotational fluid domain. The Cartesian coordinates system used in this paper is illustrated in Fig. 1. The x-axis points to the direction of the ship forward velocity, oxy coordinate plane is identical with the undisturbed water plane and z-axis is positive upward. S_H denotes the wetted hull surface and S_F denotes the free surface. The normal vector $\mathbf{n}$ is taken to point inward the fluid domain.

Assuming that the problem can be linearized, we express the total velocity potential of the flow around the ship by the superposition of three components, $\Phi(\mathbf{x},t) = \Phi^D(\mathbf{x}) + \psi(\mathbf{x}) + \phi(\mathbf{x},t)$, where Φ^D is the potential of the double-model flow around the ship, which satisfies the rigid wall condition on the undisturbed water surface, $\psi(\mathbf{x})$ represents the disturbance potential of the steady Kelvin wave and $\phi(\mathbf{x},t)$ represents the disturbence potential of the unsteady wave. The linearized free surface condition for the unsteady disturbance potential ϕ can be expressed as follows

$$\begin{aligned} &\phi_{tt} + 2\nabla\Phi^D \cdot \nabla\phi_t + g\phi_z + \nabla\Phi^D \cdot \nabla\left(\nabla\Phi^D \cdot \nabla\phi\right) + \\ &\frac{1}{2}\nabla\left(\nabla\Phi^D \cdot \nabla\Phi^D\right) \cdot \nabla\phi - \Phi^D_{zz}\left(\phi_t + \nabla\Phi^D \cdot \nabla\phi\right) = 0\,, \\ &\qquad\qquad\qquad \text{on } z = 0\,. \end{aligned} \tag{1}$$

with the error order of $O(\psi\phi,\ \phi^2)$.

In the far field, the potential of the double-model flow, Φ^D, reduces to that of the uniform flow, $-Ux$, and the modified free surface condition (1) reduces to the classical free surface condition

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)^2 \phi + g\frac{\partial \phi}{\partial z} = 0, \qquad \text{on } z = 0\,. \tag{2}$$

Therefore, the classical free surface condition (2) is the asymptotic expression of the modified free surface condition (1). On the basis of this point, CBIEM is proposed. In CBIEM, the fluid domain is decomposed into the near field Ω and the far field Ω^*, as shown in Fig. 1. These two fields are separated by an artificial control surface S_C. The modified free surface condition (1) is used in the near field and the simplified one, the classical free surface condition (2), is used in the far field. The hull surface condition for the near field potential can also be linearized consistently based on the double-model flow. The solutions of the near and far fields are matched on the control surface. The matching condition is naturally the continuity of the potential and its gradients.

By applying 3-D panel method, the far field solution can be expressed by

$$\frac{1}{2}\phi^*(\mathbf{x}) - \iint_{S_C} \phi^*(\mathbf{x}') \frac{\partial G^*(\mathbf{x};\mathbf{x}')}{\partial n^*} ds = -\iint_{S_C} \frac{\partial \phi^*(\mathbf{x}')}{\partial n^*} G^*(\mathbf{x};\mathbf{x}')\, ds, \tag{3}$$

where the line-integral has been neglected, the superscript '$*$' means the terms in the far field, and G^* is the potential of three-dimensional unsteady wave source, which satisfies the classical free surface condition (2) and the radiation condition. The use of G^* enforces the free surface condition and the radiation condition on the far field solution analytically. On the other hand, by applying the Green's second identity, the near field solution can be expressed by

$$\frac{1}{2}\phi(\mathbf{x}) - \iint_{S_H \cup S_F \cup S_C} \phi(\mathbf{x}') \frac{\partial G(\mathbf{x};\mathbf{x}')}{\partial n} ds + \iint_{S_F} G(\mathbf{x};\mathbf{x}') \frac{\partial \phi(\mathbf{x}')}{\partial n} ds$$
$$+ \iint_{S_C} G(\mathbf{x};\mathbf{x}') \frac{\phi(\mathbf{x}')}{\partial n} ds = -\iint_{S_H} \frac{\partial \phi(\mathbf{x}')}{\partial n} G(\mathbf{x};\mathbf{x}') ds, \tag{4}$$

in which $G(\mathbf{x},\mathbf{x}') = 1/4\pi|\mathbf{x}-\mathbf{x}'|$. The normal derivative term on S_H is known and that on S_C can be expressed in terms of the potential on S_C according to the matching condition and the boundary-integral equation (3). By virtue of the modified free surface condition (1), the normal derivative term on S_F can also be expressed in terms of the potential itself, but some numerical schemes must be designed to treat the first and second tangent derivatives. To avoid the trouble of instability, we design the numerical scheme by using the upstream finite difference approximation. The resultant solution scheme is unconditionally stable and of quadratic-order. The boundary-integral equation (4) can be finally discretized into a linear system of equations for determining the unknown potential ϕ.

