The diffraction of waves by an array of vertical circular cylinders in a channel

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Introduction

Experiments to determine the hydrodynamic forces on offshore structures are usually performed in wave tanks and the results used as approximations to the open-sea values. It is clearly important, therefore, to have a good understanding of how the tank walls affect the results of such experiments.

A particular, and not unrealistic, geometry that has received considerable attention in recent years is that of a vertical circular cylinder, and linear radiation and diffraction problems relating to such an obstacle are now well understood, see for example, [5],[6].

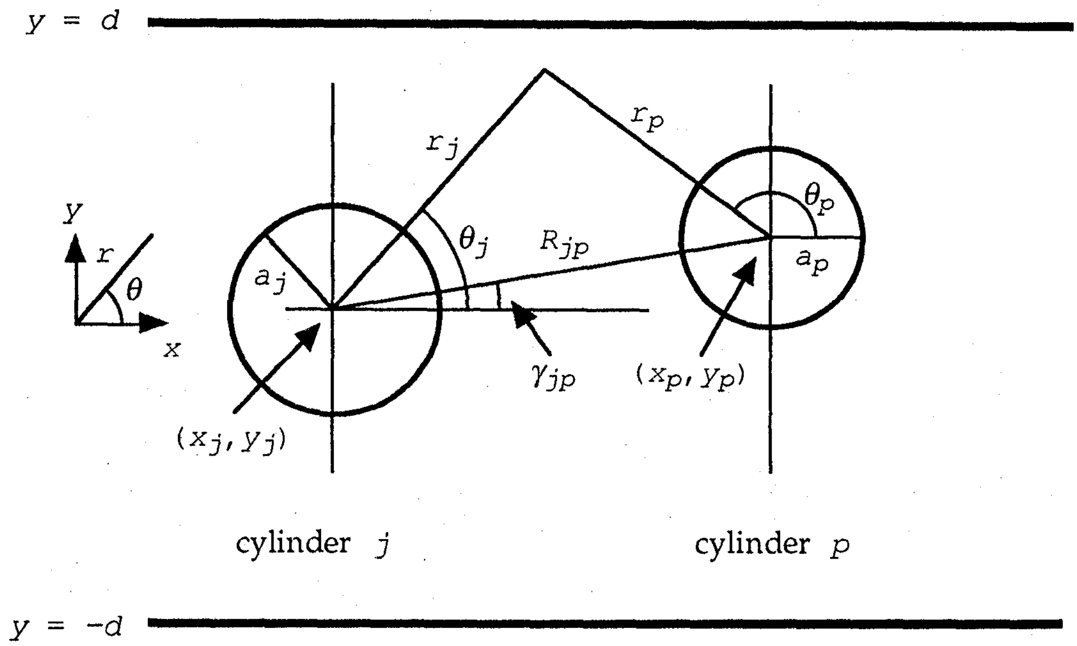
Many offshore structures however consist of not one, but a number of vertical cylindrical components and it is the purpose of this paper to investigate the hydrodynamic characteristics of such an array when it is considered fixed in the confines of a channel and in the presence of an incident plane wave.

Five papers have been published recently in which problems connected with the interaction of waves and arrays of vertical circular cylinders in channels are solved numerically, [1],[2],[3],[7],[8]. In all these papers a Green's function appropriate to channel problems is constructed and then standard integral equation techniques used.

For general-shaped obstacles this technique is extremely useful but its implementation is quite difficult due to the numerical problems that arise. It is well known that the evaluation of the Green's function is hindered by the need to evaluate slowly convergent series of Hankel functions and different authors have used different methods to deal with this problem.

However, for the case of circular cylinders another technique is available, that of multipole expansions, and this approach will be pursued here. The solution of the problem of scattering by an array of cylinders is found by combining the method of [4] where the open-sea diffraction problem was solved, and techniques described in [5],[6] where the problem of scattering by a single cylinder in a channel was solved. The resulting procedure is both fast and accurate and can be used to validate more general numerical codes as well as to produce extensive results for hydrodynamic characteristics in this interesting problem.

Formulation



PLAN VIEW

Cartesian coordinates (x, y, z) are chosen with z vertically upwards, $z = -h$ being the bottom of the channel and $z = 0$ the mean free surface. The cylinders extend throughout the water depth and so, since we are solving a diffraction problem, the dependence on z can be factored out of the problem. The wavenumber k is the real positive solution of the dispersion relation

$$k \tanh kh = \omega^2/g.$$

We assume that there are N (≥ 1) fixed vertical circular cylinders and use $N+1$ polar coordinate systems in the (x, y)-plane: (r, θ) are centred at the origin, whilst (r_j, θ_j), $j = 1, \ldots, N$, are centred at (x_j, y_j), the centre of the jth cylinder. The various parameters relating to the relative positions and sizes of the cylinders are shown in the diagram above.

The velocity potential Φ is written

$$\Phi = Re\{\phi(x,y)f(z)e^{-i\omega t}\}, \quad f(z) = -\frac{igA\cosh k(z+h)}{\omega \cosh kh}$$

and the two-dimensional potential ϕ then satisfies the Helmholtz equation

$$(\nabla^2 + k^2)\phi = 0. \tag{1}$$

An incident wave of the form $\phi_I(x,y) = e^{ikx}$ is assumed and ϕ is written as

$$\phi = \phi_I + \sum_{j=1}^{N}\sum_{n=0}^{\infty} Z_n^j(A_n^j\phi_n^j + B_n^j\psi_n^j), \quad Z_n^j = J_n'(ka_j)/H_n'(ka_j), \quad B_0^j = 0,$$

where A_n^j, B_n^j are unknown complex constants and ϕ_n^j, ψ_n^j are multipoles (singular solutions of (1) which have zero normal derivative on the channel walls and satisfy the appropriate radiation condition). For notational convenience H_n is used to denote the Hankel function $H_n^{(1)}$. Integral representations for these multipoles can be found in [6].

The development of various properties of these multipoles allows the body boundary condition ($\partial\phi/\partial r_j = 0$ on $r_j = a_j$, $j = 1, \ldots, N$) to be applied, leading to an infinite system of equations for the unknowns A_n^j, B_n^j, which can be solved efficiently by truncation.

The simplicity of the resulting formulas for hydrodynamic quantities of interest is readily seen. The dynamic pressure is obtained from

$$\phi(a_j, \theta_j) = -\frac{2\mathrm{i}}{\pi k a_j} \sum_{m=0}^{\infty} \frac{A_m^j \cos m\theta_j + B_m^j \sin m\theta_j}{H_m'(ka_j)},$$

the first-order forces from

$$\left\{ \begin{array}{c} |X^j| \\ |Y^j| \end{array} \right\} = \frac{1}{2}|F^j| \left\{ \begin{array}{c} |A_1^j| \\ |B_1^j| \end{array} \right\}, \qquad F^j = \frac{4\rho g A \tanh kh}{k^2 H_1'(ka_j)},$$

and the mean drift forces from

$$f^j \equiv f_x^j + \mathrm{i} f_y^j = \frac{2\alpha_j}{\pi(ka_j)^2} \left[-\frac{D_0^j \overline{C_1^j}}{H_0' \overline{H_1'}} - \frac{D_1^j \overline{C_0^j}}{H_1' \overline{H_0'}} + \sum_{n=1}^{\infty} \left(\frac{n(n+1)}{(ka_j)^2} - 1 \right) \left\{ \frac{C_n^j \overline{C_{n+1}^j}}{H_n' \overline{H_{n+1}'}} + \frac{\overline{D_n^j} D_{n+1}^j}{\overline{H_n'} H_{n+1}'} \right\} \right]$$

where $\alpha_j = \frac{1}{8}\rho g A^2 a_j (1 + 2kh/\sinh 2kh)$, $C_m^j = A_m^j - \mathrm{i}B_m^j$, $D_m^j = A_m^j + \mathrm{i}B_m^j$ and all the Hankel functions have argument ka_j. An overbar denotes complex conjugate.

The fundamental reflection and transmission coefficients are given by

$$R_0 = \frac{1}{kd} \sum_{j=1}^{N} \sum_{n=0}^{\infty} (-1)^n (Z_{2n}^j A_{2n}^j + \mathrm{i} Z_{2n+1}^j A_{2n+1}^j) \mathrm{e}^{\mathrm{i}kx_j}$$

$$T_0 = 1 + \frac{1}{kd} \sum_{j=1}^{N} \sum_{n=0}^{\infty} (-1)^n (Z_{2n}^j A_{2n}^j - \mathrm{i} Z_{2n+1}^j A_{2n+1}^j) \mathrm{e}^{-\mathrm{i}kx_j}.$$

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