In geometrical discretization, the grid on free surface needs a more careful consideration, because of the usage of upstream finite difference formula. In this paper, a streamline type grid is used on the free surface. This grid is orthogonal and water-line fitted.

3 Numerical Results and Conclusions

The calculation has been carried out over a wide speed range for two hull forms. One is an ellipsoid with the length-to-beam ratio of $L/B = 4.0$ and the beam-to-draft ratio of $B/d = 2.5$, the other is a spheroid with $L/B = 5.0$. As usual, the predictions of the first order hydrodynamics by CBIEM are generally more accurate than those obtained by strip theory and 3-D panel method. The results of wave patterns and added wave-resistance are presented here. The effect of different free surface linearization on the wave pattern is shown in Fig. 2. It illustrates the real component of the diffraction wave around the ellipsoid at $Fn = 0.1$. The upper part is the result with the modified free surface condition and the lower part represents that with the classical free surface condition, both of them are calculated by CBIEM. Considerable differences between the wave patterns exist even for this low speed case. Fig. 3 shows the results of heaving wave profile for the spheroid at $Fn = 0.2$. In comparison with the experimental observations, we see that CBIEM provides better prediction. The results of added wave-resistance for the spheroid at $Fn = 0.3$ are plotted in Fig. 4. The experimental data are measured by using Ohkusu's wave pattern analysis theory and the theoretical results are calculated by using Maruo's formula. The results obtained by CBIEM are better and agree with the experiments very well.

According to the comparison between the experiments and calculation we may say that considerable improvement on the theoretical predictions has been achieved by CBIEM.

We are now working on improving CBIEM by introducing the higher order curvilinear panels in the near field, and extending it to the high speed case.

Reference

[1] Iwashita, H., Lin, X., and Takaki, M., 'Combined Boundary-Integral Equation Method For Ship Motions in Waves', Trans. West-Japan Soc. Nav. Archit. No. 85, pp. 37-55, 1992.

[2] Iwashita, H., and Ohkusu, M., 'Hydrodynamic Forces on a Ship with Forward Speed in Waves', J. Soc. Nav. Archit. Japan, Vol. 166, pp. 187-205, 1989.

[3] Sclavounos, P.O., and Nakos, D.E., 'Stability Analysis of Panel Methods for Free Surface Flow with Forward Speed', 17th ONR, pp. 173-193, 1988.

[4] Nakos, D.E., and Sclavounos, P.O.,'Ship Motions by a Three-Dimensional Rankine Panel Method', Proc. 18th ONR, pp. 21-40, 1990.

[5] Kobayashi, M., 'On the Hydrodynamic Forces and Moments Acting on a Three-Dimensional Body with a Constant Forward Speed', J. Soc. Nav. Archit. Jap., Vol. 150, pp. 175-189, 1981

[6] Takagi, K., 'An Application of Rankine Source Method for Unsteady Free Surface Flows', J. Kansai Soc. Nav. Archit., Japan, Vol.213, pp. 21-29, 1990.

[7] Yasukawa, H., 'A Rankine Panel Method to Calculate Unsteady Ship Hydrodynamic Forces', J. Soc. Nav. Archit. Jap., Vol. 168, pp. 131-140, 1990.

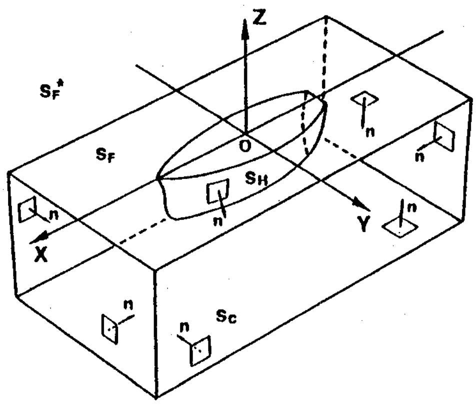

Fig. 1 Coordinate system and decomposition of the fluid domain

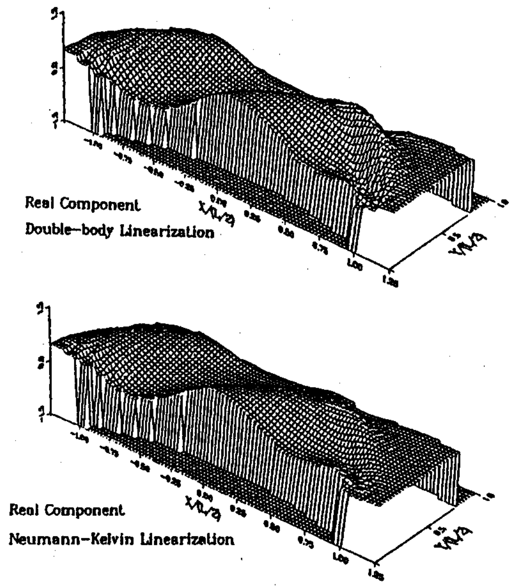

Fig. 2 The real component of the diffraction wave around the ellipsoid body of $L/B=4.0$, $B/D=2.5$ at $Fn=0.1$ in the head wave of $\lambda/L=0.5$, obtained by the CBIEM method using the Double-body(upper) and Neumann-Kelvin(lower) linearizations

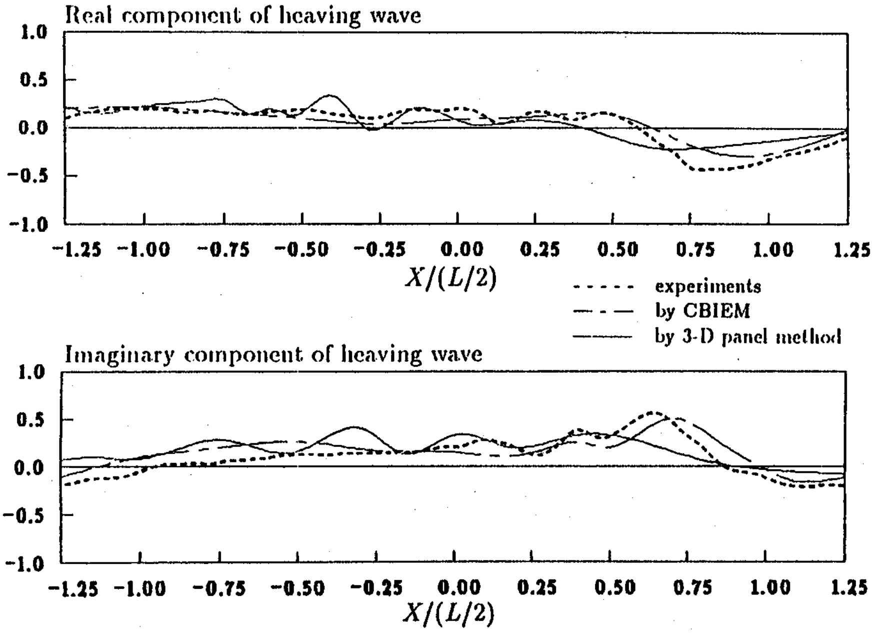

Fig. 3 Heaving wave on $Y/L = 0.2$ generated by the spheroid at $Fn = 0.2$ and oscillating at $KL = 2.0$

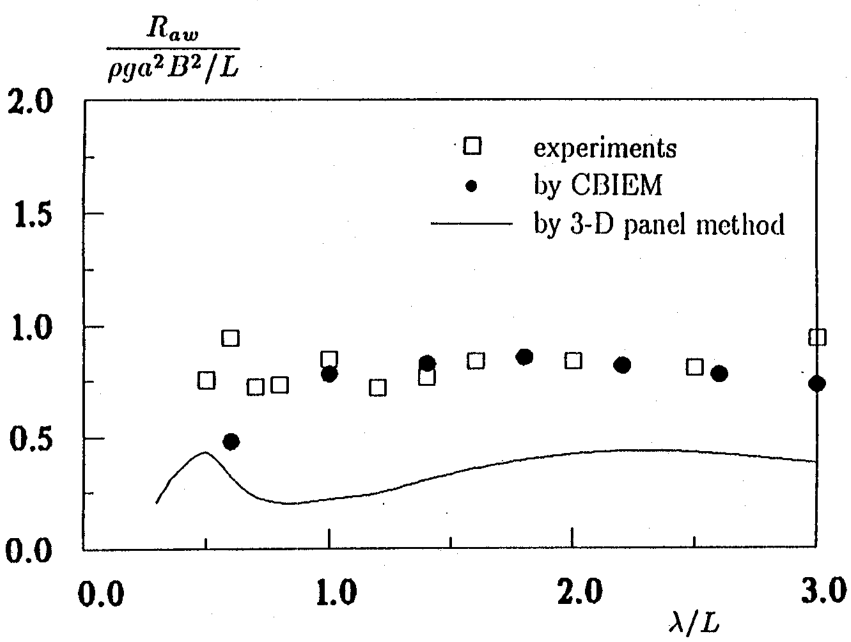

Fig. 4 Added wave-resistance of the spheroid at $Fn = 0.3$ through regular head waves

DISCUSSION

Choi H.S.: Could you comment on the influence of the aspect ratio of the panel on waves?

Lin X.: In our present study, the streamline type grid system is used. If we denote $\vec{l}$ and $\vec{s}$ the tangent and normal vector of the streamline of the double-body flow on the free surface, the aspect ratio used in this calculation is $\alpha = h_e/h_s = 1.0$ If the value of α is too large or too small, the accuracy of the results will decrease. Usually, α should be kept around 1.0

Mori K.: You use the second order approximation for the first derivative appearing in the free surface condition. My suggestion is to use higher approximation such as third-order or forth-order approximations which are commonly used in the steady flow problems.

Lin X.: Thank you very much for your suggestions. I agree that the higher-order scheme should be used. In the present calculation, because it is impossible to increase the mesh number on the free surface, we are fail to use the third-order scheme. I guess if we increase the mesh number then it will be no problem. Anyway, we will use third or even higher-order sheme in future researches.

Yeung R.W.: A very general formulation of this kind was given and tested in a paper of mine in the mid 80s (IUTAM symp. on Wave Energy Utilization, Lisbon, Portugal, 1985). This was carried out for the case of zero forward speed. In your development, a formal analysis of the exterior region will yield a waterline integral at the matching surface, which in my opinion, cannot be discarded. I would be interested in knowing the justification. Further, if included, how you might assure continuity of the solution across the waterline contour.

Lin X.: Thank you very much for your comments. Yes we shall involve the line integral terms in the expressions of outer solutions, and the involvement of the term can ensure the continuity of the solutions across the waterline contour of the control surface if the control surface is located far enough from the body.

Kagemoto H.: 1) How far should the control surface be from a body?
2) Since the free surface conditions used in the inner domain and in the outer domain are different, I suspect that the free surface elevation may not be continuous at the control point.

Lin X.: Thank you professor, the locations of the control surface is determined mainly by the decreasing rate of the coefficients of the modified free surface conditions, or in another word by ϕ^D, $\phi^D_{x_i x_j}$ and $\phi^D_{x_i}$. Fortunately, they are decreased very rapidly, so the control surface need not be located very far from the body.
For the secod quesion, I would like to agree with professor's opinions. While if the control surface is located far enough from the body so that the difference between the "modified" and "classical" free surface be very small, the difference between the surface elevations beside the control surface will be quite small.

Kashiwagi M.: 1) Could you summarize how you took account of the steady-disturbance effects in evaluating the Kochin function which must be used in computing the added resistance from Maruo's formula.

2) I suspect that the added resistance by Maruo's formula is different from that computed by the direct pressure integration over the wetted body surface, that is, the momentum conservation principle is not satisfied numerically. Could you comment on that?

Lin X.: Thank you very much for the discussions. 1) In the calculations of *Raw* in CBIEM, the integral is carried out over the control surface. It is equivalent to the integral over the ship hull surface plus the free surface of the inner domain, from this respect the steady-disturbance is taken into account mainly or partly.

2) Yes, I agree with professor's opinion. But in the present case, with the usage of constant panel, we can not get resonable results by the direct mothed.